

PQCD approach to hadronic B decays

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PQCD picture

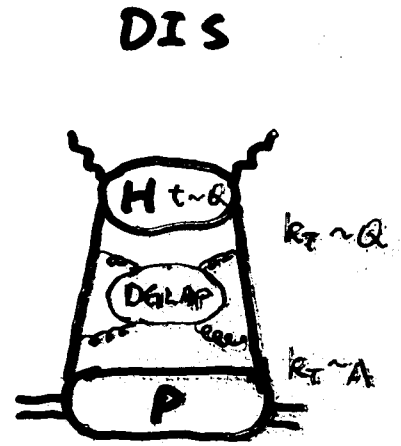
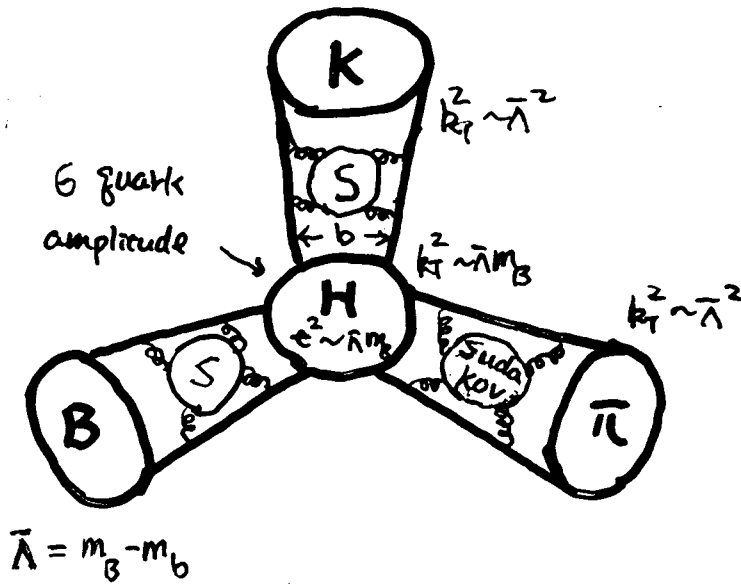
Br and CP of $B \rightarrow PP, VP, VV$ (charmless)

$B \rightarrow D^0 \pi^0$ and $\pi^+ \pi^0$

Three-body hadronic decays

Conclusion

PCPD picture



Sudakov \Leftrightarrow double log

DGLAP \Leftrightarrow single log

no real gluons during hard decay

has real gluons

\Rightarrow small color dipole is preferred (color singlet)

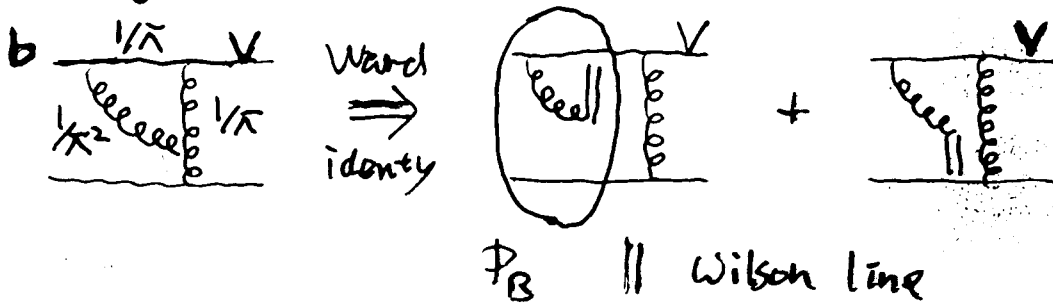
\Rightarrow Sudakov effect (no end-point singularity)

importance of $t^2 \sim \bar{\Lambda} m_B$

• power behavior of $F_B \pi$

$$F_B \pi \propto \frac{f_B f_\pi}{t^2} \quad f_B \sim \frac{\bar{\Lambda}^{3/2}}{m_B^{1/2}} \quad \left(\frac{\bar{\Lambda}}{m_B}\right)^{3/2} \Leftrightarrow \text{HQET}$$

• gauge invariance of Φ_B (nonlocal matrix element)



• dynamical enhancement of penguin

$$C_{4,6}(\bar{\Lambda} m_B) \sim 1.5 C_{4,6}(m_B)$$

inputs

- $\Phi_\pi, \Phi_K, \Phi_\rho, \Phi_{K^*}, \Phi_\omega, \Phi_\phi, \dots$ light meson DA

known from QCD sum rules

$$\Phi_\pi(x) = \frac{f_\pi}{2\sqrt{6}} x(1-x) \left[1 + a_2 C_2^{1/2}(1-2x) + a_4 C_4^{1/2}(1-2x) \right]$$

