

# CKM Quark Flavor Mixing

Implications of the Most Recent Results  
on CP Violation and Rare Decay Searches  
in the  $B$  and  $K$  Meson Systems

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FPCP – Flavor Physics & CP Violation  
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Reference for recent plots: <http://www.slac.stanford.edu/~laplace/ckmfitter.html>

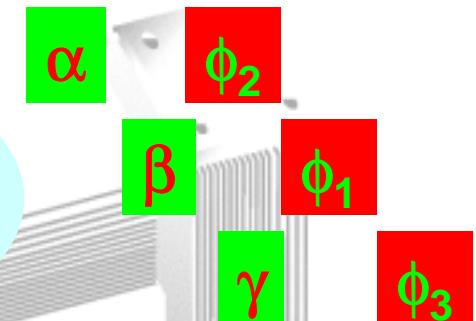
# Determining the CP-Violating CKM Phase

**CP Violation (CPV) in  $B$  and  $K$  Systems:**

CPV in interference of decays with and without mixing

CPV in mixing

CPV in interference between decay amplitudes



Cherenkov  
Detector

Tracking Chamber

Support Tube

Precise Determination of the  
Matrix Elements  $|V_{ub}|$  and  $|V_{cb}|$

Neutral  $B_d$  and  $B_s$  Mixing

**Detection of Rare Decays:**  
Search for new physics and direct CPV  
Determination of weak phases

# The CKM Matrix

Mass eigenstates  $\neq$  Flavor eigenstates  $\rightarrow$  Quark mixing

*B* and *K* mesons decay weakly

→ modified couplings for charged weak currents:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{CKM}$  unitary and complex  
4 real parameters  
(3 angles and 1 phase)

Kobayashi, Maskawa 1973

Wolfenstein Parameterization (expansion in  $\lambda \sim 0.2$ ):

$$V_{CKM} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CPV phase

“Explicit” CPV in SM, if:

$$J = \text{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*) \neq 0$$

(phase invariant!)

Jarlskog 1985

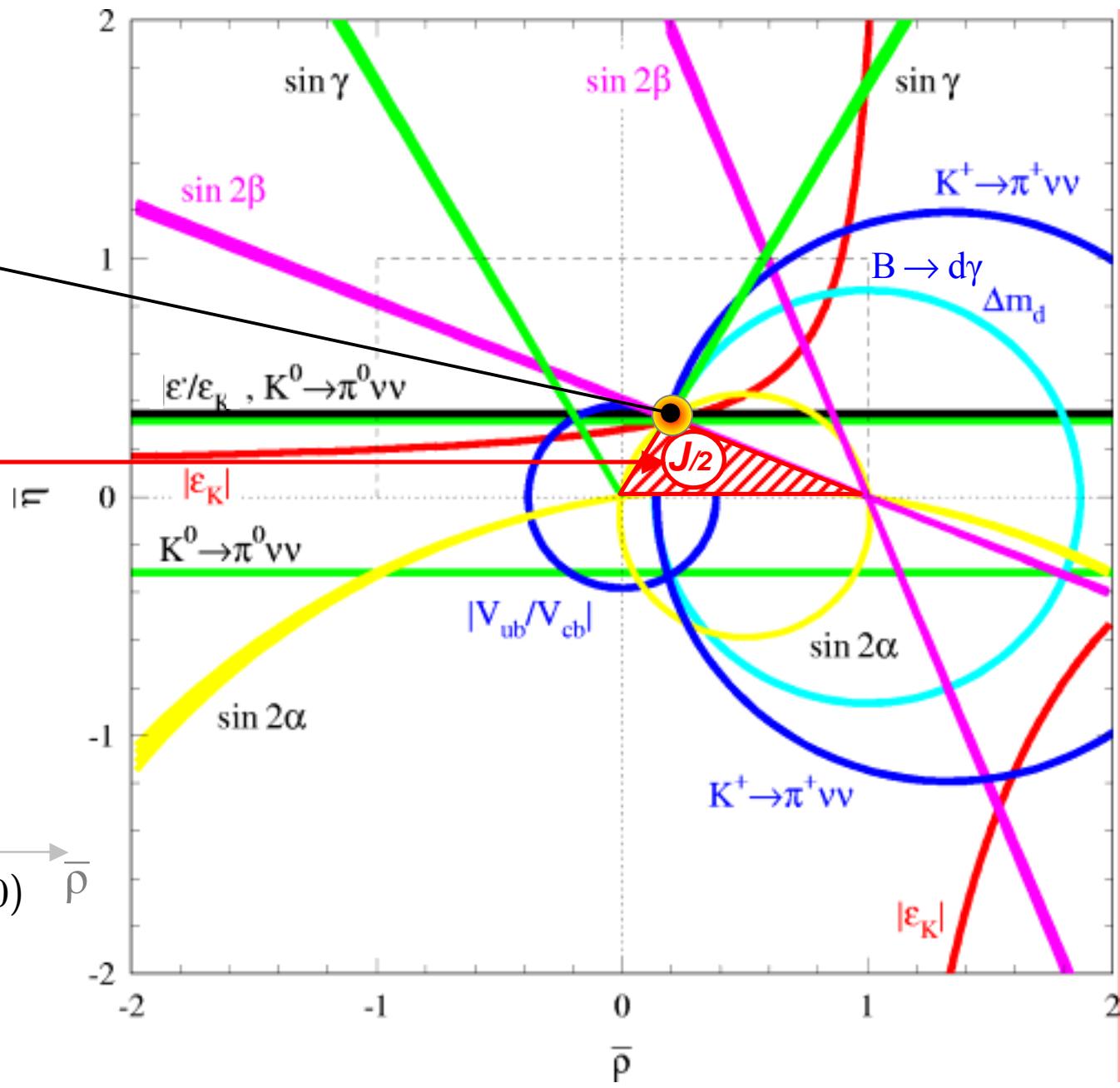
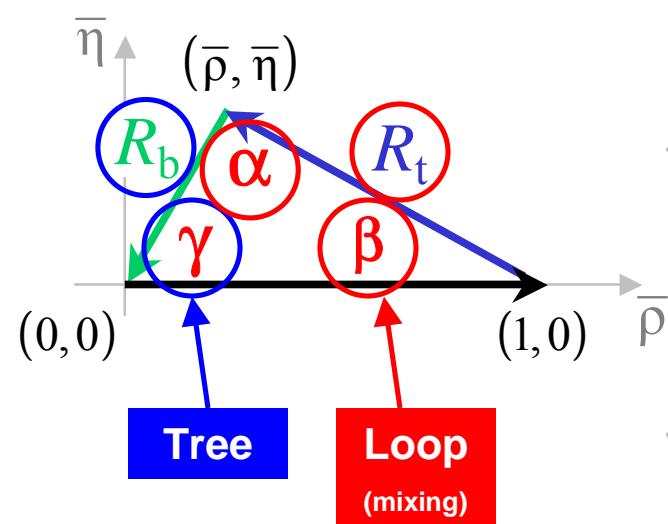
$$J \approx A^2\lambda^6\eta \quad \Rightarrow \quad \eta = 0 \Rightarrow \text{no CPV in SM}$$

# Many Ways Lead to the Unitarity Triangle

Point of Knowledge:

SM or new  
phases (fields)?

What is the value of  
 $J$  ←  
in our world?



# The CKM Matrix: Impact of non- $B$ Physics



Observables	CKM Parameters <sup>(*)</sup>	Experimental Sources	Theoretical Uncertainties	Quality
$ V_{ud} $ $ V_{us} $	$\lambda$	nuclear $\beta$ decay $K^{+(0)} \rightarrow \pi^{+(0)}$ ev	small	* * *
$\varepsilon_K$	$\eta \propto (1-\rho)^{-1}$	$K^0 \rightarrow \pi^+\pi^-$ , $\pi^0\pi^0$	$B_K$ , $\eta_{cc}$	*
$\varepsilon'/\varepsilon_K$	$\eta$	$K^0 \rightarrow \pi^+\pi^-$ , $\pi^0\pi^0$	$B_{6(\text{QCD-peng})}$ , $B_{8(\text{EW-peng})}$	?
$\text{Im}^2[V_{ts}^* V_{td} \dots]$	$\propto (\lambda^2 A)^4 \eta^2$	$K_L^0 \rightarrow \pi^0 v\bar{v}$	small (but: $(\lambda^2 A)^4$ )	* * (*)
$ V_{td} $	$(1-\rho)^2 + \eta^2$	$K^+ \rightarrow \pi^+ v\bar{v}$	charm loop (and: $(\lambda^2 A)^4$ )	* (*)

(\*) Observables may also depend on  $\lambda$  and  $A$  - not always explicitly noted

# The CKM Matrix: Present Impact of $B$ Physics

Observables	CKM Parameters <sup>(*)</sup>	Experimental Sources	Theoretical Uncertainties	Quality
$\Delta m_d$ ( $ V_{td} $ )	$(1-\rho)^2 + \eta^2$	$B_d \bar{B}_d \rightarrow f^+ f^- + X, X_{\text{RECO}}$	$f_{B_d} \sqrt{B_d}$	*
$\Delta m_s$ ( $ V_{ts} $ )	$A$	$B_s \rightarrow f^+ + X$	$\xi = f_{B_s} \sqrt{B_s} / f_{B_d} \sqrt{B_d}$	* *
$\sin 2\beta$	$\rho, \eta$	$B_d \rightarrow c\bar{c} s\bar{d}$	small	* * *
$\sin 2\alpha$	$\rho, \eta$	$B_d \rightarrow \pi^+(\rho^+) \pi^-$	Strong phases, penguins	?
$\gamma$	$\rho, \eta$	$B^+ \rightarrow D^0 K^+$	small	* *
		$b \rightarrow u$ , Direct CPV	Strong phases, penguins	?
$ V_{cb} $	$A$	$b \rightarrow cl\nu$ (excl. / incl.)	$F_{D^*}(1) / \text{OPE}$	* *
$ V_{ub} $	$\rho^2 + \eta^2$	$b \rightarrow ul\nu$ (excl. / incl.)	Model / OPE	*
$ V_{td} $	$(1-\rho)^2 + \eta^2$	$B_d \rightarrow \rho \gamma$	Model (QCD FA)	?
$ V_{ts} $	$A$ (NP)	$B_d \rightarrow X_s (K^{(*)}) \gamma,$ $K^{(*)} l^+ l^- (\text{FCNC})$	Model	?
$ V_{ub} , f_{B_d}$	$\rho^2 + \eta^2$	$B^+ \rightarrow \tau^+ \nu$	$f_{B_d}$	* *

# Extracting the CKM Parameters

Measurement



Constraints on  
theoretical parameters

$x_{\text{exp}}$

Theoretical predictions

$y_{\text{theo}} = (A, \lambda, \rho, \eta, m_t, \dots)$

$$x_{\text{theo}}(y_{\text{model}} = y_{\text{theo}}, y_{\text{QCD}})_{y_{\text{QCD}} = (B_K, f_B, B_{Bd}, \dots)}$$

$$\chi^2 = -2 \ln L(y_{\text{model}})$$

$$L(y_{\text{model}}) = L_{\text{exp}} [x_{\text{exp}} - x_{\text{theo}}(y_{\text{model}})] \times L_{\text{theo}}(y_{\text{QCD}})$$

Assumed  
to be  
Gaussian



« Guesstimates »

Frequentist:  $R_{\text{fit}}$

Uniform likelihoods:  
*Ranges*

Bayesian

Probabilities

# Three Step CKM Analysis

## Probing the SM

Test: “Goodness-of-fit”

- Evaluate global minimum

$$\chi^2_{\min; y_{\text{mod}}} (y_{\text{mod-opt}})$$

- Fake perfect agreement:

$$x_{\text{exp-opt}} = x_{\text{theo}}(y_{\text{mod-opt}})$$

generate  $x_{\text{exp}}$  using  $\mathcal{L}_{\text{exp}}$

- Perform many toy fits:

$$\chi^2_{\min-\text{toy}} (y_{\text{mod-opt}}) \rightarrow F(\chi^2_{\min-\text{toy}})$$



$$CL(\text{SM}) \leq \int_{\chi^2 \geq \chi^2_{\min; y_{\text{mod}}}}^{\infty} F(\chi^2) d\chi^2$$

## Metrology

- Define:

$$\begin{aligned} y_{\text{mod}} &= \{a; \mu\} \\ &= \{\rho, \eta, A, \lambda, y_{\text{QCD}}, \dots\} \end{aligned}$$

- Set Confidence Levels in  $\{a\}$  space, irrespective of the  $\mu$  values

- Fit with respect to  $\{\mu\}$

$$\chi^2_{\min; \mu}(a) = \min_{\mu} \{ \chi^2(a, \mu) \}$$

$$\Delta \chi^2(a) = \chi^2_{\min; \mu}(a) - \chi^2_{\min; y_{\text{mod}}}$$



$$CL(a) = \text{Prob}(\Delta \chi^2(a), N_{\text{dof}})$$

## Test New Physics

- If  $CL(\text{SM})$  good



Obtain limits on New Physics parameters

- If  $CL(\text{SM})$  bad



Hint for New Physics ?!

# Inputs Before FPCP'02 (status: Moriond 2002)

Tree process  
→ no New Physics

$ V_{ud} $	$0.97394 \pm 0.00089$	<i>neutron &amp; nuclear <math>\beta</math> decay</i>
$ V_{us} $	$0.2200 \pm 0.0025$	<i><math>K \rightarrow \pi l\nu</math></i>
$ V_{cd} $	$0.224 \pm 0.014$	<i>dimuon production: <math>\nu N</math> (DIS)</i>
$ V_{cs} $	$0.969 \pm 0.058$	<i><math>W \rightarrow X c X</math> (OPAL)</i>
$ V_{ub} $	$(4.08 \pm 0.61 \pm 0.47) \times 10^{-3}$	<i>LEP inclusive</i>
$ V_{ub} $	$(4.08 \pm 0.56 \pm 0.40) \times 10^{-3}$	<i>CLEO inclusive &amp; moments <math>b \rightarrow s\gamma</math></i>
$ V_{ub} $	$(3.25 \pm 0.29 \pm 0.55) \times 10^{-3}$	<i>CLEO exclusive</i>
$ V_{cb} $	$(40.4 \pm 1.3 \pm 0.9) \times 10^{-3}$	$\rightarrow$ product of likelihoods for $\langle  V_{ub}  \rangle$
$\varepsilon_K$	$(2.271 \pm 0.017) \times 10^{-3}$	<i>Excl./Incl.+CLEO Moment Analysis</i>
$\Delta m_d$	$(0.496 \pm 0.007) \text{ ps}^{-1}$	<i>PDG 2000</i>
$\Delta m_s$	Amplitude Spectrum'02	<i>BABAR, Belle, CDF, LEP, SLD (2002)</i>
$\sin 2\beta$	$0.78 \pm 0.08$	<i>LEP, SLD, CDF (2002)</i>
		<i>WA, Updates Moriond'02 BABAR and Belle included</i>

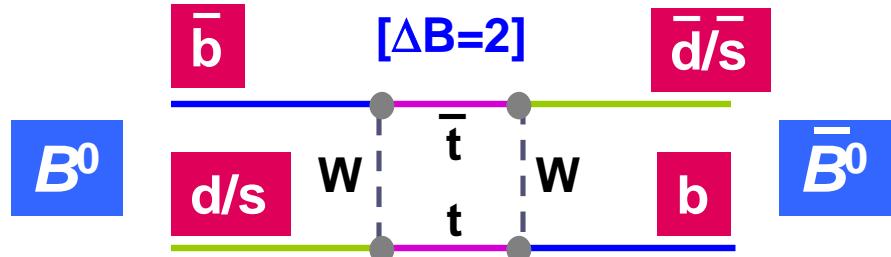
Standard CKM fit in  
hand of lattice QCD

$m_t(\overline{\text{MS}})$	$(166 \pm 5) \text{ GeV}/c^2$	<i>CDF, D0, PDG 2000</i>
$f_{B_d}\sqrt{B_d}$	$(230 \pm 28 \pm 28) \text{ MeV}$	<i>Lattice 2000</i>
$\xi$	$1.16 \pm 0.03 \pm 0.05$	<i>Lattice 2000</i>
$B_K$	$0.87 \pm 0.06 \pm 0.13$	<i>Lattice 2000</i>

+ other parameters with less relevant errors...

# $B^0 \bar{B}^0$ Mixing

Effective FCNC Processes  
(CP conserving):



whose oscillation frequencies  $\Delta m_{d/s}$  are computed by:

$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} m_W^2 \eta_B S(x_t) f_{B_q}^2 B_q |V_{tq} V_{tb}^*|^2 \approx 0.5 \text{ ps}^{-1}$$

mit:  $q = s, d$

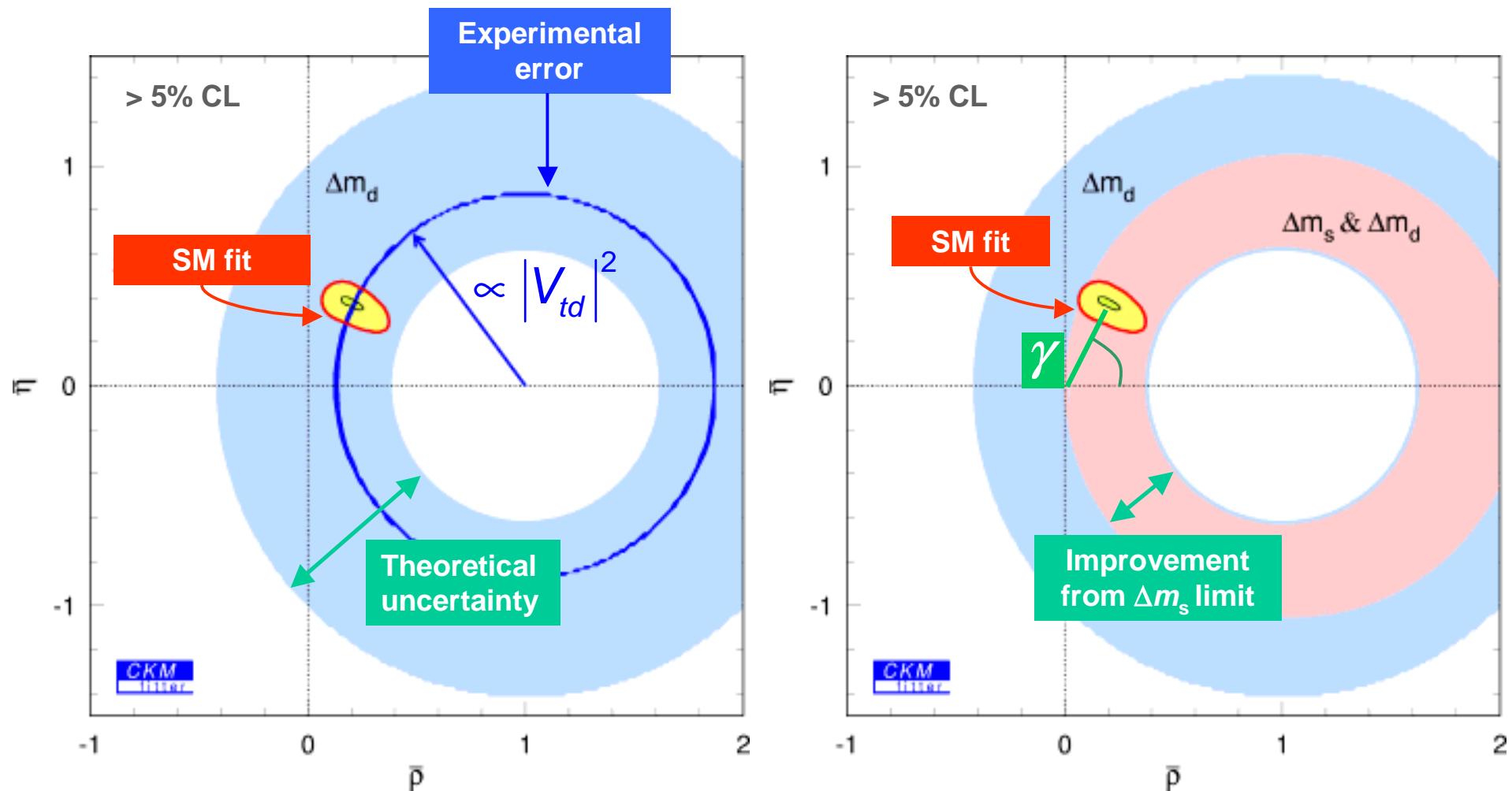
Lattice QCD (eff. 4 fermion operator)