Gaugenbauer
↓

$$a_2 = 0.44 \pm 30\%$$

$$a_4 = 0.09 \pm 30\%$$

- Φ_B is unknown choose quark model

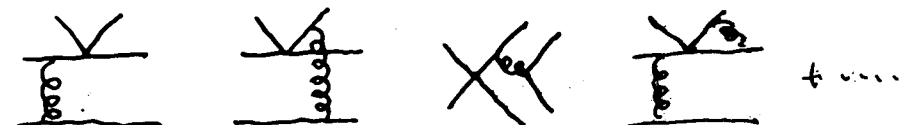
$$\Phi_B(x, b) = \frac{f_B}{2\sqrt{6}} N_B x^2 (1-x)^2 e^{-\frac{1}{2} \left(\frac{x m_B}{\omega_B} \right)^2} e^{-\frac{1}{2} (\omega_B b)^2}$$

$$\omega_B = 0.40 \pm 0.04 \text{ determined from } F^{B\pi}(0) = 0.30 \pm 0.03$$

- $m_0 = \frac{m_{\pi(K)}^2}{m_u + m_{d(s)}} = 1.4 \pm 0.4 \text{ GeV}$

calculable

- Wilson coefficients C

- hard part H 

- Sudakov factor S 

formula

$$\frac{1}{b} \rightarrow t$$

$$t \rightarrow M\omega$$

$$A \propto \int [dx] [db] \Phi(x, b) S(x, t, b) H(x, t) C(t)$$

hard scale t determined by vanishing $O(\alpha_s^2)$ corrections

$$\frac{1}{b} = \alpha x m_B$$

$$\geq \frac{1}{b}$$

α : $O(1)$ constant, $\frac{1}{b}$: factorization scale

$x m_B$: virtuality of internal particles

Quantity	CLEO	PQCD	BBNS
$\frac{Br(\pi^+\pi^-)}{Br(\pi^\pm K^\mp)}$	0.25 ± 0.10	<u>0.30 - 0.69</u>	0.5 - 1.9
$\frac{Br(\pi^\pm K^\mp)}{2Br(\pi^0 K^0)}$	0.59 ± 0.27	0.78 - 1.05	0.9 - 1.4
$\frac{2 Br(\pi^0 K^\pm)}{Br(\pi^\pm K^0)}$	1.27 ± 0.47	0.77 - 1.60	0.9 - 1.3
$\frac{\tau(B^+) Br(\pi^\mp K^\pm)}{\tau(B^0) Br(\pi^\pm K^0)}$	1.00 ± 0.30	0.70 - 1.45	0.6 - 1.0

Table 13: Ratios of CP-averaged rates in $B \rightarrow K\pi, \pi\pi$ decays with $\phi_3 = 80^\circ$, $R_b = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV.

ponguin is dynamical enhanced,

$Br(\pi K)$ increase

$\frac{Br(\pi\pi)}{Br(\pi K)}$ is lower

chiral enhancement ($m_0 \sim 3$ GeV) in BBNS?

use $B \rightarrow \phi K$ to test PQCD and QCDF (Chen, Keum)
(Li, Mishima)

$m_0 \rightarrow m_\phi \sim 1$ GeV, no chiral enhancement

$Br(B \rightarrow \phi K) \sim 10 \times 10^{-6}$, PQCD may be correct

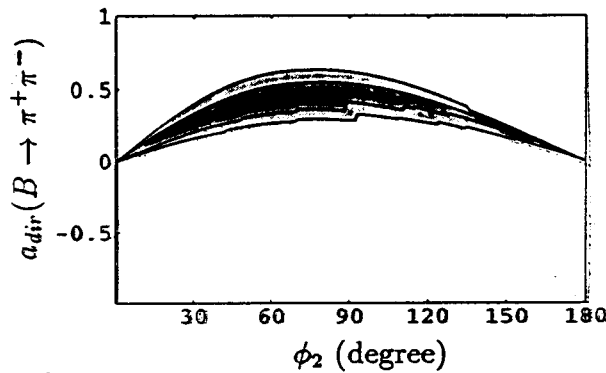
$Br(B \rightarrow \phi K) \sim 4 \times 10^{-6}$, QCDF may be correct
(Cheng et al., He et al.)

leading strong phases come from annihilation

we predict $A_{CP}(K\pi) < 0$, $A_{CP}(\pi^+\pi^-) > 0$ $|A_{CP}(K\pi)| < |A_{CP}(\pi\pi)|$

+ subleading
(vertex corr.
charm penguin)

$|A_{CP}|$ decreases
(see Brodsky, Gardner)



Belle: $+0.94^{+0.25}_{-0.31} \pm 0.09$

BaBar: $+0.02^{+0.29}_{-0.09} \pm 0.09$

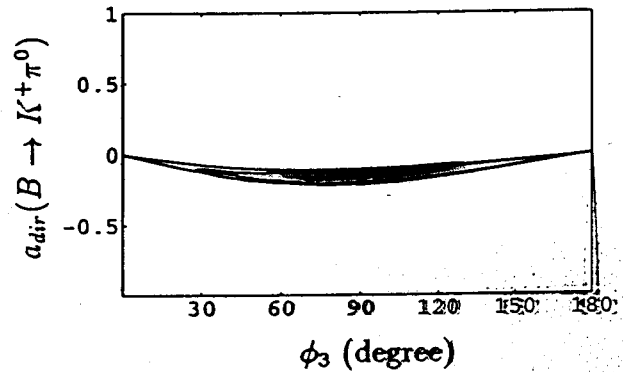
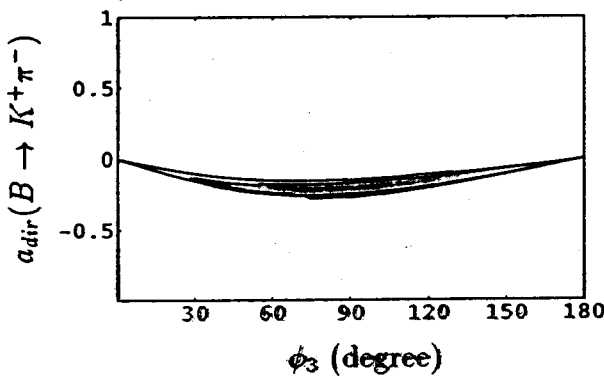
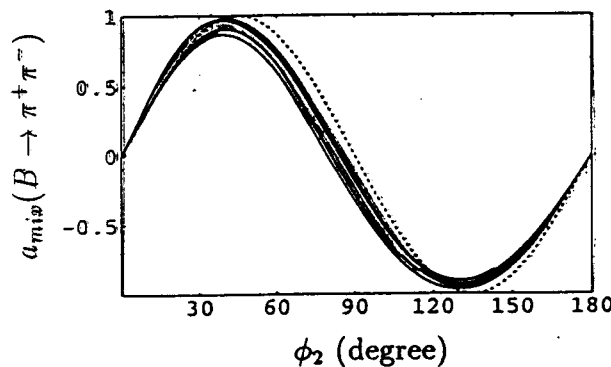


FIG. 3. Direct CP asymmetry for $B \rightarrow \pi\pi, K\pi$ decay modes. The central value of KM factors [19,20] gives the darker shaded regions, and the lighter shaded regions include the error of KM factors. $a_{dir}(\pi^+\pi^0)$ is almost zero for any ϕ_2 . $a_{dir}(K^0\pi^+)$ is almost zero for any ϕ_3 . $a_{dir}(K^0\pi^0)$ becomes maximum at $\phi_3 = 90^\circ$, $a_{dir}(K^0\pi^0) = -0.04$.



(Sanda, Uka;)

FIG. 4. Mixing induced CP asymmetry for $B \rightarrow \pi^+\pi^-$. The difference from the dotted line ($\sin 2\phi_2$) shows sizeable penguin pollution.

BaBar

consistent with
PQCD

Mode	Signal yield	$B (10^{-6})$
$\pi^+\pi^-$ [6]	41 ± 10	$4.1 \pm 1.0 \pm 0.7$
$K^+\pi^-$	169 ± 17	$16.7 \pm 1.6 \pm 1.3$
K^+K^-	$8.2^{+7.2}_{-6.4}$	< 2.5 (90% C.L.)
$\pi^+\pi^0$	37 ± 14	< 9.6 (90% C.L.)
$K^+\pi^0$	75 ± 14	$10.8^{+2.1}_{-1.9} \pm 1.0$
$K^0\pi^+$	59^{+11}_{-10}	$18.2^{+3.3}_{-3.0} \pm 2.0$
\bar{K}^0K^+	$-4.1^{+4.5}_{-3.8}$	< 2.4 (90% C.L.)
$K^0\pi^0$	$17.9^{+6.8}_{-5.8}$	$8.2^{+3.1}_{-2.7} \pm 1.2$
$K^0\bar{K}^0$ [7]	$3.4^{+3.4}_{-2.4}$	< 7.3 (90% C.L.)
ϕK^+ [8]	$31.4^{+6.7}_{-5.9}$	$7.7^{+1.6}_{-1.4} \pm 0.8$
ϕK^0	$10.8^{+4.1}_{-3.3}$	$8.1^{+3.1}_{-2.5} \pm 0.8$
ϕK^{*+}	—	$9.7^{+4.2}_{-3.4} \pm 1.7$
$\phi K^{*+}_{K^+\pi^0}$	$7.1^{+4.3}_{-3.4}$	$12.8^{+7.7}_{-6.1} \pm 3.2$
$\phi K^{*+}_{K^0\pi^+}$	$4.4^{+2.7}_{-2.0}$	$8.0^{+5.0}_{-3.7} \pm 1.3$
ϕK^{*0}	$20.8^{+5.9}_{-5.1}$	$8.7^{+2.5}_{-2.1} \pm 1.1$
$\phi\pi^+$	$0.9^{+2.1}_{-0.9}$	< 1.4 (90% C.L.)
ωK^+ [9]	$6.4^{+5.6}_{-4.4}$	< 4 (90% C.L.)
ωK^0	$8.1^{+4.6}_{-3.6}$	< 13 (90% C.L.)
$\omega\pi^+$	$27.6^{+8.8}_{-7.7}$	$6.6^{+2.1}_{-1.8} \pm 0.7$
$\omega\pi^0$	$-0.9^{+5.0}_{-3.2}$	< 3 (90% C.L.)
ηK^{*0} [10]	20.5 ± 6.0	$19.8^{+6.5}_{-5.6} \pm 1.7$
ηK^{*+}	14.3 ± 6.6	< 33.9 (90% C.L.)
$\eta' K^+$ [9]	—	$70 \pm 8 \pm 5$
$\eta'_{\eta\pi\pi} K^+$	$49.5^{+8.1}_{-7.3}$	63^{+10}_{-9}
$\eta'_{\eta\gamma} K^+$	$87.6^{+13.4}_{-12.5}$	80^{+12}_{-11}
$\eta' K^0$	—	$42^{+13}_{-11} \pm 4$
$\eta'_{\eta\pi\pi} K^0$	$6.3^{+3.3}_{-2.5}$	28^{+15}_{-11}
$\eta'_{\eta\gamma} K^0$	$20.8^{+7.4}_{-6.5}$	61^{+22}_{-19}
$\eta'\pi^+$	—	< 12 (90% C.L.)
$\eta'_{\eta\pi\pi} \pi^+$	$5.7^{+3.5}_{-2.8}$	$7.1^{+4.8}_{-3.5}$
$\eta'_{\eta\gamma} \pi^+$	$-0.9^{+7.8}_{-6.2}$	$-0.7^{+6.7}_{-5.3}$
$a_0^\pm \pi^\mp$ [12]	$18.1^{+8.7}_{-7.4}$	< 11.5 (90% C.L.)
$\rho^\pm \pi^\mp$ [13]	89 ± 16	$28.9 \pm 5.4 \pm 4.3$
$\rho^0 \pi^0$	6.1 ± 5.8	< 10.6 (90% C.L.)
$K^{*0} \pi^+$ [14]	34.8 ± 7.6	$15.5 \pm 3.4 \pm 1.5$