Important theoretical uncertainties:

$$\sigma_{\text{rel}}(f_{B_{d/s}}^2 B_{d/s}) \square 36\%$$

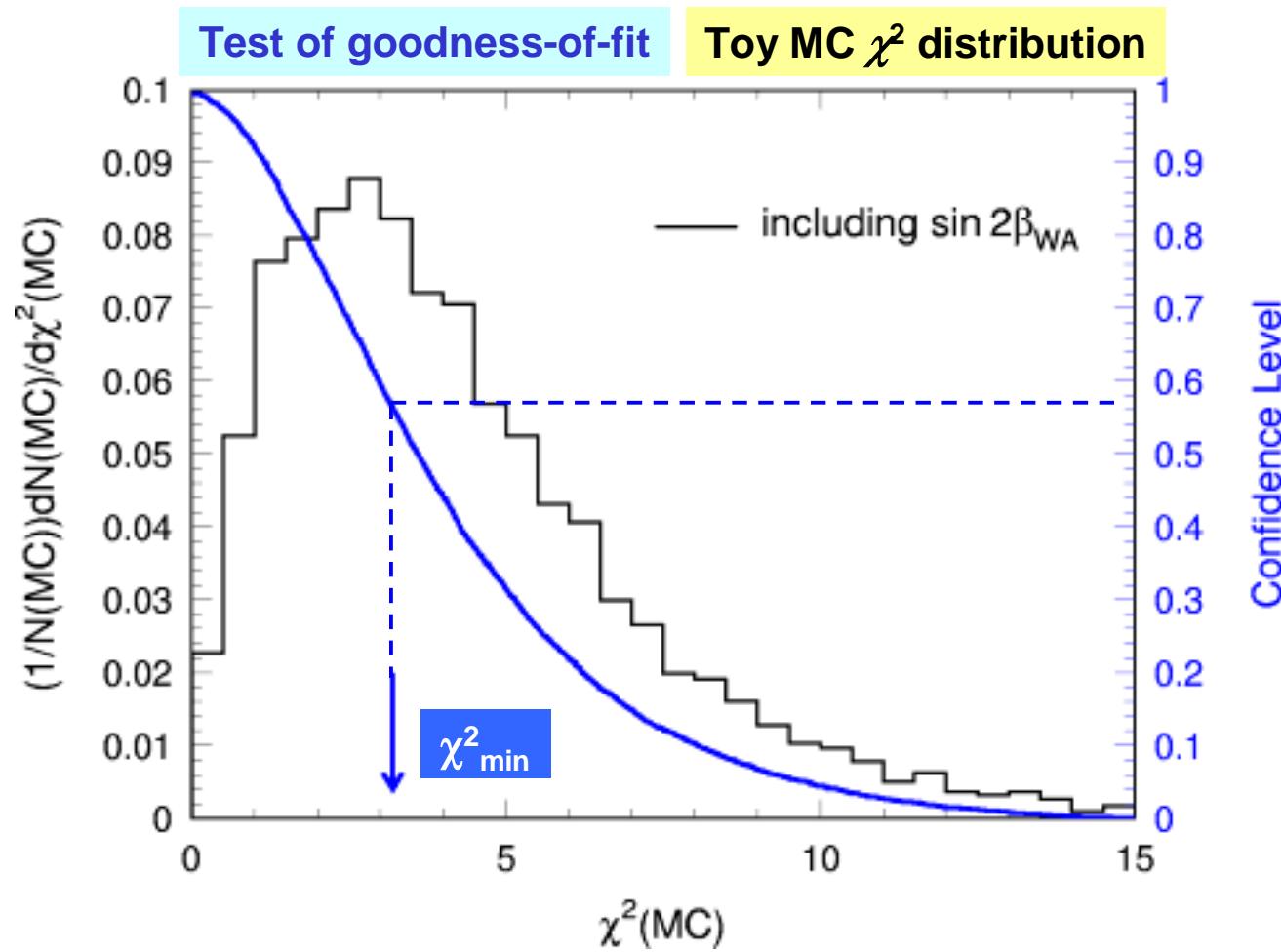
Improved error from  $\Delta m_s$  measurement:  $\sigma_{\text{rel}}(\xi^2 = f_{B_s}^2 B_s / f_{B_d}^2 B_d) \square 10\%$

# Using $\Delta m_s$



Waiting for a  $\Delta m_s$  measurement at Tevatron...

# Probing the Standard Model

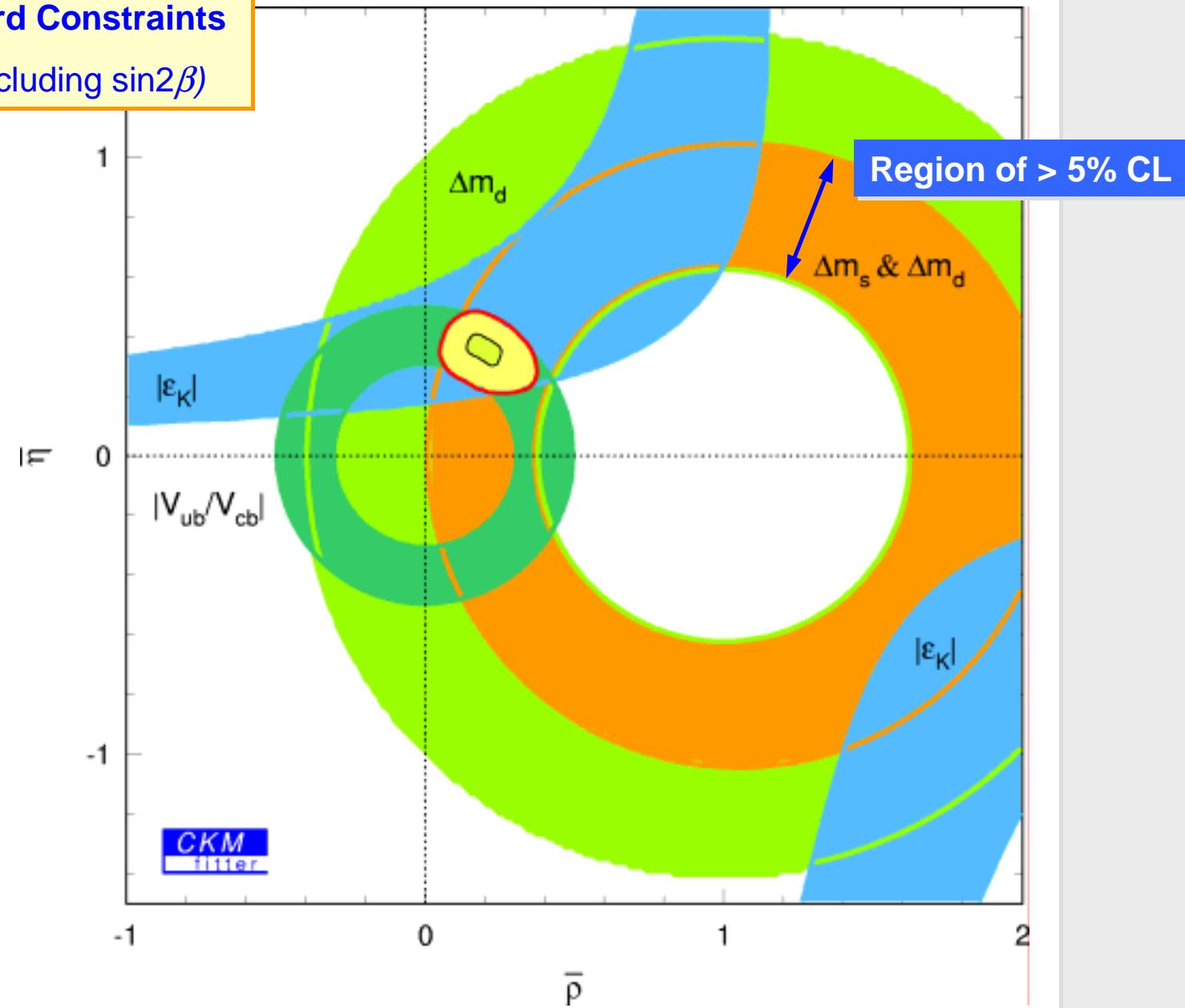


Confidence Level of Standard Model:  $\text{CL}(\text{SM}) = 57\%$

# Metrology (I)

Standard Constraints

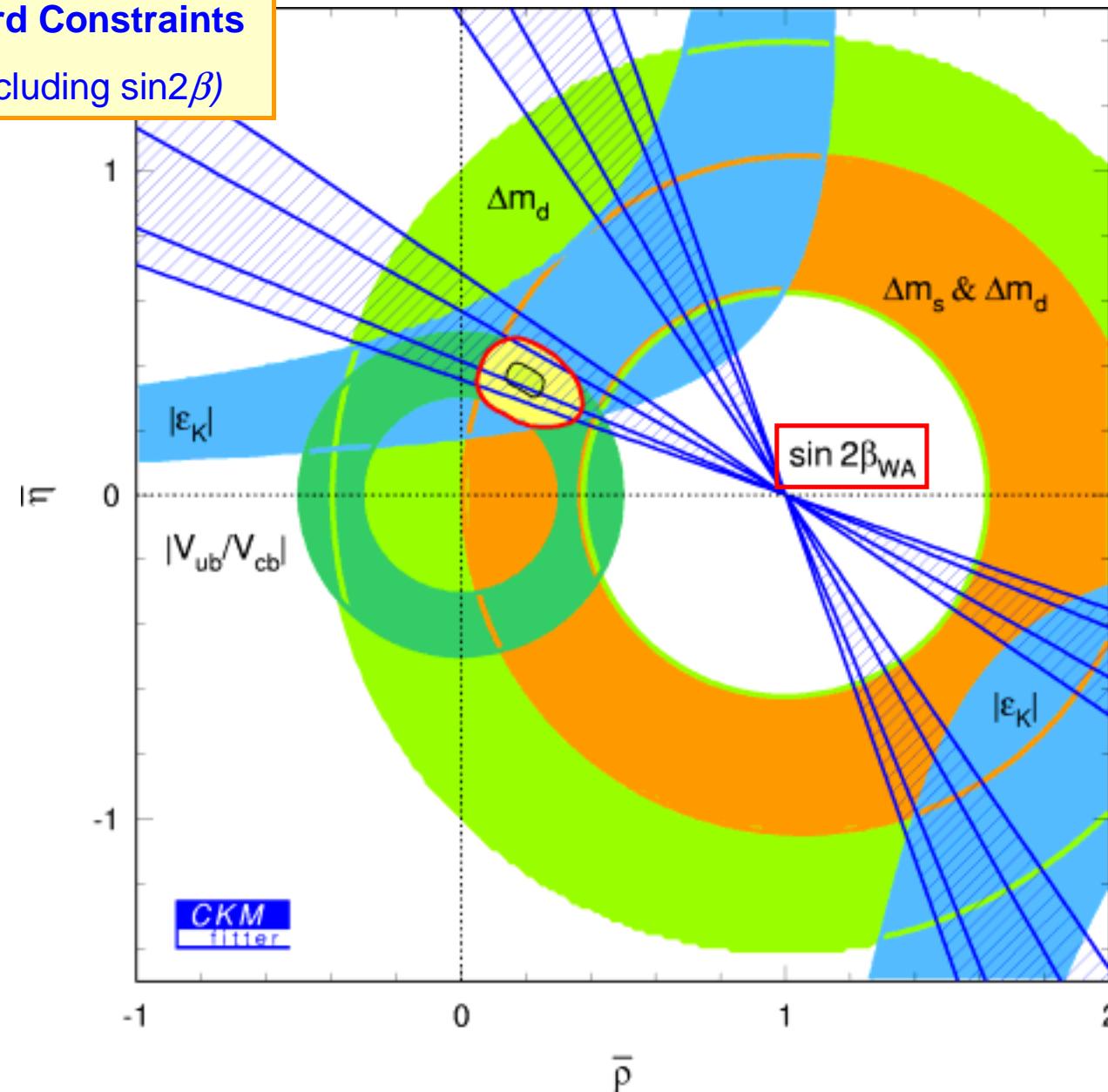
(not including  $\sin 2\beta$ )



# Metrology (I)

Standard Constraints

(not including  $\sin 2\beta$ )



A TRIUMPH  
FOR THE  
STANDARD  
MODEL AND  
THE KM  
PARADIGM !

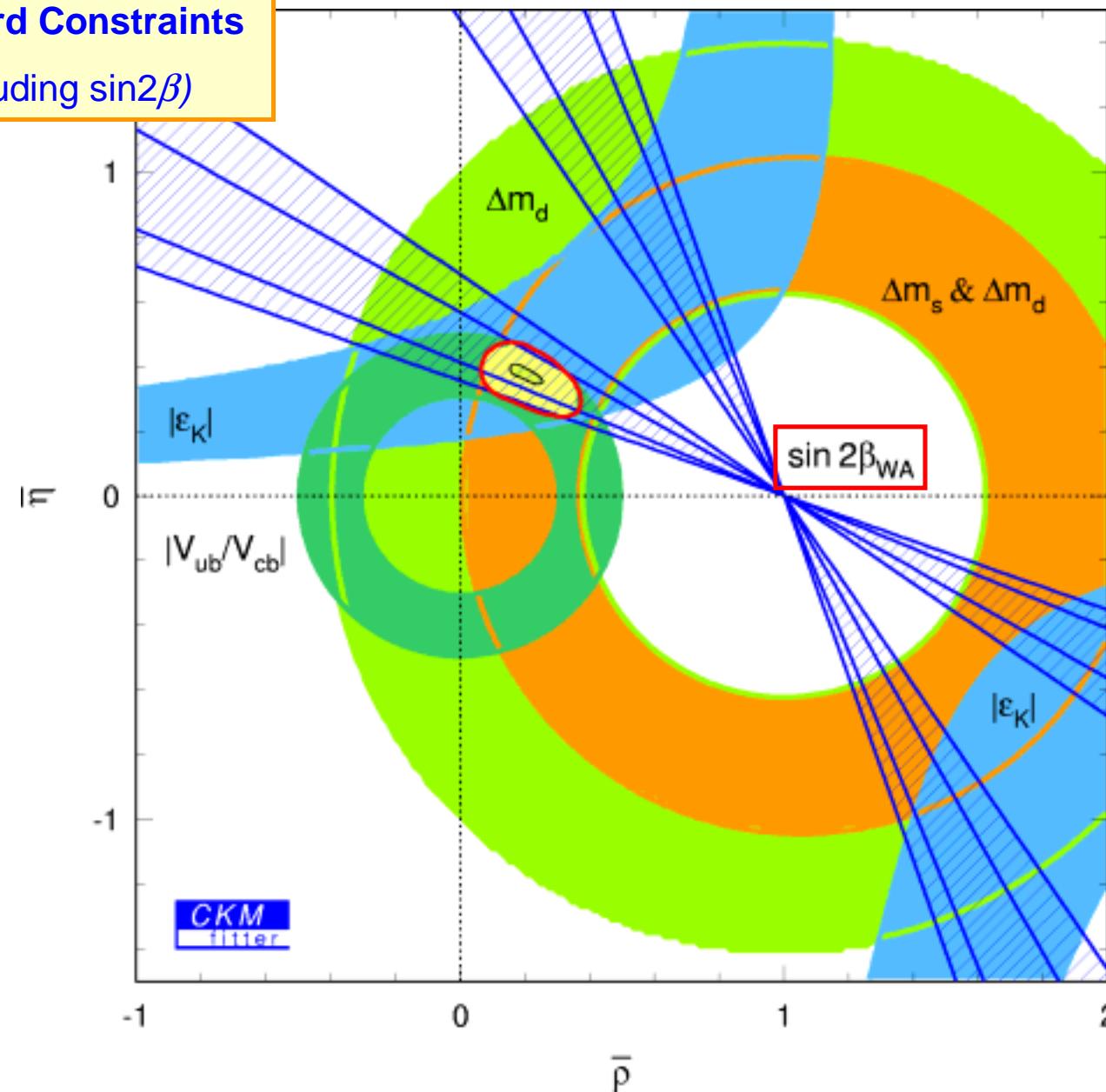


KM mechanism  
most probably  
dominant  
source of CPV  
at EW scale

# Metrology (I)

## Standard Constraints

(including  $\sin 2\beta$ )



$\sin 2\beta$  already provides one of the most precise and robust constraints

- How to improve these constraints?
- How to measure the missing angles ?