gluon content ?

(Melic)

(Chen)

(Chen, Sandha)

consistent

(Lu, Yang)

consistent

Mode	A_{CP}
$K^* \gamma$ [3]	$-0.044 \pm 0.076 \pm 0.012$
$K^+\pi^-$ [15]	$-0.07 \pm 0.08 \pm 0.04$
$K^+\pi^0$ [6]	$0.00 \pm 0.18 \pm 0.04$
$K^0\pi^+$ [6]	$-0.21 \pm 0.18 \pm 0.03$
ϕK^+ [8]	$-0.05 \pm 0.20 \pm 0.03$
ϕK^{*+} [8]	$-0.43^{+0.36}_{-0.30} \pm 0.06$
ϕK^{*0} [8]	$0.00 \pm 0.27 \pm 0.03$
$\omega\pi^+$ [16]	$-0.01^{+0.29}_{-0.31} \pm 0.03$
$\eta' K^+$ [16]	$-0.11 \pm 0.11 \pm 0.02$
$\rho^\pm \pi^\mp$ [13]	$-0.04 \pm 0.18 \pm 0.02$

Results of $B \rightarrow K\pi, \pi\pi$ Belle

in PQCD

mode	signal yield	Σ	$B(10^{-5})$	
• $K^+\pi^-$	218 ± 18	16.4	$2.18 \pm 0.18 \pm 0.15$	1.55 ± 0.30
$K^+\pi^0$	58 ± 11	6.3	$1.25 \pm 0.24 \pm 0.12$	
$K^0\pi^+$	66 ± 10	8.2	$1.88 \pm 0.30 \pm 0.15$	
$K^0\pi^0$	19 ± 8	2.7	$0.77 \pm 0.32 \pm 0.16$	
$\pi^+\pi^-$	51 ± 11	5.4	$0.51 \pm 0.11 \pm 0.04$	
• $\pi^+\pi^0$	36 ± 11	3.5	$0.70 \pm 0.22 \pm 0.08$	0.37 ± 0.15
$\pi^0\pi^0$	12 ± 6	2.2	< 0.56	
K^+K^-	0 ± 2	0	< 0.05	
K^+K^0	0 ± 2	0	< 0.38	
K^0K^0	1 ± 3	0	< 1.3	

- ωK^-
 - $2B(K^+\pi^0)/B(K^0\pi^+) = 1.33 \pm 0.33 \pm 0.14$ ~ 0.32
 - $B(K^+\pi^-)/2B(K^0\pi^0) = 1.43 \pm 0.60 \pm 0.28$
 - $\tau_+/\tau_0 B(K^+\pi^-)/B(K^0\pi^+) = 1.27 \pm 0.23 \pm 0.09 \pm 0.04$
 - $B(\pi^+\pi^-)/B(K^+\pi^-) = 0.24 \pm 0.06 \pm 0.02$
- $\tau_+/\tau_0 B(\pi^+\pi^-)/2B(\pi^+\pi^0) = 0.40 \pm 0.15 \pm 0.05 \pm 0.01$
- $\tau_+/\tau_0 B(\pi^0\pi^0)/B(\pi^+\pi^0) < 0.77$ (90% C.L.)