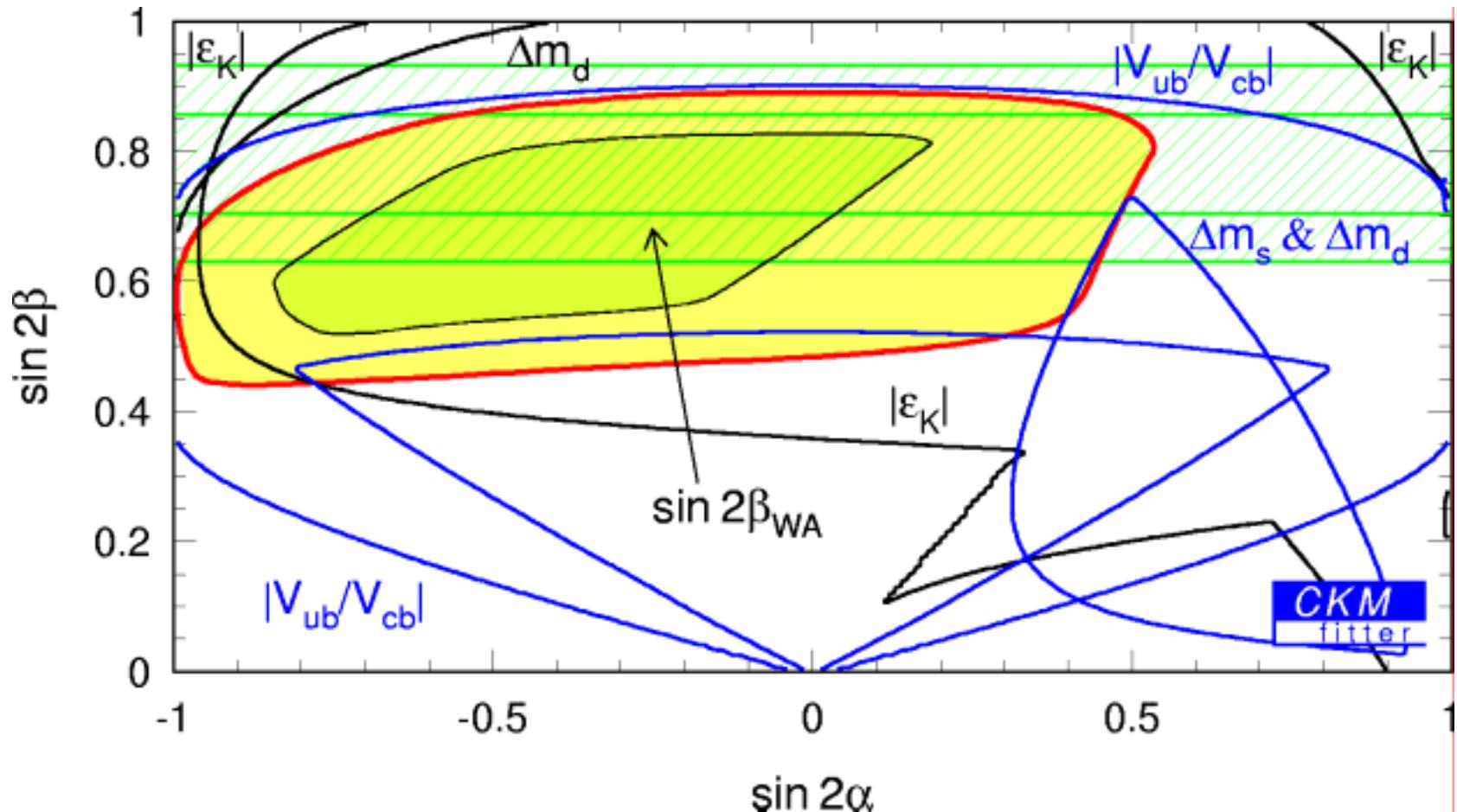


# Metrology (II): the $\sin(2\alpha)$ - $\sin(2\beta)$ Plane

Standard Constraints

(not including  $\sin 2\beta$ )

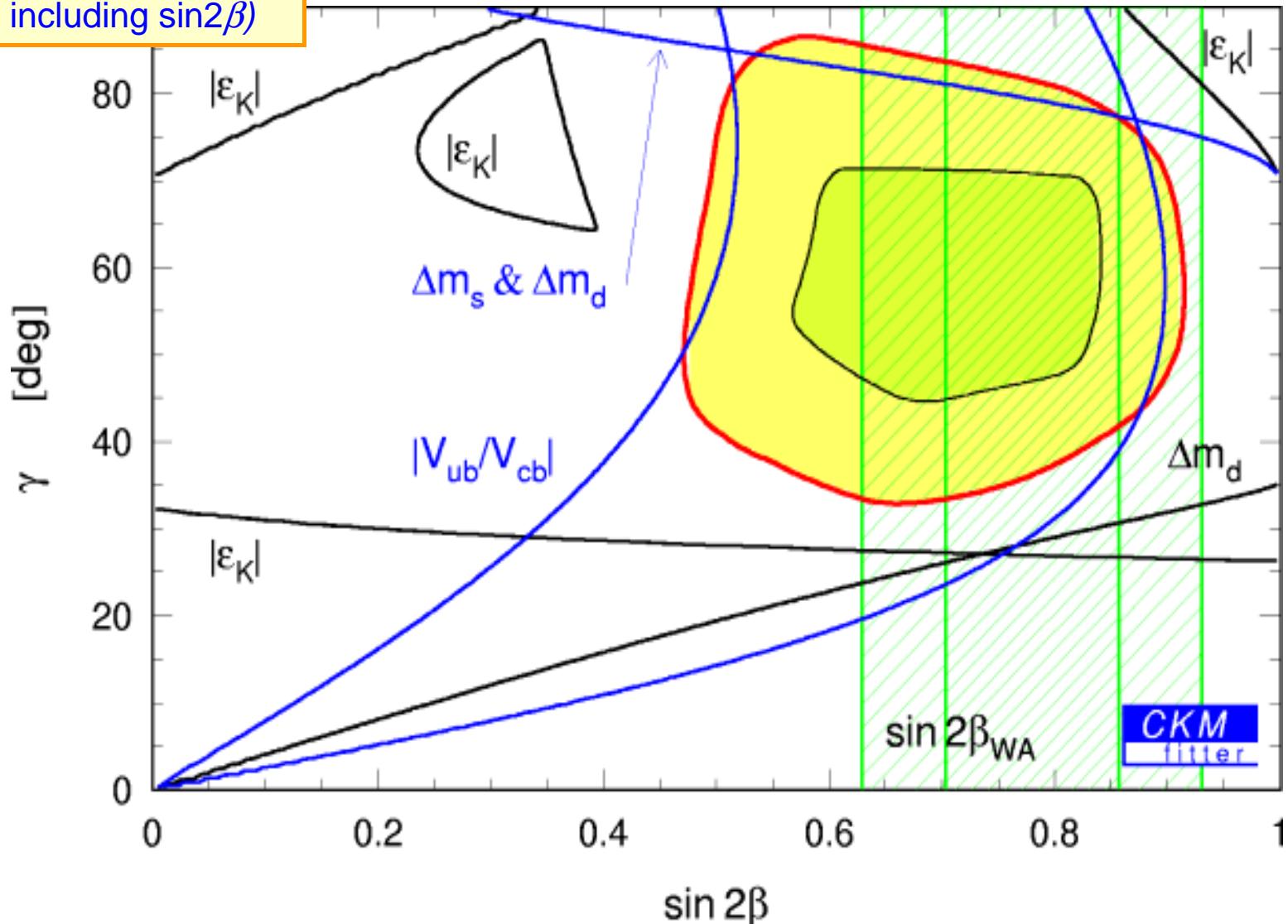
Be aware of  
ambiguities !



# Metrology (II): the $\sin(2\beta)$ - $\gamma$ Plane

## Standard Constraints

(not including  $\sin 2\beta$ )



# Metrology (III): Selected Numerical Results

## CKM and UT Parameters

Parameter	95% CL region
$\lambda$	$0.2221 \pm 0.0041$
$A$	$0.76 - 0.90$
$\rho$	$0.08 - 0.35$
$\eta$	$0.28 - 0.45$
$J$	$(2.2 - 3.5) \times 10^{-5}$
$\sin(2\alpha)$	$-0.81 - 0.43$
$\sin(2\beta)$	$0.64 - 0.84$
$\alpha$	$77^\circ - 117^\circ$
$\beta$	$19.9^\circ - 28.6^\circ$
$\gamma$	$40^\circ - 78^\circ$

## Rare Branching Fractions

Observable	95% CL region
$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$(1.6 - 4.2) \times 10^{-11}$
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(5.1 - 8.4) \times 10^{-11}$
$\text{BR}(B^+ \rightarrow \tau^+ \nu)$	$(7.2 - 22.1) \times 10^{-5}$
$\text{BR}(B^+ \rightarrow \mu^+ \nu)$	$(2.9 - 8.7) \times 10^{-7}$

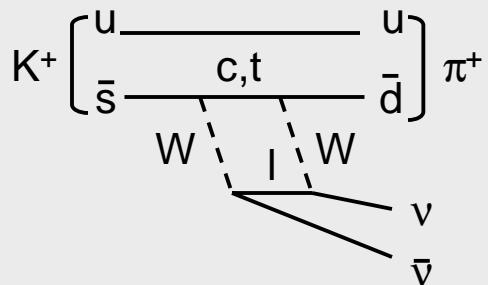
## Theory Parameters<sup>(\*)</sup>

Observable	95% CL region
$m_t$	$(104 - 380) \text{ GeV}/c^2$
$f_{B_d} \sqrt{B_d}$	$(199 - 282) \text{ MeV}$
$B_K$	$0.59 - 1.55$

(\*) Without using a priori information

# Constraint from Rare Kaon Decays: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

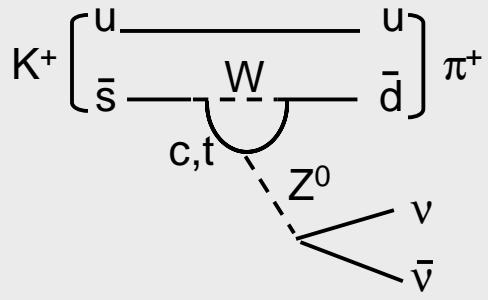
**Box:**



Buchalla, Buras, Nucl.Phys. B548 (1999) 309

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \propto \frac{BR(K^+ \rightarrow \pi^0 e^+ \nu)}{|V_{us}|^2} \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X'_{NL} + V_{ts}^* V_{td} X(x_t)|^2$$

**Penguin:**



**top contribution**

**charm contribution**

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \propto \lambda^8 A^4 \frac{X^2(x_t)}{\sigma} \left[ (\sigma \bar{\eta})^2 + (\rho_0 - \bar{\rho})^2 \right]$$

ellipse

Main theoretical uncertainty  
comes from charm contribution

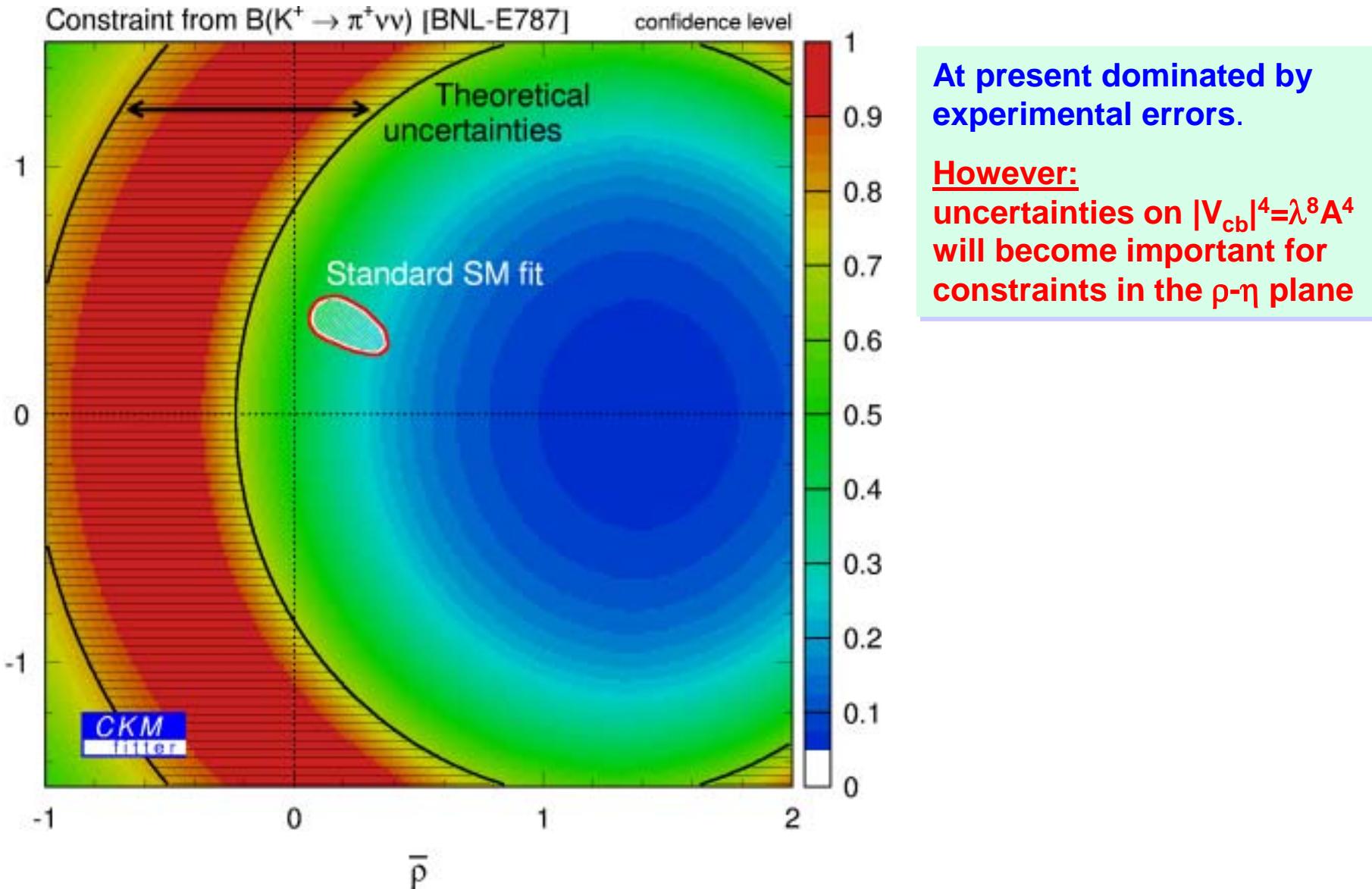
**Experiment:**

Two events observed at BNL (E787), yielding:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.57^{+1.75}_{-0.82}) \times 10^{-10}$$

E787 (BNL-68713)  
hep-ex/0111091

# Constraint from Rare Kaon Decays: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



# Rare Charmless $B$ Decays

We distinguish two Categories:

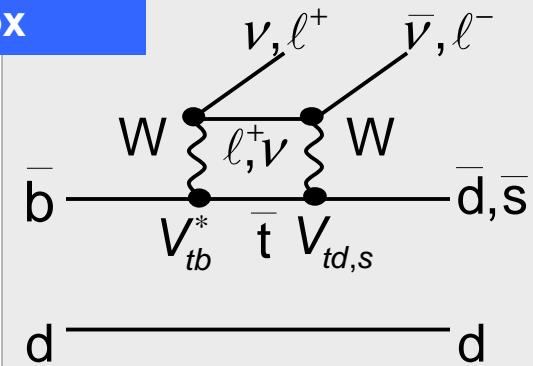
## ■ Semileptonic (FCNC) and radiative decays

- ◆  $(G_F)^2 \alpha$  increased compared to loop-induced non-radiative decays  $\propto (G_F \alpha)^2$
- ◆ Sensitive sondes for new physics (SUSY, right-handed couplings, ...)
- ◆ Determination of  $|V_{td}|$  and  $|V_{ts}|$
- ◆ Determination of HQET parameters
- ◆ Search for direct CP asymmetry

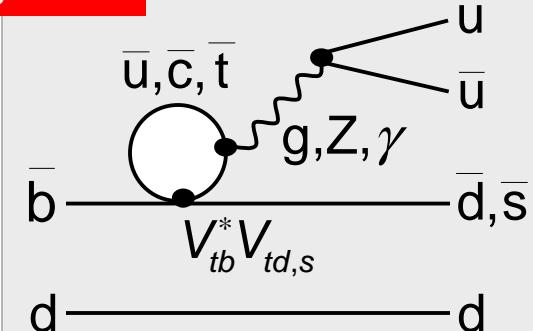
## ■ Hadronic $b \rightarrow u(d)$ decays

- ◆ Measurement of CPV
- ◆ Determination of UT angles  $\alpha$  and  $\gamma$
- ◆ Test der  $B$  decay dynamics (Factorization)

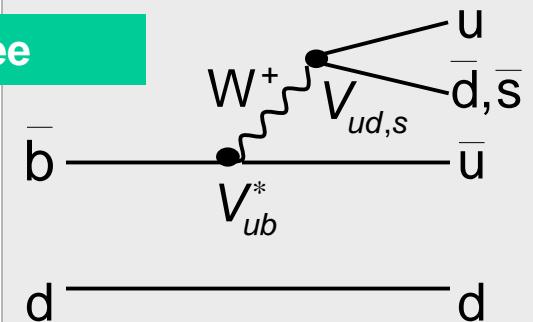
Box



Penguin



Tree



# Radiative $B$ Decays

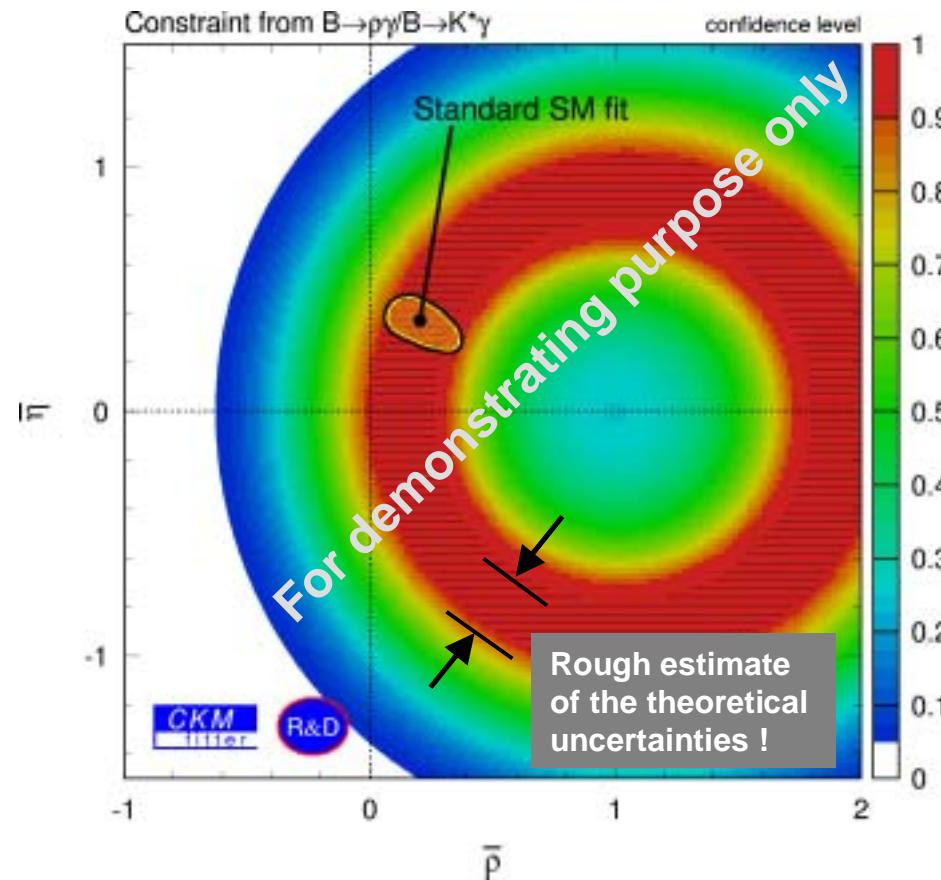
The ratio of the rates  $B \rightarrow \rho\gamma$  to  $B \rightarrow K^*\gamma$  can be predicted more cleanly than the individual rates: determines  $|V_{td}|$

$$\frac{\text{BR}(B \rightarrow \rho\gamma)}{\text{BR}(B \rightarrow K^*\gamma)} \propto \left| \frac{V_{td}}{V_{ts}} \right|^2 \zeta^2 (1 + \Delta R_{\text{NP}})$$

$$\zeta = 0.76 \pm 0.06, \quad \Delta R_{\text{NP}} < 0.15$$

Ali, Parkhomenko, EPJ C23 (2002) 89  
see also :  
Bosch, Buchalla, NP B621 (2002) 459

Source	$B^0 \rightarrow \rho^0\gamma$ ( $\text{BR} \times 10^{-6}$ )	$B^+ \rightarrow \rho^+\gamma$ ( $\text{BR} \times 10^{-6}$ )
Ali, Parkhomenko	$0.5 \pm 0.2$	$0.9 \pm 0.4$
Bosch, Buchalla	$0.8 \pm 0.3$	$1.5 \pm 0.5$
BABAR	$< 1.5$	$< 2.8$
Belle	$< 1.0$	$< 1.1$
CLEO	$< 1.7$	$< 1.3$



Standard SM fit

# Charmless $B$ Decays into two Pseudoscalars

[ Constraining  $\alpha$  and  $\gamma$ ?! ]

# $B \rightarrow K\pi$ and the Determination of $\gamma$

Interfering contributions of tree and penguin amplitudes:

$$A_{K\pi} \propto P + \lambda^2 e^{i\gamma} T$$

→ Potential for significant direct CPV

CP averaged BRs and measurements of direct CPV determine the angle  $\gamma$

Theoretical analysis deals with:

- SU(3) breaking
- Rescattering (FSI)
- EW penguins

The tool is: QCD Factorization...

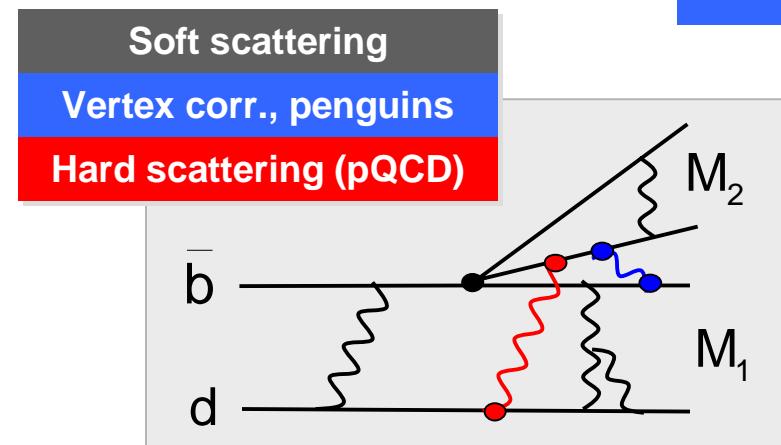
... based on Color Transparency

- Large energy release
- soft gluons do not interact with small  $q\bar{q}$ -bar color dipole of emitted mesons
- non-fact. contributions are calculable in pQCD perfect for  $m_b \rightarrow \infty$ .

Higher order corrections:  $(\Lambda_{\text{QCD}}/m_b)$

Fleischer, Mannel (98)  
Gronau, Rosner, London (94, 98)  
Neubert, Rosner (98)  
Buras, Fleischer (98)  
Beneke, Buchalla, Neubert, Sachrajda (01)  
Keum, Li, Sanda (01)  
Ciuchini et al. (01)  
...list by far not exhaustive!

→ see contributions at this conference



# Branching Fractions for $B \rightarrow \pi\pi / K\pi$

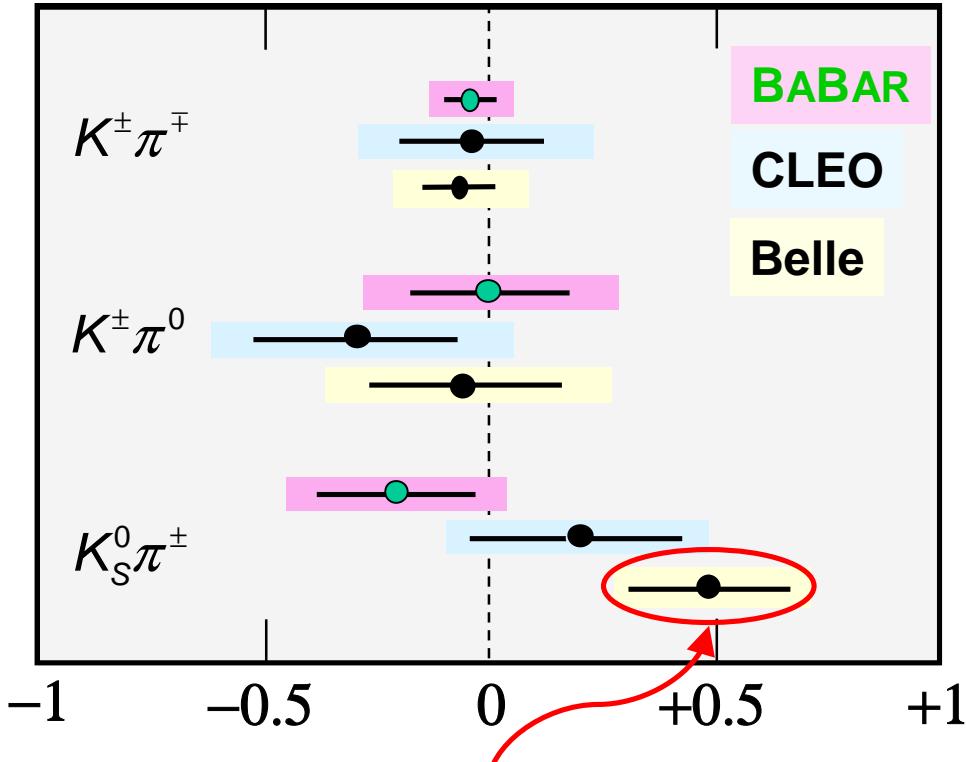
Updated Belle (La Thuile'02)  
Updated BABAR (Moriond EW'02)

BR ( $\times 10^6$ )	CLEO $9 \text{ fb}^{-1}$	BABAR up to $56 \text{ fb}^{-1}$	Belle $32 \text{ fb}^{-1}$	World average
$B^0 \rightarrow \pi^+ \pi^-$	$4.3^{+1.6}_{-1.4} \pm 0.5$	$5.4 \pm 0.7 \pm 0.4$	$5.1 \pm 1.1 \pm 0.4$	$5.17 \pm 0.62$
$B^0 \rightarrow K^+ \pi^-$	$17.2^{+2.5}_{-2.4} \pm 1.2$	$17.8 \pm 1.1 \pm 0.8$	$21.8 \pm 1.8 \pm 1.5$	$18.6 \pm 1.1$
$B^0 \rightarrow K^+ K^-$	$< 1.9$ (90%)	$< 1.1$ (90%)	$< 0.5$ (90%)	
$B^+ \rightarrow \pi^+ \pi^0$	$5.6^{+2.6}_{-2.3} \pm 1.7$	$5.1 \pm 2.0 \pm 0.8$	$7.0 \pm 2.2 \pm 0.8$	$5.9 \pm 1.4$
$B^+ \rightarrow K^+ \pi^0$	$11.6^{+3.0}_{-2.7} {}^{+1.4}_{-1.3}$	$10.8 \pm 2.1 \pm 1.0$	$12.5 \pm 2.4 \pm 1.2$	$11.5 \pm 1.5$
$B^+ \rightarrow K^0 \pi^+$	$18.2^{+4.6}_{-4.0} \pm 1.6$	$18.2 \pm 3.3 \pm 2.0$	$18.8 \pm 3.0 \pm 1.5$	$18.5^{+2.3}_{-2.2}$
$B^0 \rightarrow K^0 \pi^0$	$14.6^{+5.9}_{-5.1} {}^{+2.4}_{-3.3}$	$8.2 \pm 3.1 \pm 1.2$	$7.7 \pm 3.2 \pm 1.6$	$8.9 \pm 2.3$
$B^0 \rightarrow \pi^0 \pi^0$	$< 5.7$ (90%)	$< 3.4$ (90%)	$< 5.6$ (90%)	



Agreement among experiments. Most rare decay channels discovered

# Direct CP Asymmetries in $K\pi$ Modes



Are annihilation contributions important?



Agreement among experiments.  
No significant deviation from zero.

**BABAR:**

BABAR Moriond'02

$$A_{CP}(K^+\pi^-) = -0.05 \pm 0.06 \pm 0.01$$

$$A_{CP}(K^+\pi^0) = +0.00 \pm 0.18 \pm 0.04$$

$$A_{CP}(K^0\pi^+) = -0.21 \pm 0.18 \pm 0.03$$

**Belle:**

BELLE La Thuile'02

$$A_{CP}(K^+\pi^-) = -0.06 \pm 0.08 \pm 0.08$$

$$A_{CP}(K^+\pi^0) = -0.04 \pm 0.19 \pm 0.03$$

$$A_{CP}(K^0\pi^+) = +0.46 \pm 0.15 \pm 0.02$$

**CLEO:**

CLEO PRL 85 (2000) 525

$$A_{CP}(K^+\pi^-) = -0.04 \pm 0.16$$

$$A_{CP}(K^+\pi^0) = -0.29 \pm 0.23$$

$$A_{CP}(K^0\pi^+) = +0.18 \pm 0.24$$

**World averages:**

$$A_{CP}(K^+\pi^-) = -0.05 \pm 0.05$$

$$A_{CP}(K^+\pi^0) = -0.09 \pm 0.12$$

$$A_{CP}(K^0\pi^+) = +0.18 \pm 0.10$$

# Bounds on $\gamma$

Ratios of CP averaged branching fractions can lead to bounds on  $\gamma$ :

**FM bound:**

$$R = \frac{\tau(B^+)}{\tau(B^0)} \cdot \frac{\text{BR}(K^\pm \pi^\mp)}{\text{BR}(K^0 \pi^\pm)} = 1.07^{+0.15}_{-0.12} < 1 ? \rightarrow \text{no constraint}$$

Fleischer, Mannel PRD D57 (1998) 2752

**BF bound:**

$$R_n = \frac{1}{2} \frac{\text{BR}(K^\pm \pi^\mp)}{\text{BR}(K^0 \pi^0)} = 1.04^{+0.37}_{-0.22} \neq 1 ? \rightarrow \text{no constraint}$$

Buras, Fleischer EPJ C11 (1998) 93

**NR bound:**

$$R_*^{-1} = 2 \frac{\text{BR}(K^\pm \pi^0)}{\text{BR}(K^0 \pi^\pm)} = 1.24^{+0.24}_{-0.21} \neq 1 ? \rightarrow \text{no constraint}$$

Neubert, Rosner PL B441 (1998) 403



See also recent *Bayesian* analysis: Bargiotti et al. hep-ph/0204029

# Neubert-Rosner Bound

a)

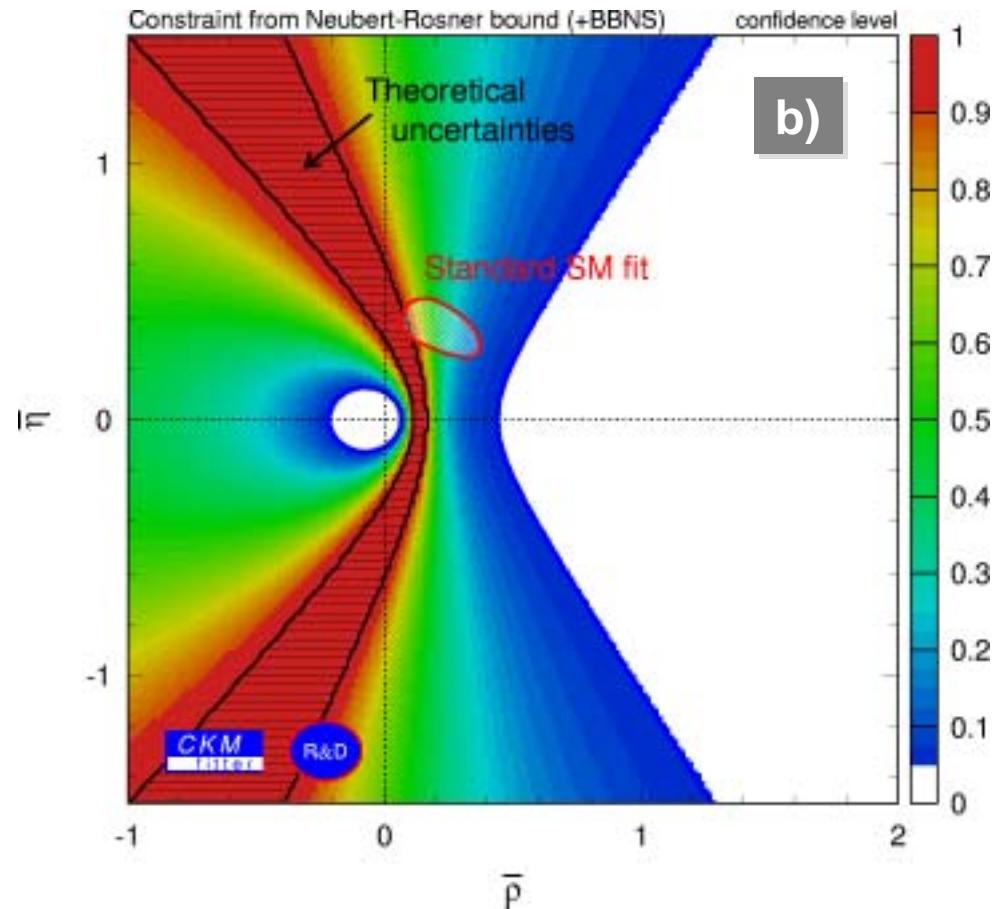
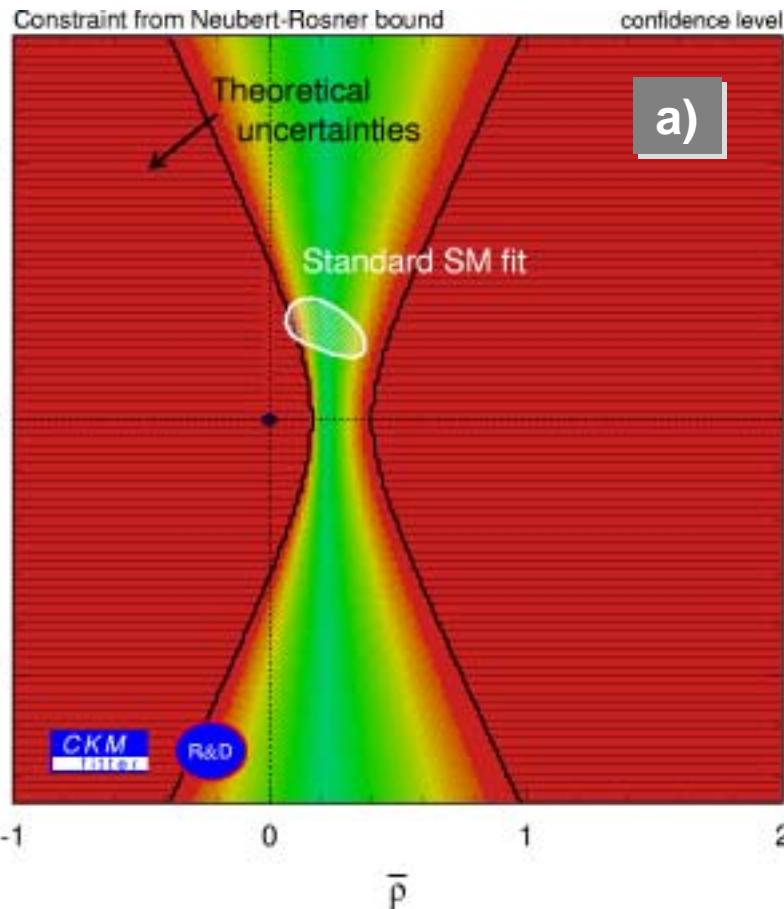
$$T / P \rightarrow \bar{e}_{3/2} = R_{th} \cdot \tan \theta_c \frac{f_K}{f_\pi} \sqrt{\frac{2 \cdot BR(\pi^\pm \pi^0)}{BR(K^0 \pi^\pm)}} = R_{th}(\text{SU}(3), \text{BBNS}) \cdot (0.221 \pm 0.028)$$

Tree

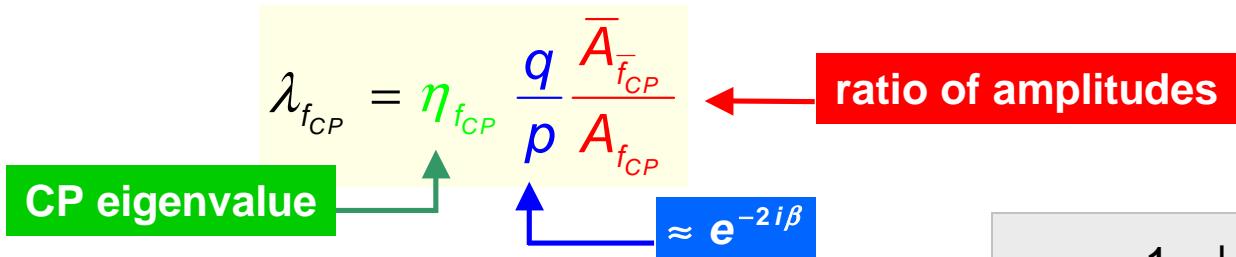
Penguin

b)

QCD FA: small relative strong phases



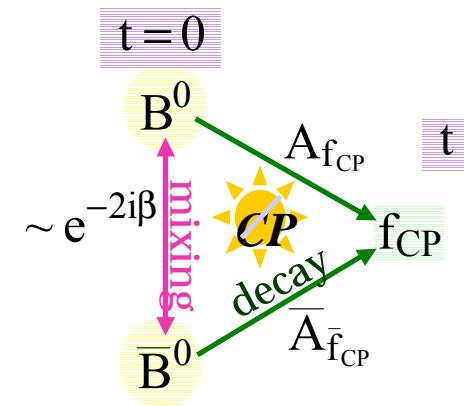
# CP Violation in $B^0 \rightarrow \pi^+ \pi^-$ Decays



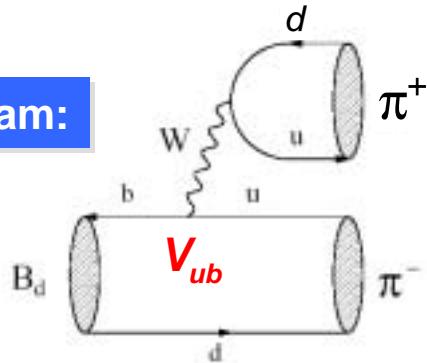
$$A_{f_{CP}}(t) \propto C_{f_{CP}} \cos(\Delta m_d t) + S_{f_{CP}} \sin(\Delta m_d t)$$

$$C_{f_{CP}} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}$$

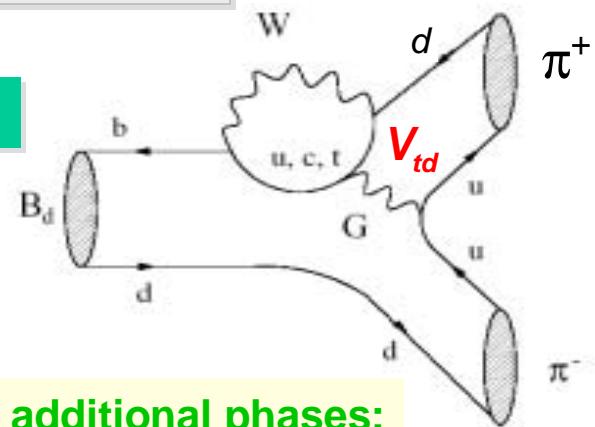
$$S_{f_{CP}} = \frac{2 \operatorname{Im} \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2}$$