mode	$N(B)$	$N(B)$	A_{CP}	90% C.L.
$K^+\pi^-$	103 ± 12	115 ± 14	$-0.06 \pm 0.08 \pm 0.01$	$-0.20:0.09$
$K^+\pi^0$	28 ± 8	30 ± 8	$-0.04 \pm 0.19 \pm 0.03$	$-0.39:0.30$
• $K_S^0\pi^+$	49 ± 8	18 ± 6	$0.46 \pm 0.15 \pm 0.02$	$0.18:0.73$
$\pi^+\pi^0$	24 ± 8	13 ± 7	$0.31 \pm 0.31 \pm 0.05$	$-0.25:0.89$

Preliminary

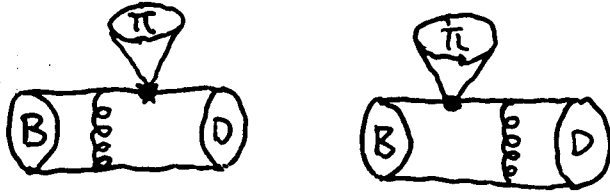
- No clear evidence for partial rate asymmetries

(Kurimoto, Li, Lu)

$B \rightarrow D\pi$ ($|a_2|/a_1$)

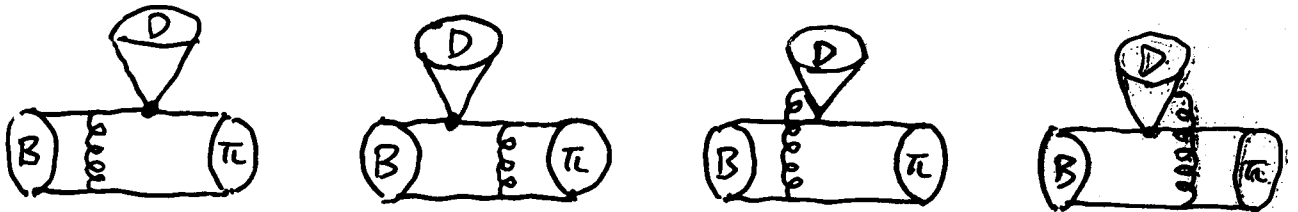
$$\phi_0(x) = \frac{f_D}{2\sqrt{6}} x(1-x) [1 + 0.3(1-2x)]$$

a_1 :



dominate

a_2 :



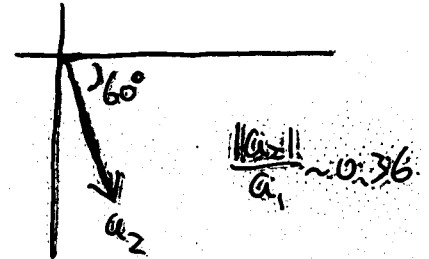
small Wilson coeff.

no soft cancellation

$B(B^- \rightarrow D^0 \pi^-) \propto |a_1 + a_2|^2 \sim 1.5$

$B(B^0 \rightarrow D^+ \pi^-) \propto |a_1|^2 \sim 1$

$B(B^0 \rightarrow D^0 \pi^0) \propto |a_2|^2/2 \sim 0.13/2$



fact: nonfact = $1 : \frac{m_D}{m_B} \xrightarrow{m_B \rightarrow m_\pi} 1 : \frac{\bar{\Lambda}}{m_B}$
 \parallel
 0.3

	PCAD (10^{-3})	Exp (10^{-3})
$B(D^0 \pi^-)$	~ 4.8	5.3 ± 0.5
$B(D^+ \pi^-)$	~ 3.0	3.0 ± 0.4
$B(D^0 \pi^0)$	~ 0.2	0.3 ± 0.1 (Belle)
(preliminary)		0.27 ± 0.05 (CLEO)

$$r = \frac{B(\pi^+\pi^-)}{2B(\pi^+\pi^0)} \sim 0.4 \quad (\text{Belle})$$

is difficult to understand

a_2 is small in $B \rightarrow \pi\pi$

the Wilson coefficient $C_1 + \frac{C_2}{N_c}$ vanishes
nonfactorizable contributions cancel

PQCD prefers $r = \frac{|a_1|^2}{2|a_1 + a_2|^2/2} \leq 1$

if data persist, PQCD can not explain r

\Rightarrow something new

final state interaction can lower r (Hou et al)

but the upper bound of $B(B \rightarrow K^+K^-)$ puts a strong

constraint on FSI

if Belle's bound is taken, $B(B \rightarrow K^+K^-) < 5 \times 10^{-7}$
 $\lesssim 1 \times 10^{-7}$ in PQCD

FSI is not sufficient to explain r

BaBar's result?