Tree diagram:



Penguin diagram:



For a single weak phase (tree):

$$\lambda = \frac{q}{p} \frac{\bar{A}_f}{A_f} = \eta_f e^{-2i(\beta+\gamma)} = \eta_f e^{2i\alpha}$$

$$C_{\pi\pi} = 0, S_{\pi\pi} = \sin(2\alpha)$$

For additional phases:

$|\lambda| \neq 1 \Rightarrow$  must fit for direct CP  
 $\operatorname{Im}(\lambda) \neq \sin(2\alpha) \Rightarrow$  need to relate asymmetry to  $\alpha$

$$C_{\pi\pi} \neq 0, S_{\pi\pi} = \sin(2\alpha_{\text{eff}})$$

# $\sin(2\alpha_{\text{eff}})$ & Gronau-London Isospin Analysis

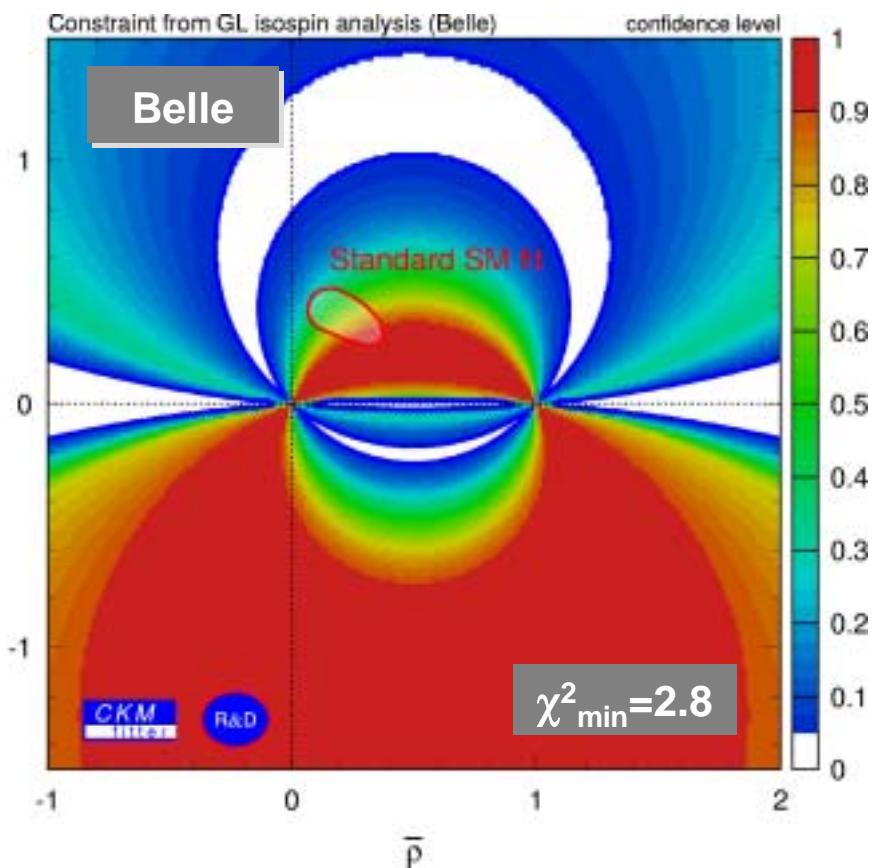
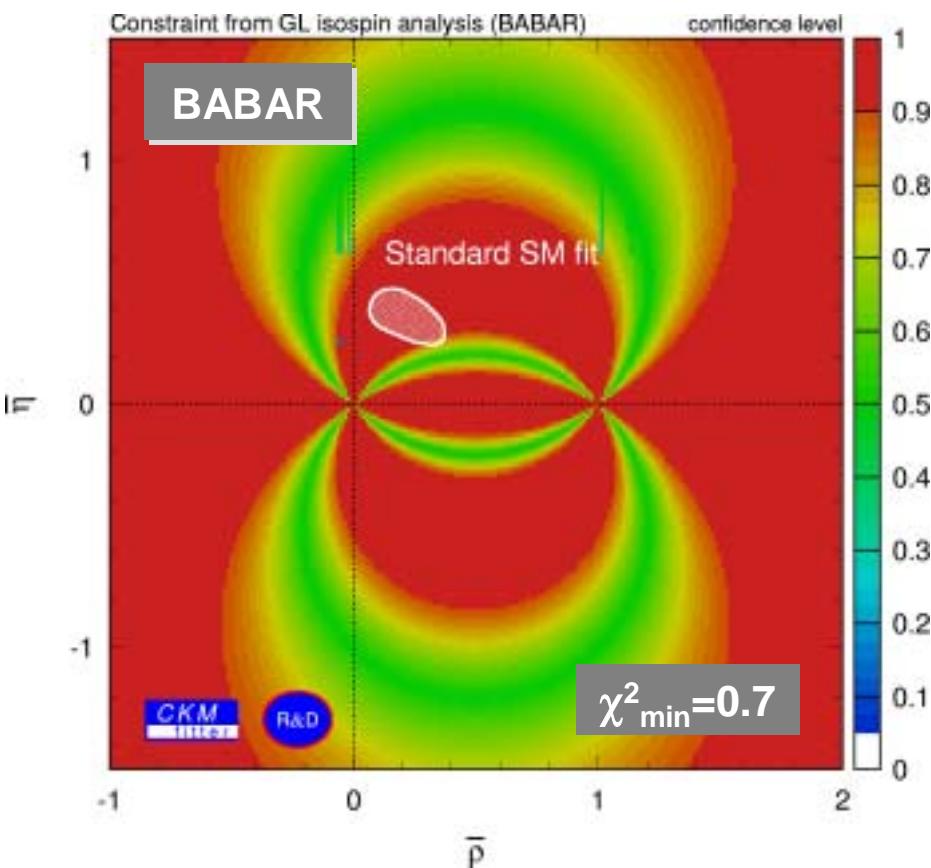
Using the BRs :  $\pi^+\pi^-$ ,  $\pi^\pm\pi^0$ ,  $\pi^0\pi^0$  (limit)

and the CP asymmetries :  $A_{\text{CP}}(\pi^\pm\pi^0)$ ,  $S_{\pi\pi}$ ,  $C_{\pi\pi}$

and the amplitude relations:  $A^{+-}/\sqrt{2} + A^{00} = A^{+0}$ ,  
 $(A \leftrightarrow \bar{A})$  and  $|A^{+0}| = |\bar{A}^{+0}|$

	BABAR	Belle
$S_{\pi\pi}$	$-0.01 \pm 0.38$	$-1.21^{+0.41}_{-0.30}$
$C_{\pi\pi}$	$-0.02 \pm 0.30$	$-0.94^{+0.32}_{-0.27}$

sign convention changed!

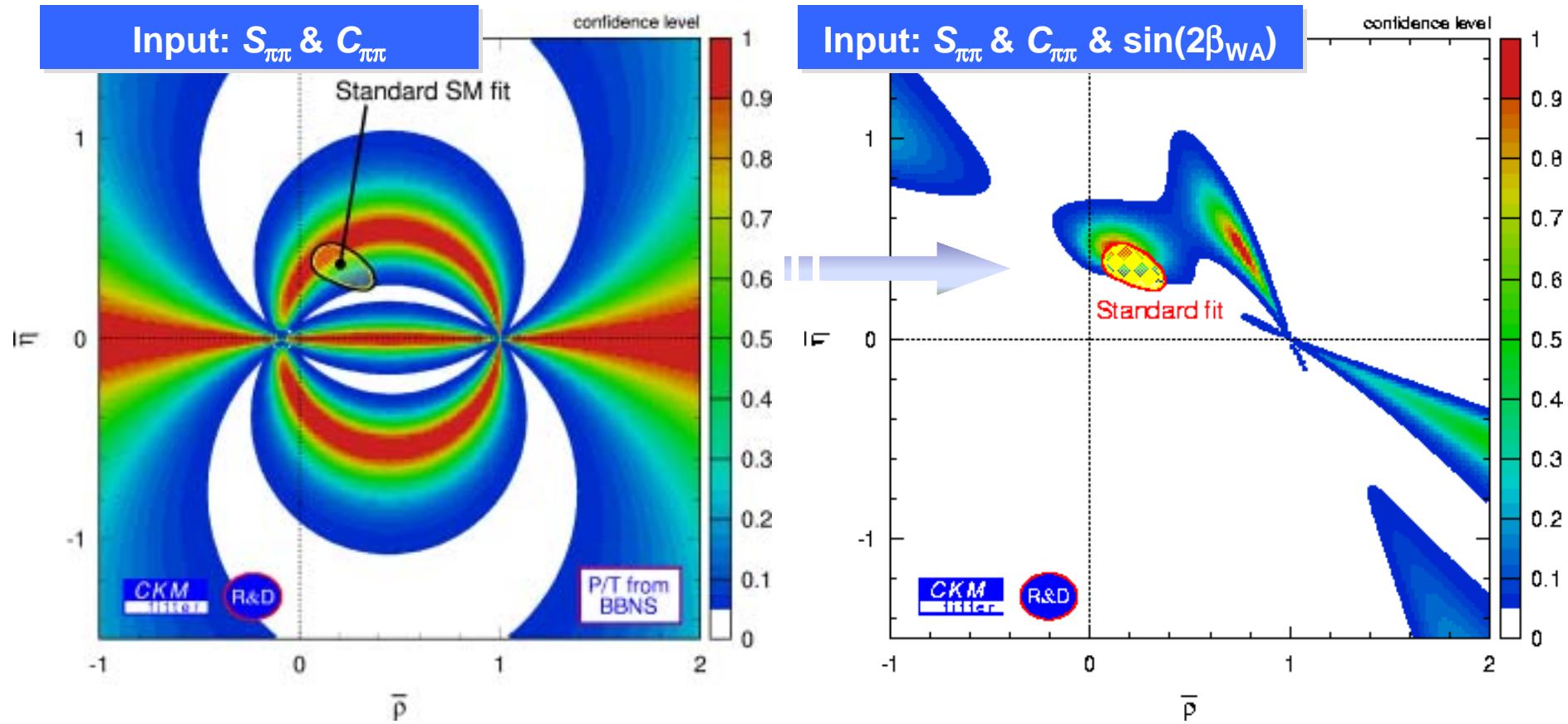


# BABAR: $\sin(2\alpha_{\text{eff}})$ & Theory (QCD FA)

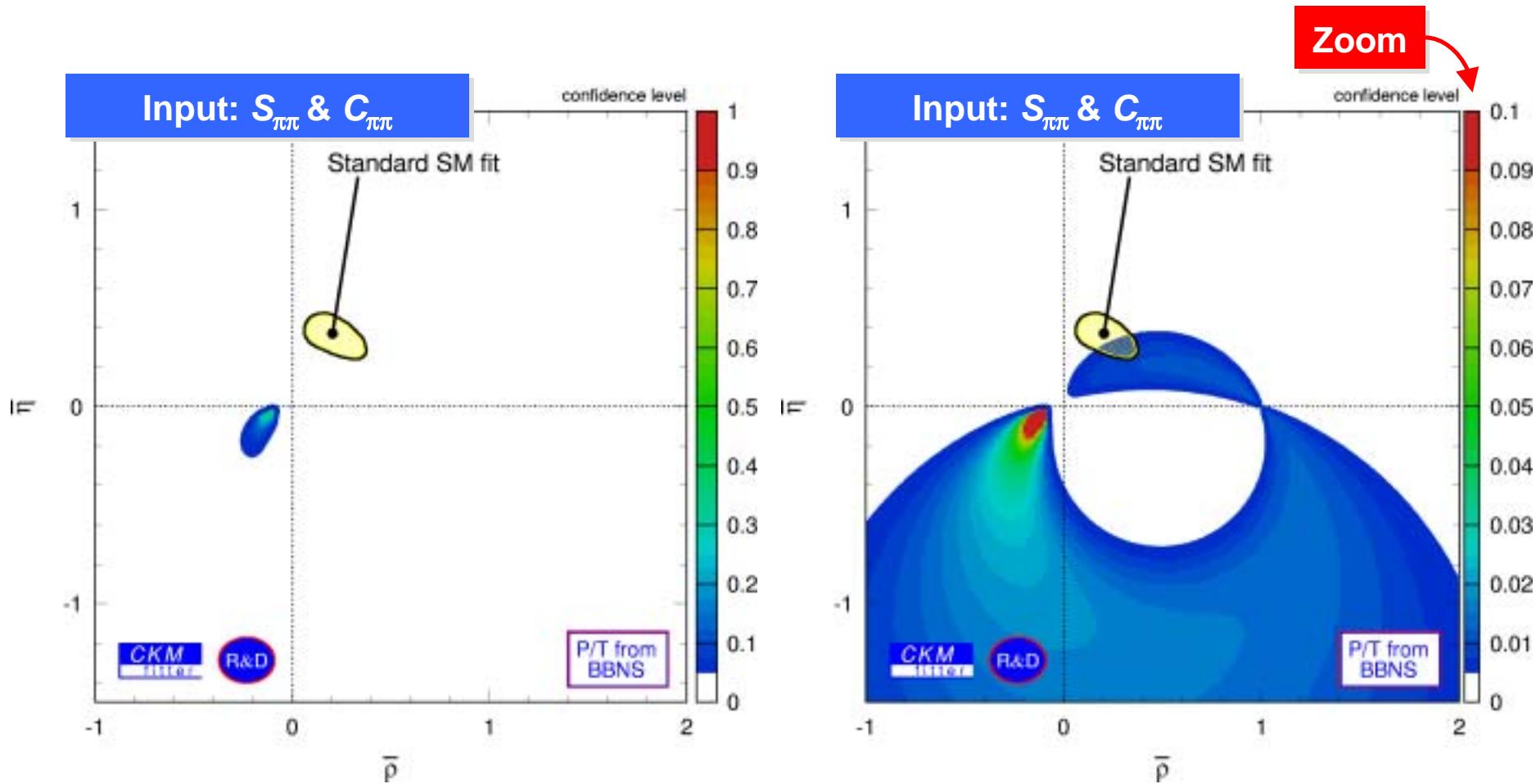
$$S_{\pi\pi} = \frac{2\text{Im}\lambda_{\pi\pi}}{1+|\lambda_{\pi\pi}|^2}, \quad C_{\pi\pi} = \frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2}$$

$$\lambda_{\pi\pi} = e^{-2i\beta} \frac{e^{-i\gamma} + \frac{P_{\pi\pi}/T_{\pi\pi}}{P_{\pi\pi}'/T_{\pi\pi}'}}{e^{+i\gamma} + \frac{P_{\pi\pi}/T_{\pi\pi}}{P_{\pi\pi}'/T_{\pi\pi}'}}$$

& QCD FA (BBNS)

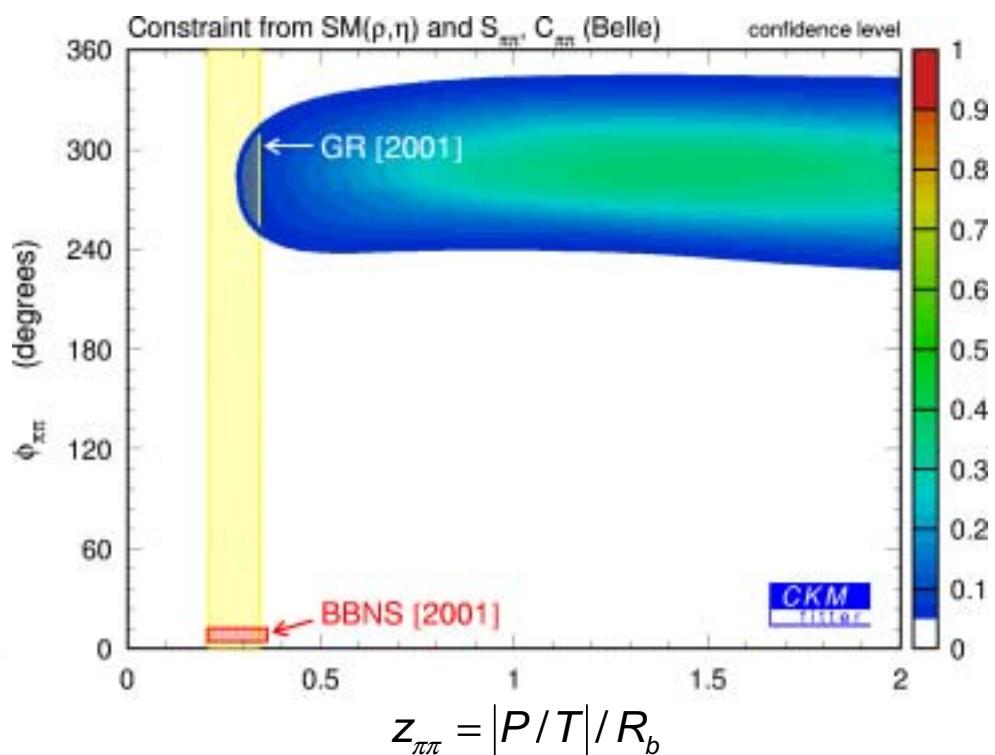
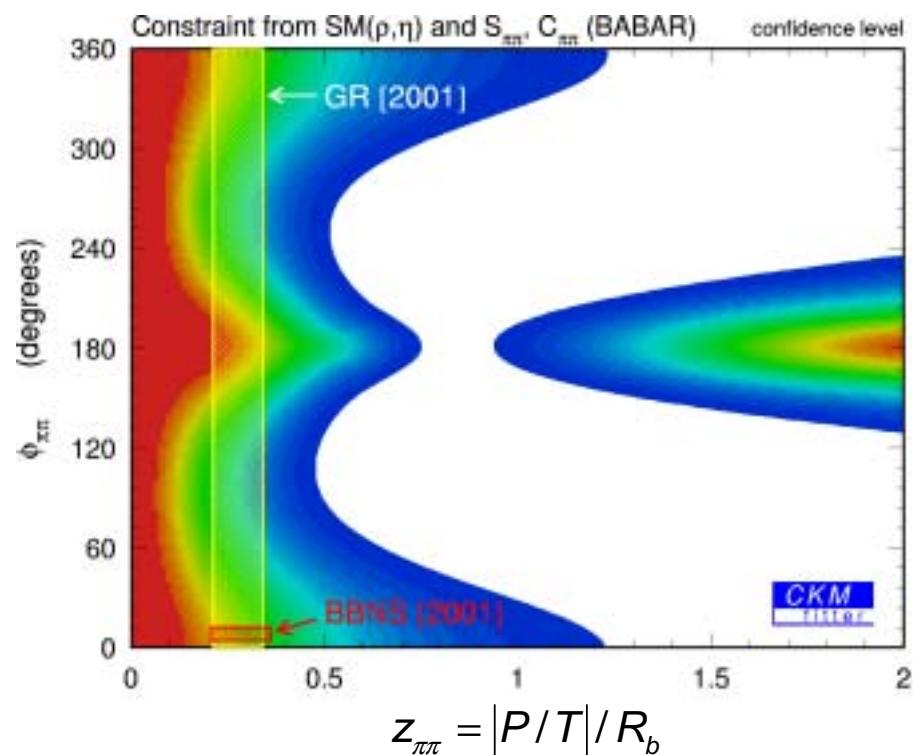