Three-body hadronic decays

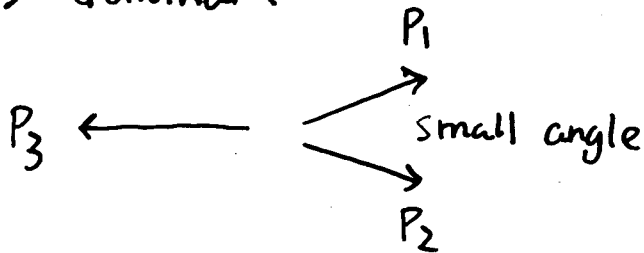
• kinematics of $B \rightarrow P_1 P_2 P_3$

all P_i carry $O(m_B)$ momentum

\Rightarrow power suppressed

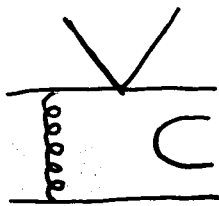
one carries $O(\bar{\Lambda})$, two carry $O(m_B)$

\Rightarrow dominant

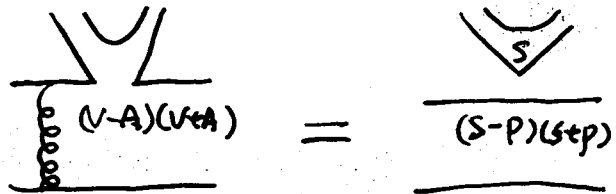


invariant mass of $P_1 + P_2$
 $\sim O(\bar{\Lambda} m_B)$

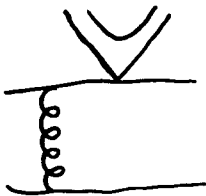
• 4 topologies



$$I \propto \frac{1}{\bar{\Lambda} m_B} \frac{1}{\bar{\Lambda} m_B}$$

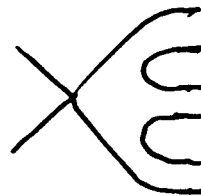


$$II E \propto \frac{1}{\bar{\Lambda} m_B} \frac{1}{\bar{\Lambda} m_B} \frac{m_0}{m_B}$$



$$III A \propto \frac{1}{\bar{\Lambda} m_B} \frac{1}{\bar{\Lambda} m_B} \frac{\bar{\Lambda}}{m_B}$$

helicity
suppression
 \downarrow



$$IV \propto \frac{1}{\bar{\Lambda} m_B} \frac{1}{m_B^2}$$

$$I : II E : III A : IV = 1 : \frac{m_0}{m_B} : \frac{\bar{\Lambda}}{m_B} : \frac{\bar{\Lambda}}{m_B}$$

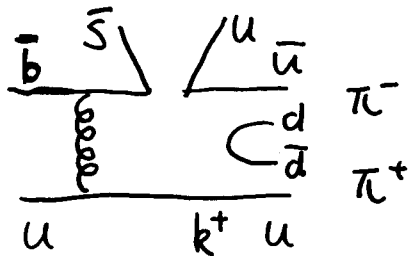
I dominates in the heavy quark limit.

II E may be important for physical value of m_B

2π distribution amplitude (DA)

concentrate on leading topology I,

still too many diagrams $2 \times 2 \times 8 = 32$



2π system has the invariant

$$\text{mass } \omega^2 \sim O(\bar{\Lambda} m_B)$$

$$\omega^2 = (P_1 + P_2)^2 \equiv P^2$$

P_k in minus direction, 2π mainly in plus dir.
(-z)

$$P^+ \sim O(m_B), \quad P^- \sim O(\bar{\Lambda}), \quad |\vec{P}_T| = |\vec{P}_{1T}| = |\vec{P}_{2T}| \sim O(\sqrt{\bar{\Lambda} m_B})$$

$P^+ \gg |\vec{P}_T| \gg P^- \Leftrightarrow$ similar to the collinear configuration

$B \rightarrow \pi$ at fast recoil, $P_\pi^+ \sim O(m_B), P_\pi^- \sim O(\frac{\bar{\Lambda}^2}{m_B}), |\vec{P}_{\pi T}| \sim O(\bar{\Lambda})$

$$P_\pi^2 \sim O(\bar{\Lambda}^2), \quad P_\pi^+ \gg |\vec{P}_{\pi T}| \gg P_\pi^- \Rightarrow \text{pion DA}$$

hint: can introduce 2π DA twist-3 $\left\{ \begin{array}{l} \delta^{\mu\nu} \\ \mathbf{I} \end{array} \right.$

$$\Phi_{2\pi}(z, \zeta, \omega^2) = \int \frac{dy^-}{2\pi} e^{-izp^+ y^-} \langle \pi^+(P_1) \pi^-(P_2) | \bar{q}(y) \gamma^+ q(0) | 0 \rangle$$

parton momentum fraction $z = \frac{k^+}{p^+}$, $\zeta = \frac{P_1^+}{p^+}$ (pion mem fraction)

all other kinematic variables

$$P^- = \frac{\omega^2}{2p^+}, \quad P_1^- = \frac{(1-\zeta)\omega^2}{2p^+}, \quad P_2^- = \zeta(1-\zeta)\omega^2$$

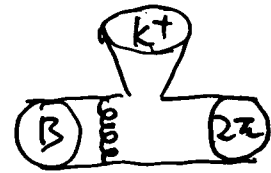
$$P_2^+ = (1-\zeta)p^+, \quad P_1^+ = \frac{\zeta\omega^2}{2p^+}$$

All four topologies can be calculated.

End-point singularity

with 2π DA, 3-body decays reduce to 2-body decays.

same number of diagrams



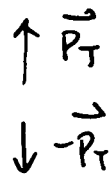
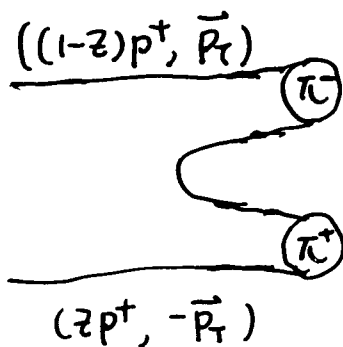
also the same end-point singularity

$$\int dx dz \frac{\phi_B(x) \phi_{2\pi}(z, \zeta, \omega^2)}{x z^2 m_B^2} \quad \text{diverges like} \quad \int_0^1 \frac{dz}{z}$$

in 2-body decays, introduce parton k_T and Sudakov suppression (k_T distribution)

$$\frac{1}{x z^2 m_B^2} \rightarrow \frac{S(k_T)}{(x z m_B^2 + (k_T - k_{3T})^2)^2} \quad \text{size of pion} \Leftrightarrow k_T^2$$

in 3-body decays, the invariant mass of the 2π system provides a natural smearing



$$\frac{1}{x z^2 m_B^2} \rightarrow \frac{1}{(x z m_B^2 + P_T^2)(z m_B^2 + P_T^2)}$$

$$\text{size of } 2\pi \text{ system} \Leftrightarrow P_T^2 = 3(1-z)\omega^2$$

$$\Gamma \propto \int d\gamma d\zeta \ln \zeta(1-\zeta) \gamma \quad \text{finite!} \quad \gamma = \frac{\omega^2}{m_B^2}$$

no end-point singularity in the leading contribution to

3-body decays

formalism applicable to the whole kinematic range

$$0 < \omega^2 < m_B^2$$

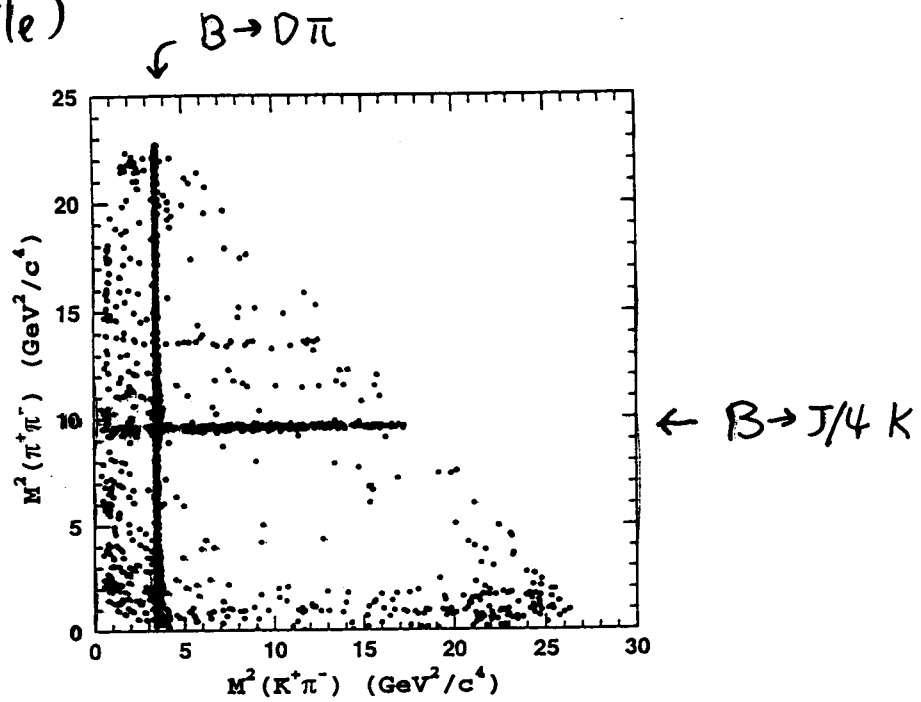


FIG. 1: The Dalitz plot for $B^+ \rightarrow K^+ \pi^+ \pi^-$ candidates from the B signal region.