# Belle: $\sin(2\alpha_{\text{eff}})$ & Theory (QCD FA)



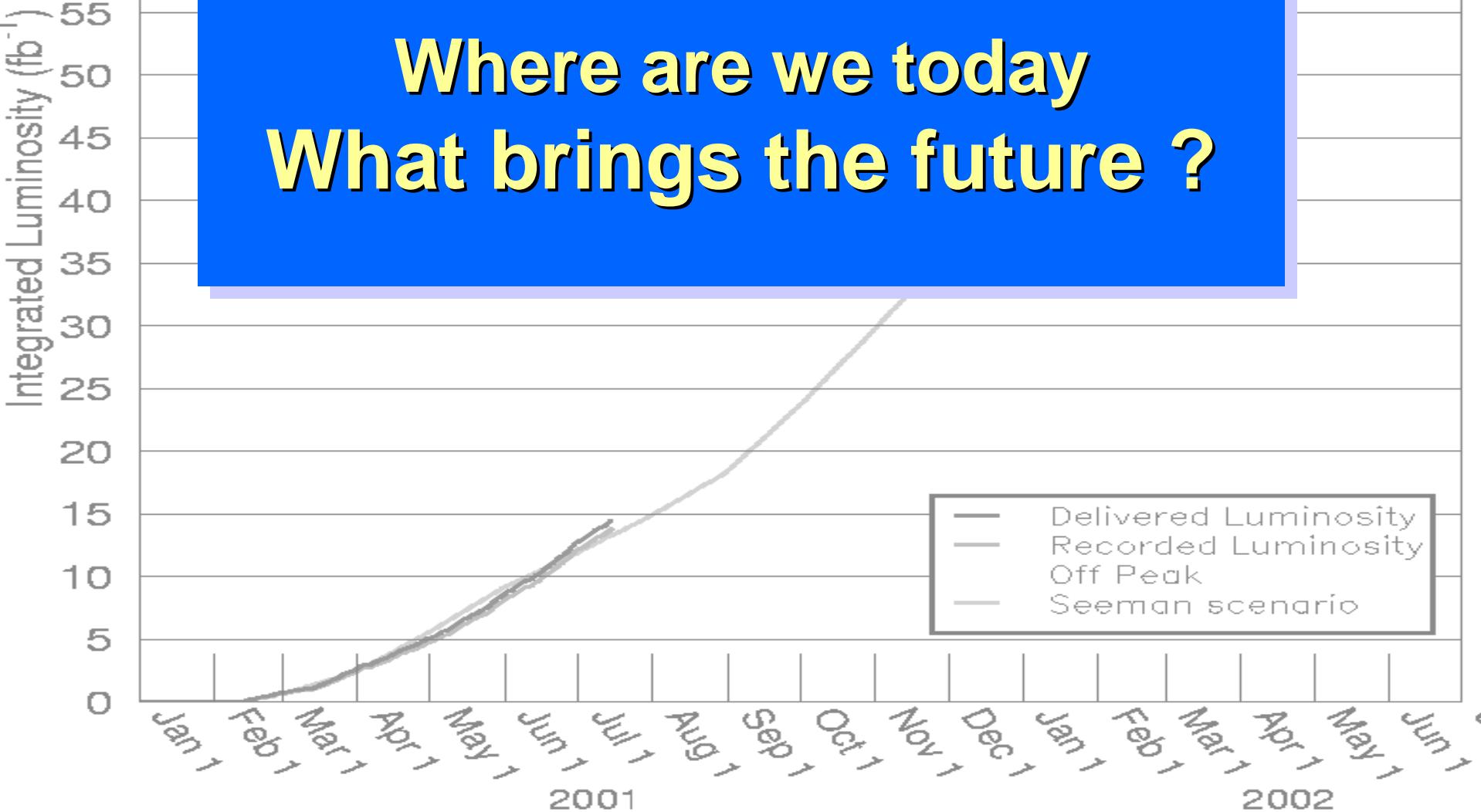
# The Reverse: $\sin(2\alpha_{\text{eff}}, 2\beta)$ & SM fit → THEORY

- The theory provides tree and penguin contributions and their relative phases
- The global fit determines the agreement between experiment and theory, using all measured BRs and CP asymmetries (also time-dependent)
- Determine also the free parameters of the theory (*i.e.*, the CKM elements)

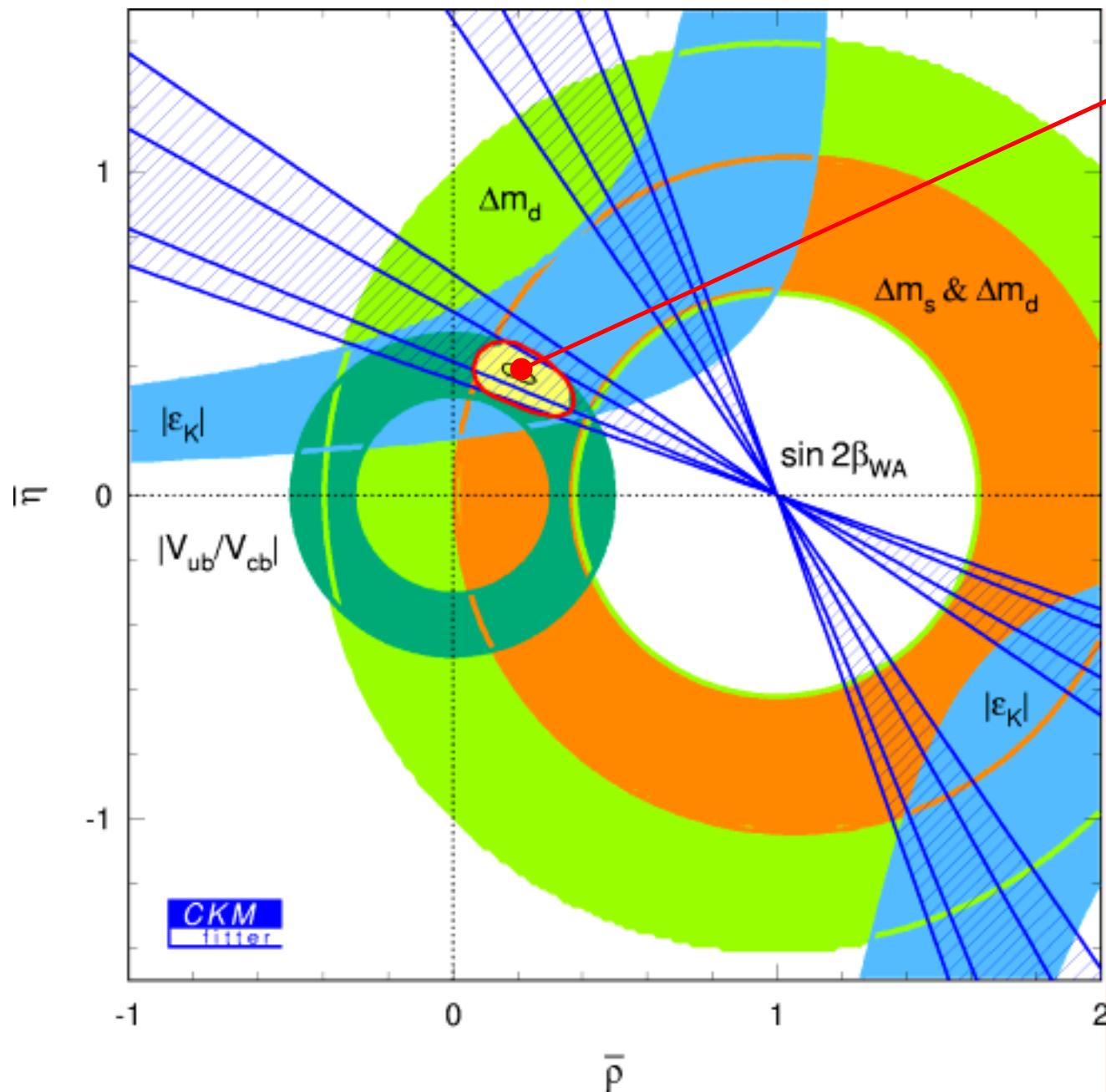


GR: Gronau, Rosner, Phys.Rev.D65:013004,2002  
BBNS: Beneke et al., Nucl.Phys.B606:245-321,2001

# Where are we today What brings the future ?



# The Standard Model holds the castle:

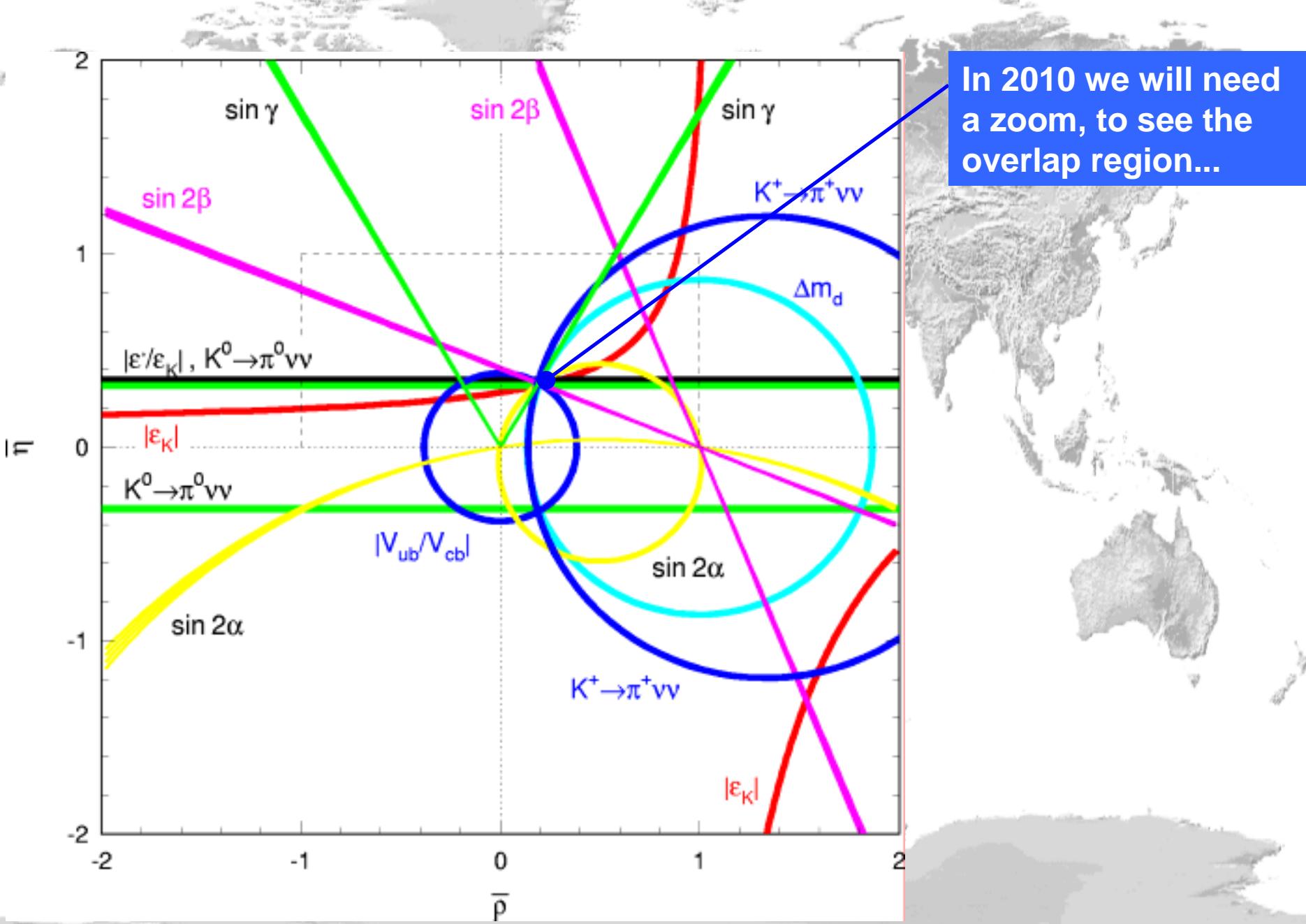


We know the center already quite well... but it is too large!

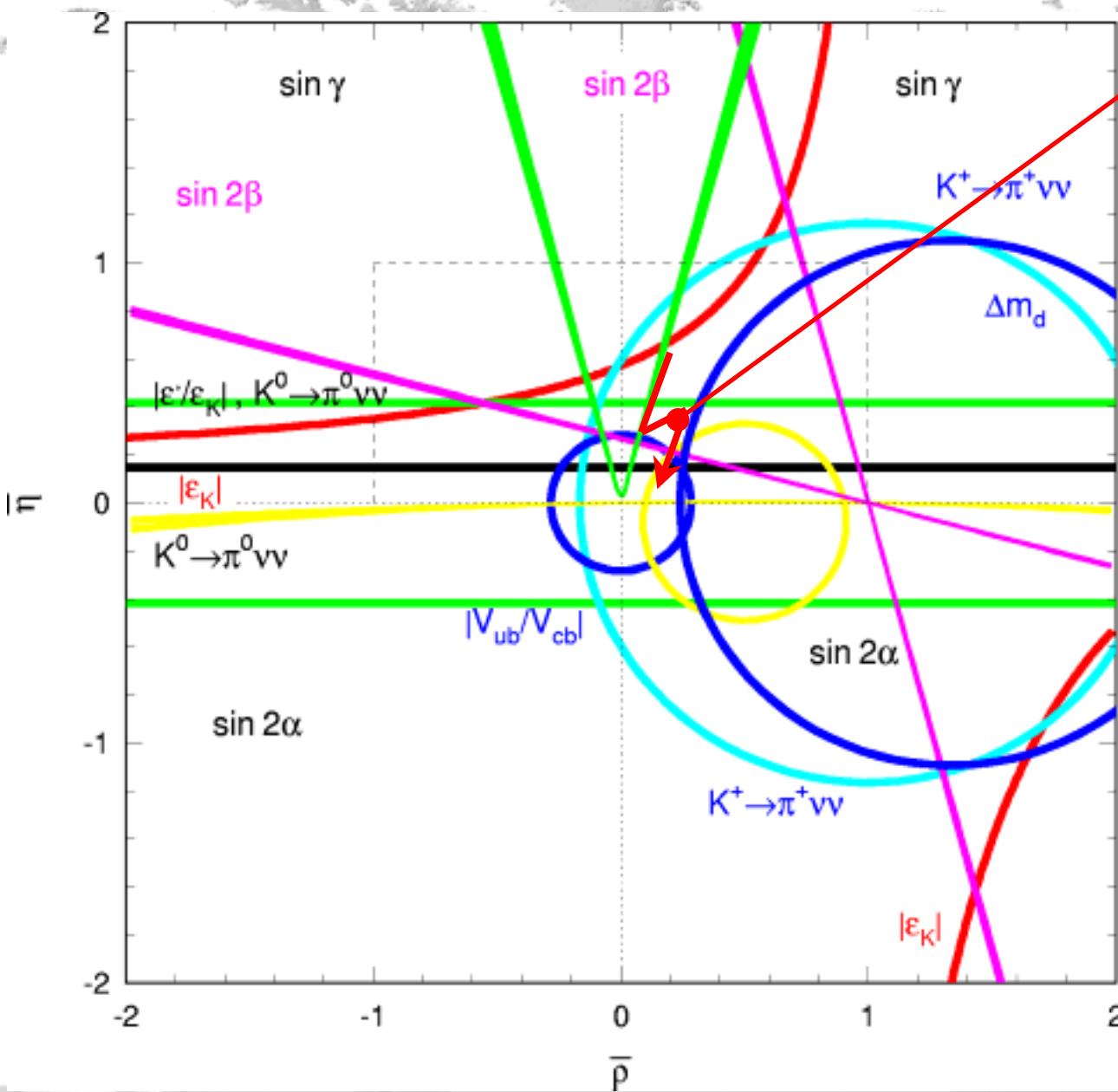
A better understanding of long distance QCD opens the shrine to a full exploitation of the huge data samples currently produced at KEKB and PEPII.

...and the incredible data quantities that will be produced at the Tevatron & LHC

And in the far future ?

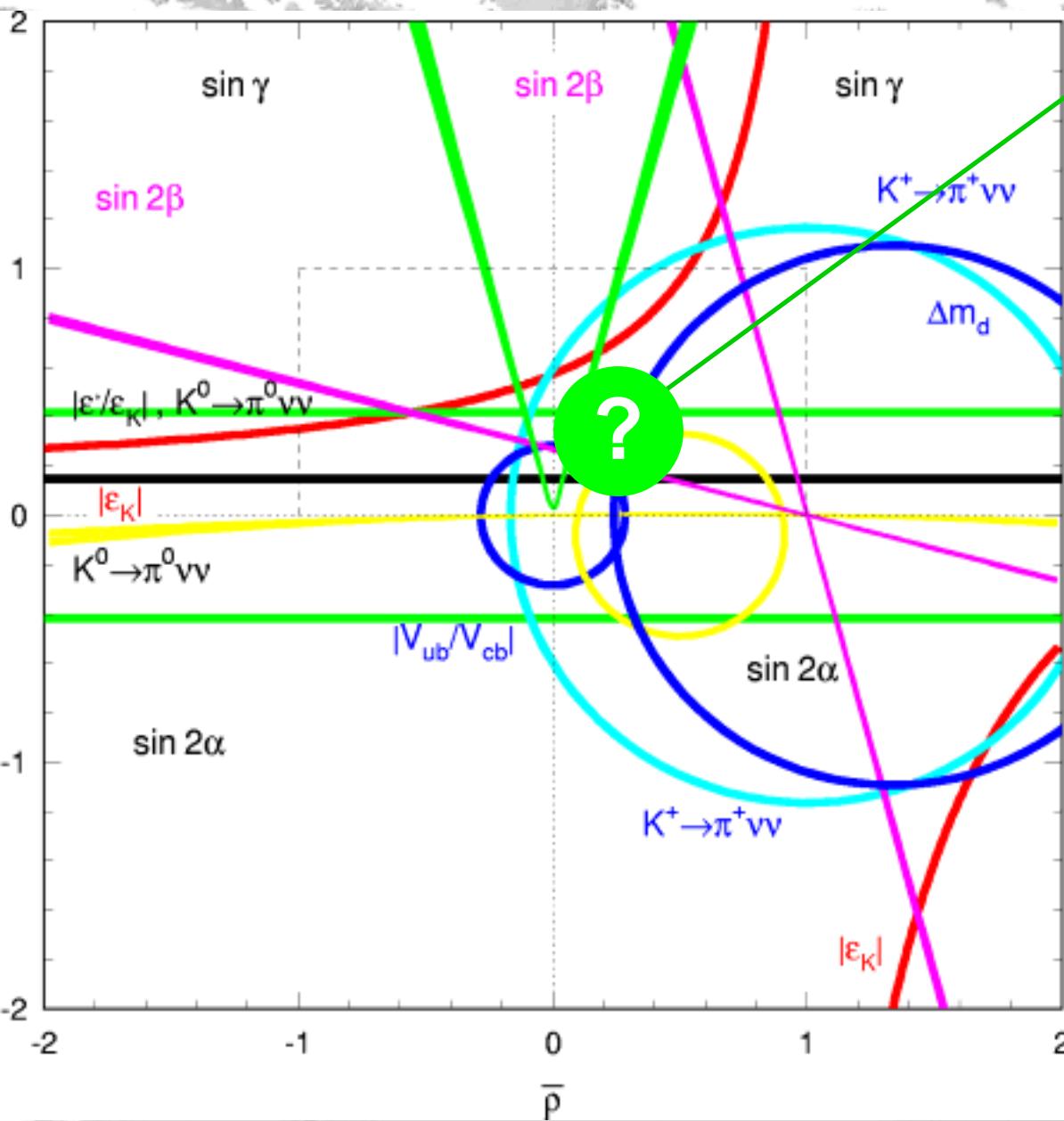


And in the far future ?



Will there still be an overlap region ?

And in the far future ?



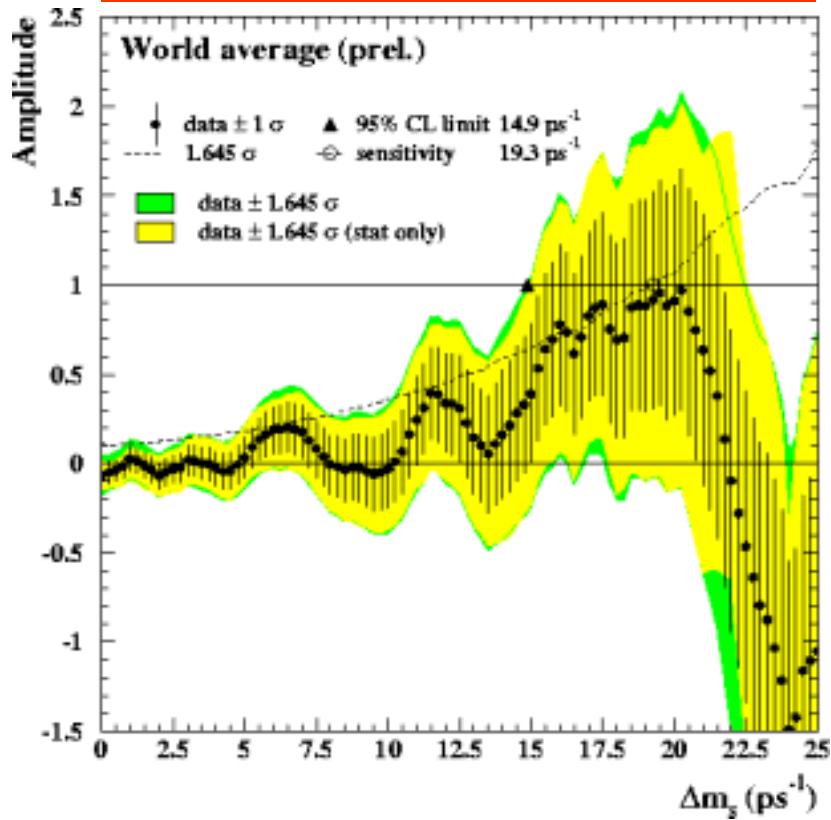
... maybe we can establish new physics before the LHC finds it ???

# **Backup Material**

# Using $\Delta m_s$

$\Delta m_s$  not yet measured. How to use the available experimental inform.?

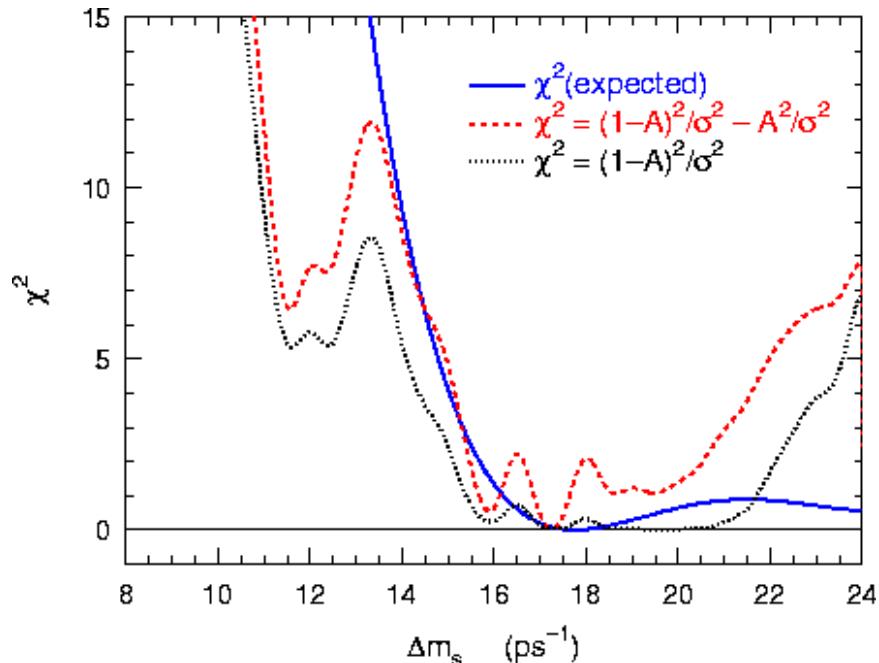
Amplitude spectrum:  
LEP/SLD/CDF



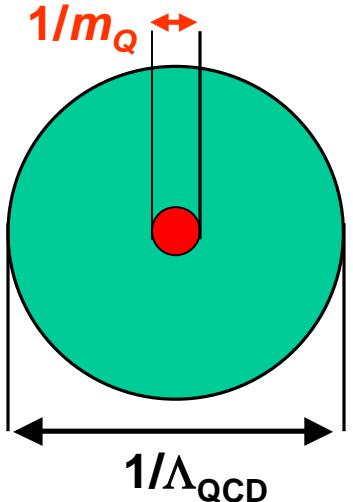
Preferred value:  $17.2 \text{ ps}^{-1}$

Following a presentation of F. Le Diberder  
at the CERN CKM workshop (Feb. 02)

- compute the expected PDF for the current preferred value
- compute the CL
- infer an equivalent  $\chi^2$



# Determination of the Matrix Elements $|V_{cb}|$ and $|V_{ub}|$



Symmetry of heavy quarks [=SU(2n<sub>Q</sub>)]:

in the limit  $m_Q \rightarrow \infty$  of a Qq system, the heavy quark represents a static color source with fixed 4-momentum.

The light degrees of freedom become insensitive to spin and flavor of the quark.

For both,  $|V_{cb}|$  and  $|V_{ub}|$ , exist exclusive and inclusive semileptonic approaches.

The theoretical tools is *Heavy Quark Effective Theory* (HQET) and the Operator Product Expansion (OPE)

- $|V_{ub}| (\rightarrow \rho^2 + \eta^2)$  is important for the SM prediction of  $\sin(2\beta)$
- $|V_{cb}| (\rightarrow A)$  is crucial for the interpretation of kaon decays ( $\varepsilon_K$ ,  $\text{BR}(K \rightarrow \pi \nu \bar{\nu})$ , ...)

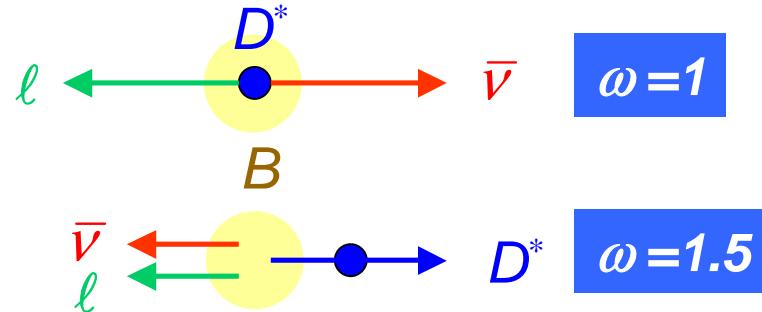
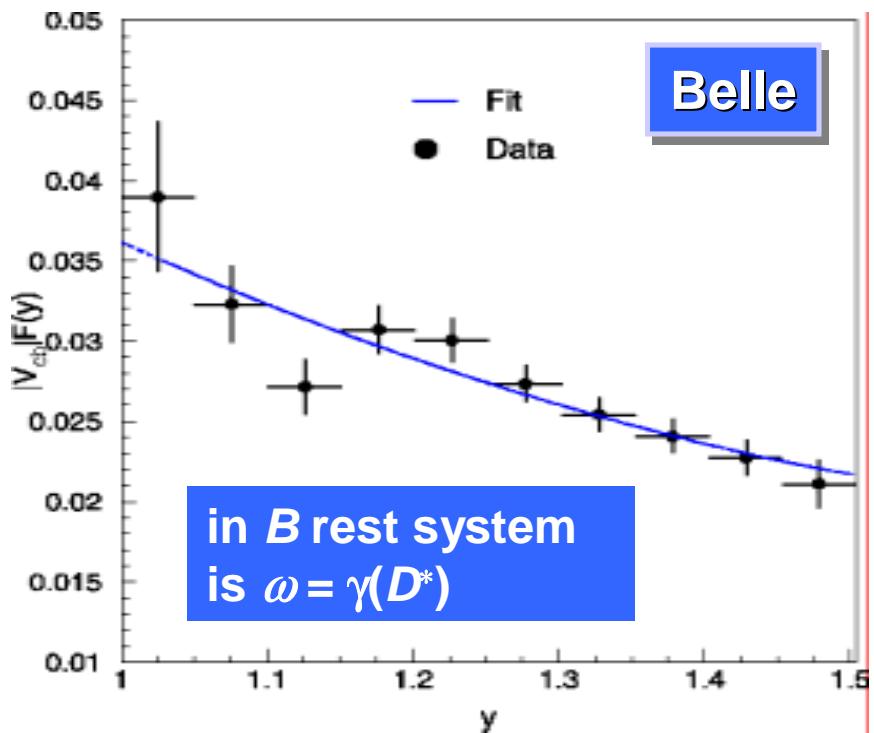
# Exclusive Semileptonic $B \rightarrow D^* \ell \bar{\nu}$ Decays

- Measurement of  $B \rightarrow D^* \ell \bar{\nu}$  rate as fct. of  $B \rightarrow \ell \bar{\nu}$  momentum transition  $\omega$
- Determination of  $|V_{cb}|$  from extrapolation to  $\omega \rightarrow 1$  (theory is most restrictive)

$$\frac{d\Gamma(B \rightarrow D^* \ell \bar{\nu})}{d\omega} \propto F_*^2(\omega) |V_{cb}|^2$$

HQ Symmetry:  
 $F_*(1) \approx 0.9$  ( $\pm 5\%$ )

Bigi, Uraltsev;  
Neubert;  
...;  
Lattice QCD



$$F_*(1)|V_{cb}| = 10^{-3} \times \begin{cases} 35.6 \pm 1.7 & (\text{LEP}) \\ 42.2 \pm 2.2 & (\text{CLEO}) \\ 36.2 \pm 2.3 & (\text{Belle}) \end{cases}$$

Belle, PLB 526, 247 (2002)

# Inclusive Semileptonic $B \rightarrow X_c l \bar{\nu}$ Decays

- OPE: expansion of decay rate in  $\Lambda_{\text{QCD}} / m_b$  und  $\alpha_s(m_b)$
- Model-independent results for sufficiently inclusive observables:

Bigi, Shifman, Uraltsev; Hoang, Ligeti, Manohar

$$|V_{cb}| \simeq 0.0419 \sqrt{\frac{\text{BR}(B \rightarrow X_c \ell \bar{\nu})}{0.105}} \frac{1.55 \text{ ps}}{\tau_B} \left( 1 \pm 0.015_{\text{pQCD}} \pm 0.010_{m_b} \pm 0.012_{1/m_b^3} \right)$$

## Experimental strategy

- Identify  $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$  by tagging one of the  $B$ s:
  - ◆ Full reconstruction of the high energetic lepton
- Select leptons from the semileptonic decay of the other  $B$

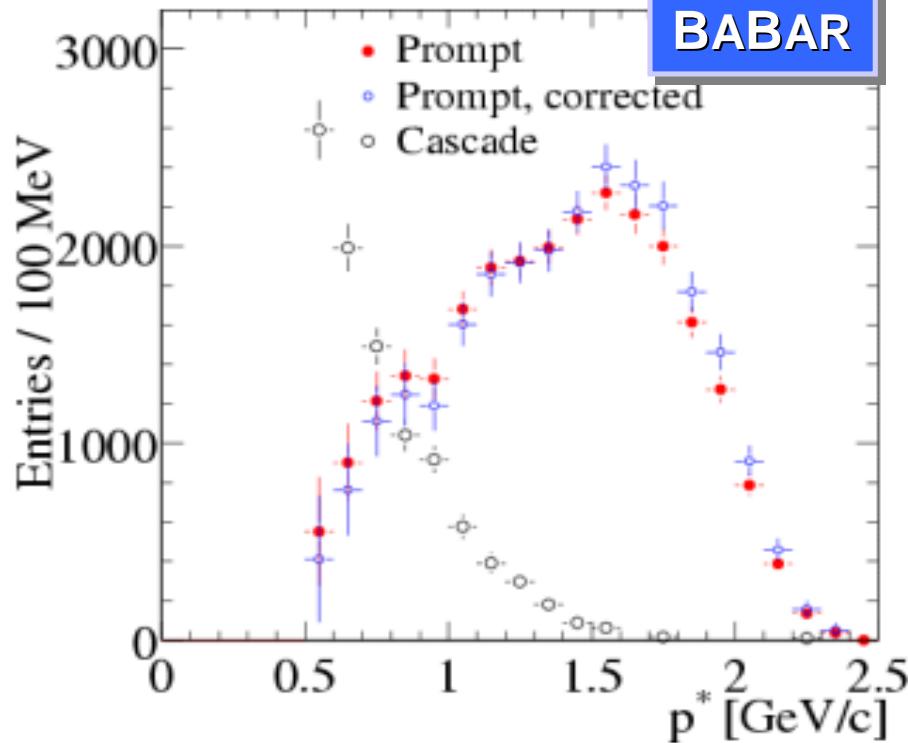
$$B^0 \bar{B}^0 \text{ tag: } \begin{cases} B^{0/+} \rightarrow X_{\bar{c}, \bar{u}} e^+ \nu_e & \text{Fast } e^+: \text{„right-sign“} \\ B^{0/+} \rightarrow X_{\bar{c}, \bar{u}} Y, \quad X_{\bar{c}} \rightarrow X' e^- \bar{\nu}_e & \text{Cascade } e^-: \text{„wrong-sign“} \end{cases}$$

- $\text{BR}(B \rightarrow X \ell \nu_\ell) \propto N_{\text{fast}} / N_{\text{tag}}$

## **BR( $B \rightarrow X l(e) \nu$ ):**

**BABAR**

BABAR:  $(10.82 \pm 0.21 \pm 0.38)$  %  
 Belle:  $(10.86 \pm 0.14 \pm 0.47)$  %  
 CLEO:  $(10.49 \pm 0.17 \pm 0.43)$  %  
 LEP:  $(10.65 \pm 0.23)$  %  
 ARGUS :  $(9.7 \pm 0.5 \pm 0.4)$  %



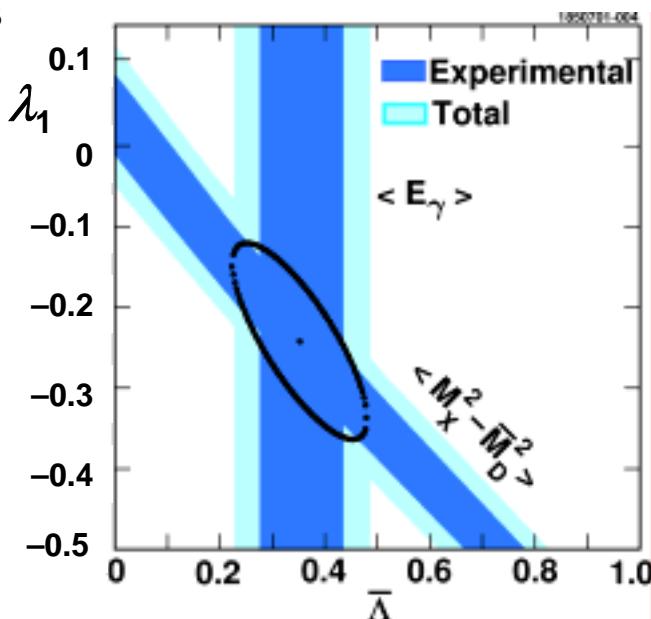
**BABAR** preliminär:

→ z.B.:  $|V_{cb}|(\text{BABAR}) \cong (40.8 \pm 1.7 \pm 1.5) \times 10^{-3}$

A promising approach for a theoretically improved analysis is the combined fit of the HQET parameters  $\Lambda$  und  $\lambda_1$  (CLEO) by means of  $b \rightarrow s\gamma$ . Allows to test Quark-Hadron Duality. (See also spectral moments analysis of hadronic Tau decays).