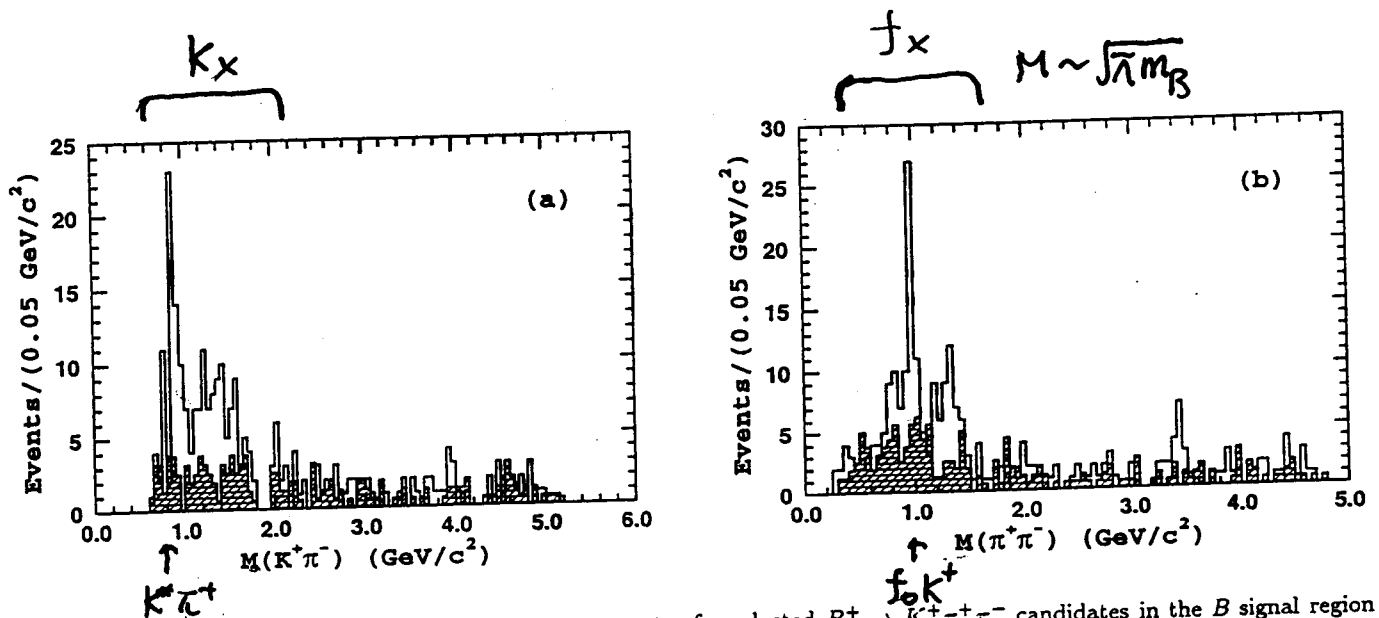


FIG. 3: The (a) $K^+ \pi^-$ and (b) $\pi^+ \pi^-$ invariant mass spectra for selected $B^+ \rightarrow K^+ \pi^+ \pi^-$ candidates in the B signal region (open histograms) and for background events in the ΔE sidebands (hatched histograms).

nonresonant contributions strongly overlap with the resonant $K_x \pi^+$ and $f_x K^+$ channels.

they have similar mass spectra

the rapid decrease of nonresonant contributions at

large $M(K^+ \pi^-, \pi^+ \pi^-)$ is due to $1/m_B$ suppression

no clear evidence for $B \rightarrow K^+ G$

Preliminary Results (Brodsky, Chen, Li)

$$\text{Belle: } \text{Br}(B^+ \rightarrow K^+ \pi^+ \pi^-) = (55.6 \pm 5.8 \pm 7.7) \times 10^{-6}$$

$$\text{Br}(B^+ \rightarrow K^+ K^+ K^-) = (35.3 \pm 3.7 \pm 4.5) \times 10^{-6}$$

resonant $B^+ \rightarrow K_S^* \pi^+$ $\sim 12.9^{+2.8+1.4+2.3}_{-2.6-1.4-4.5}$

through $B^+ \rightarrow K^+ f_0$ $\sim 9.6^{+2.5+1.5+3.4}_{-2.3-1.5-0.8}$

$$B^+ \rightarrow K_x \pi^+ \sim 14.5^{+3.5+1.8+3.3}_{-3.3-1.8-6.5}$$

$$B^+ \rightarrow K^+ f_x \sim 11.1^{+3.4+1.4+7.2}_{-3.1-1.4-2.9}$$

nonresonant contribution $3 \sim 10$

Dalitz plot of $B^+ \rightarrow K^+ K^+ K^-$ indicates stronger

nonresonant contribution

Conclusion

- leading PQCD picture : $\Phi_B \otimes H \otimes \Phi_{h_1} \otimes \Phi_{h_2}$
- PQCD has the correct power behavior for form factors
- k_T factorization is gauge invariant (to appear)
- PQCD can explain most of data of Br and ϕ penguin dynamical enhancement and annihilation play important roles
- PQCD predicts helicity amplitudes and phases among them in $B \rightarrow VV$ unambiguously ($B \rightarrow \phi K^*$ by Chen, Keum, Li)
- PQCD can explain $B \rightarrow D\pi$, $a_2/a_1 \sim 0.4 e^{-i60^\circ}$
- $\frac{B(\pi^+\pi^-)}{2B(\pi^+\pi^0)}$ is tough, which violates power counting rules
- PQCD can be extended to three-body decays
- the formalism is simpler and has no end-point singularity
- nonresonant contribution can be calculated by introducing 2-meson distribution amplitudes
- need to investigate subleading contributions (Fukushima, Sarda, to appear)