→  $|V_{cb}|(\text{CLEO}) \cong (40.4 \pm 1.3) \times 10^{-3}$

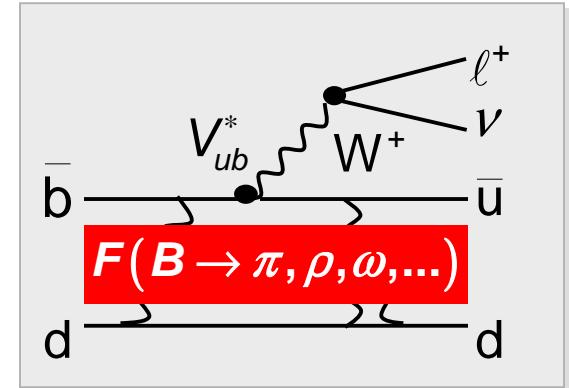
CLEO, Phys. Rev. Lett. 87, 251808 (2001)



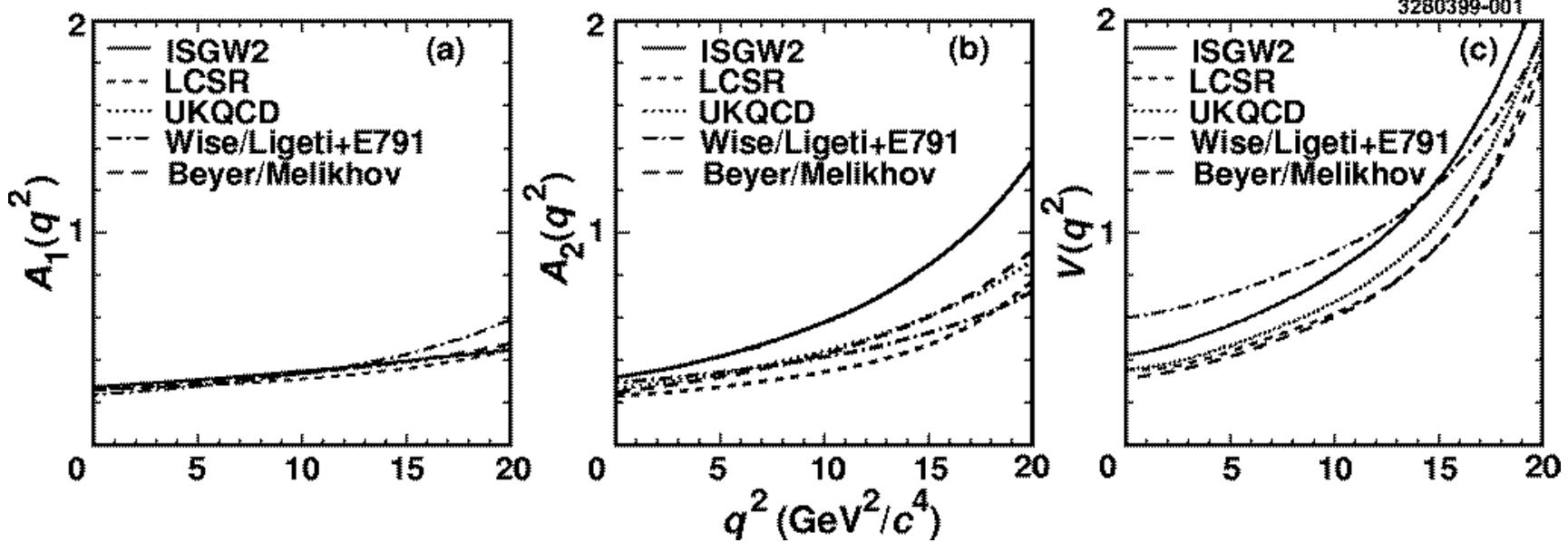
# $|V_{ub}|$ from exclusive Decays (I)

Pure tree decay. The decay rate is proportional to the CKM element  $|V_{ub}|^2$

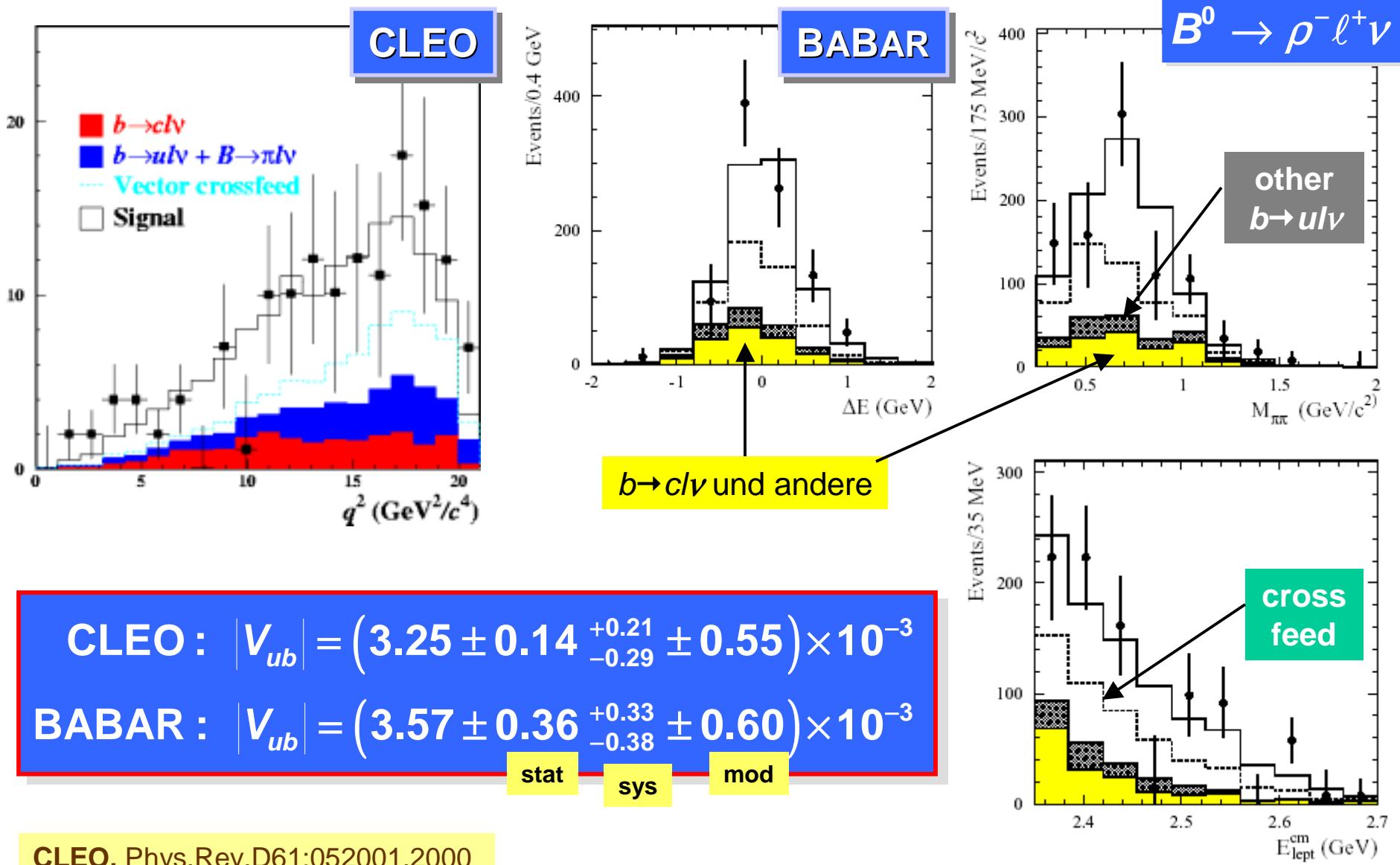
$$\text{BR}(B^0 \rightarrow h^- \ell^+ \nu) \propto |V_{ub}|^2 F_B^2(q^2)$$



Problem: form factor is model dependent



# $|V_{ub}|$ from exclusive Decays (II)



# $|V_{ub}|$ from inclusive Decays

CLEO

Suppression of the dominant charm background by cutting on the  $B \rightarrow X_u/\nu$  lepton momentum beyond the kinematic limit of  $B \rightarrow X_c/\nu$

**Problem:** strong model dependence of  $|V_{ub}|$

- Reduction of model dependence by using HQE and the “shape function” measured in  $B \rightarrow X_s \gamma$

CLEO, hep-ex/0202019

$$|V_{ub}| = (4.08 \pm 0.34 \pm 0.44 \pm 0.16 \pm 0.24) \times 10^{-3}$$

stat      fu       $1/m_b$       HQE

Possible “violation” of quark-hadron duality?

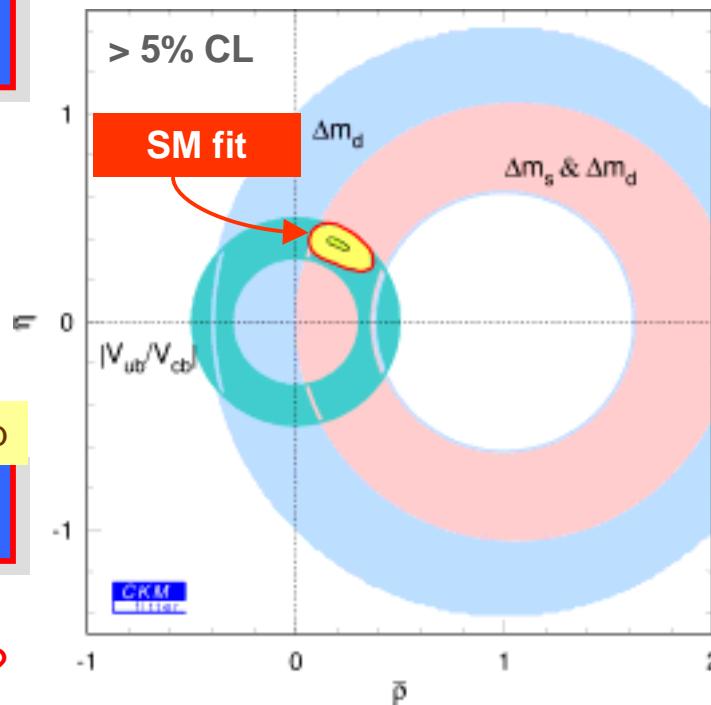
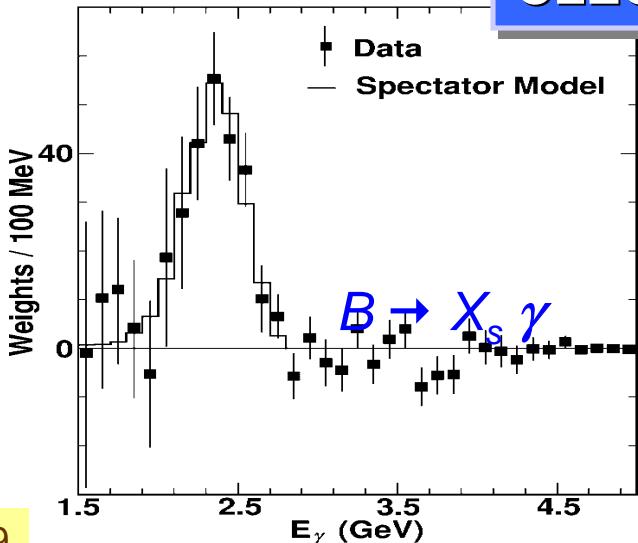
- Measurement of the whole spectrum (→ Theorie under control)  $B \rightarrow X_u/\nu$  (Neural Net for Signal)

LEP  $B$  Working group

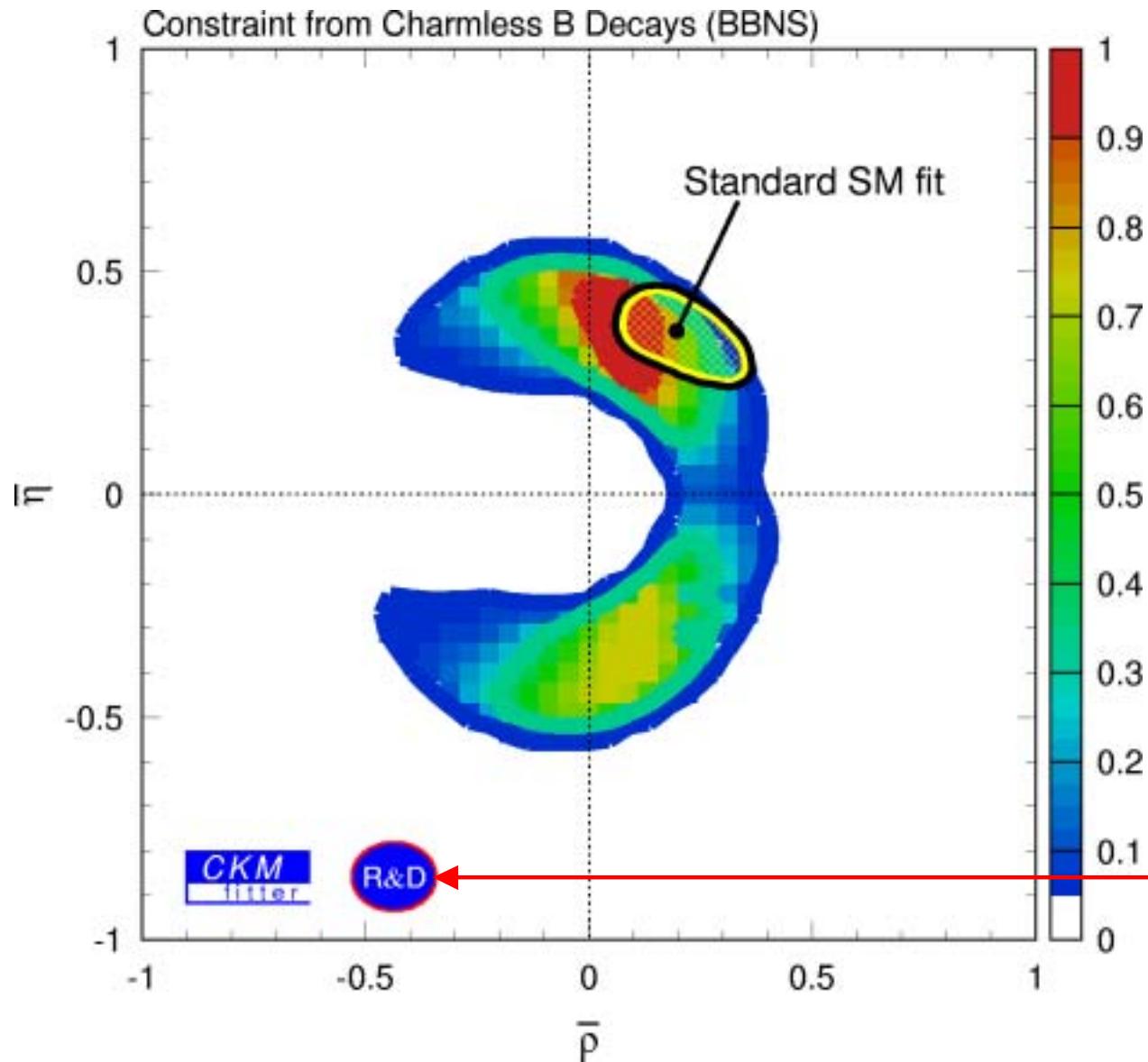
$$|V_{ub}| = (4.09^{+0.36 +0.42 +0.24}_{-0.39 -0.47 -0.26} \pm 0.01 \pm 0.17) \times 10^{-3}$$

exp     $b \rightarrow c$      $b \rightarrow u$        $\tau_b$       HQE

Knowledge of  $b \rightarrow c$  background, incl. measurement ?



# $\text{BR}(B \rightarrow \pi\pi / K\pi) & A_{\text{CP}} & \text{Theory (QCD FA)}$



Beneke, Buchalla, Neubert,  
Sachrajda (BBNS)  
Nucl.Phys.B606:245-321,2001

## Theoretical uncertainties:

- $m_s, m_c, \lambda_B, R_{\pi K}$
- Renorm. scale  $\mu$
- Gegenbauer moms:  
 $a_1(K), a_2(K), a_2(\pi)$
- $F(B \rightarrow \pi), f_B$
- $X_H, X_A$

This means:  
error estimation not settled yet !!!

# Frequentist Approach: R $\chi^2$

## Three main analysis steps:

AH, H. Lacker, S. Laplace, F. Le Diberder  
EPJ C21 (2001) 225, [hep-ph/0104062]

### Probing the SM

Test: “Goodness-of-fit”

- Evaluate global minimum  
 $\chi^2_{\min; y_{\text{mod}}} (y_{\text{mod-opt}})$
- Fake perfect agreement:  
 $x_{\text{exp-opt}} = x_{\text{theo}}(y_{\text{mod-opt}})$   
 generate  $x_{\text{exp}}$  using  $L_{\text{exp}}$
- Perform many toy fits:  
 $\chi^2_{\min-\text{toy}}(y_{\text{mod-opt}}) \rightarrow F(\chi^2_{\min-\text{toy}})$



$$CL(\text{SM}) \leq \int_{\chi^2 \geq \chi^2_{\min; y_{\text{mod}}}}^{\infty} F(\chi^2) d\chi^2$$

### Metrology

- Define:  
 $y_{\text{mod}} = \{a; \mu\}$   
 $= \{\rho, \eta, A, \lambda, y_{\text{QCD}}, \dots\}$
- Set Confidence Levels in  $\{a\}$  space, irrespective of the  $\mu$  values
- Fit with respect to  $\{\mu\}$   
 $\chi^2_{\min; \mu}(a) = \min_{\mu} \{\chi^2(a, \mu)\}$
- $\Delta\chi^2(a) = \chi^2_{\min; \mu}(a) - \chi^2_{\min; y_{\text{mod}}}$



$$CL(a) = \text{Prob}(\Delta\chi^2(a), N_{\text{dof}})$$

### Test New Physics

- If  $CL(\text{SM})$  good



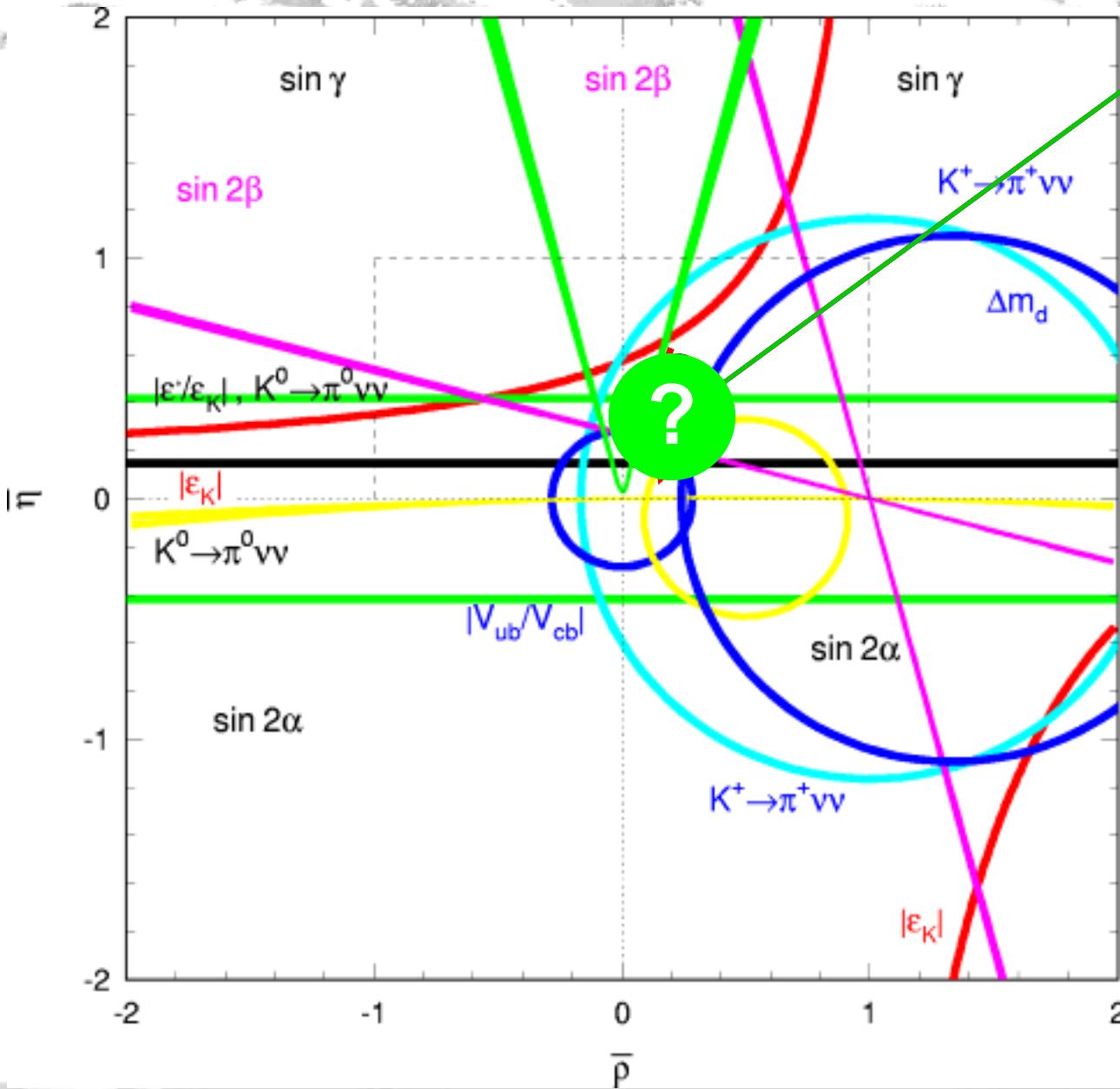
Obtain limits on New Physics parameters

- If  $CL(\text{SM})$  bad



Hint for New Physics ?!

And in the far future ?



... maybe we can establish new physics before the LHC finds it ???