The Soft-Collinear Effective Field Theory

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Outline

- Motivation
- Soft-Collinear Effective Theory
- Power counting, Symmetries, and Factorization
- Factorization for $B \to X_s \gamma$ and $B \to D\pi$
- Sudakov Logarithms, Reparameterization Invariance
- Large N_c versus Large E Factorization
- Conclusions

Based on work with:

C. Bauer, D. Pirjol,

S. Fleming, I. Rothstein

T. Mehen, A. Manohar, B. Grinstein

Motivation

To understand hadronic uncertainties we need to separate short ($p \sim Q$) and long ($p \sim \Lambda_{\rm QCD}$) distance contributions [this is Factorization]

Effective Field Theory: Useful for separating physics at different momentum scales

HQET, $m_B \gg \Lambda_{\rm QCD}$

- systematic expansion in $\Lambda_{\rm QCD}/m_Q$
- new symmetries for $m_Q \to \infty$
- universal non-perturbative parameters

What about processes with energetic particles, $Q \gg \Lambda_{\rm QCD}$?

$B \to \pi e\nu$, $B \to \rho e\nu$, $B \to K^*\gamma$, $B \to K e^+e^-$,
$B \to D\pi$, $B \to \pi\pi$, $B \to K\pi$,
$B \to X_u e\nu$, $B \to X_s\gamma$, ...
DIS, Drell Yan, $\gamma^*\gamma \to \pi^0$, $\gamma^*\pi \to \pi$, ...

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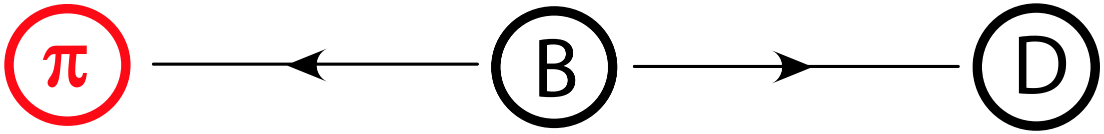
For $Q \gg \Lambda_{\rm QCD}$ the SCET helps to:

- simplify power counting in $\lambda = \Lambda_{\rm QCD}/Q$
- make symmetries explicit
- sum IR logarithms
- understand factorization in a universal way

Kinematics

eg.



Pion has: $p_\pi^\mu = (2.310\,\mathrm{GeV}, 0, 0, -2.306\,\mathrm{GeV})$

$$= Q\, n^\mu \qquad n^\mu = (1,0,0,-1)\,, \qquad Q \gg \Lambda_{QCD}, m_\pi$$

Let $p^+ = n\cdot p$, $p^- = \bar{n}\cdot p$ then pion constituents:

$$(p^+, p^-, p^\perp) \sim \left(\frac{\Lambda_{QCD}^2}{Q}, Q, \Lambda_{QCD}\right)$$

$$\sim Q(\lambda^2, 1, \lambda) \qquad \lambda \ll 1$$

Soft-Collinear Effective Theory

Bauer, Fleming, Luke
B.F.P.S., B.S., B.P.S.

Introduce fields for infrared degrees of freedom

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_{n,p},\ A^\mu_{n,q}$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_{s,p},\ A^\mu_{s,q}$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A^\mu_{us}$

Typically $\lambda = \frac{\Lambda_{\rm QCD}}{Q}$ or $\lambda = \sqrt{\frac{\Lambda_{QCD}}{Q}}$ and B is soft or usoft

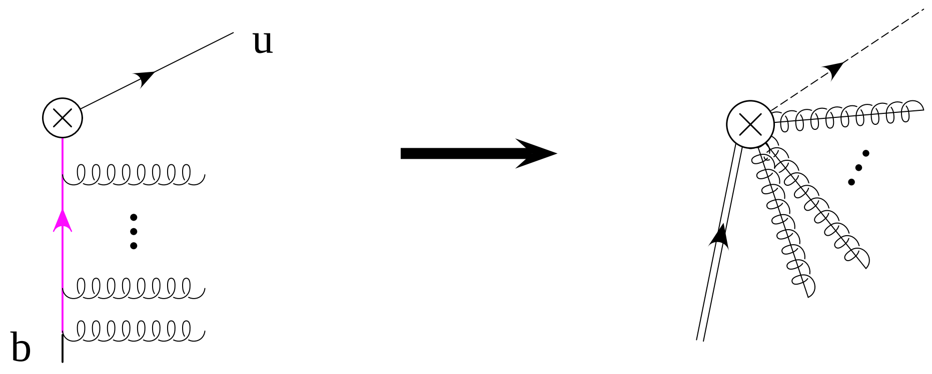
Offshell modes with $p^2 \gg Q^2\lambda^2$ are integrated out

- SCET operators and diagrams are homogeneous in λ
- $\mathcal{L}^{(0)}$ and $\mathcal{O}^{(0)}$ $\longrightarrow$ leading order results

Collinear Gauge Invariance

integrate out
offshell quarks

$\bar{n}\cdot A_{n,q} \sim 1$
$\bar{n}\cdot q_i \sim 1$



Consider matching for heavy-to-light current, $\bar{u}\,\Gamma\,b$

no gluons: $\bar{\xi}_{n,p}\,\Gamma\,h_v$

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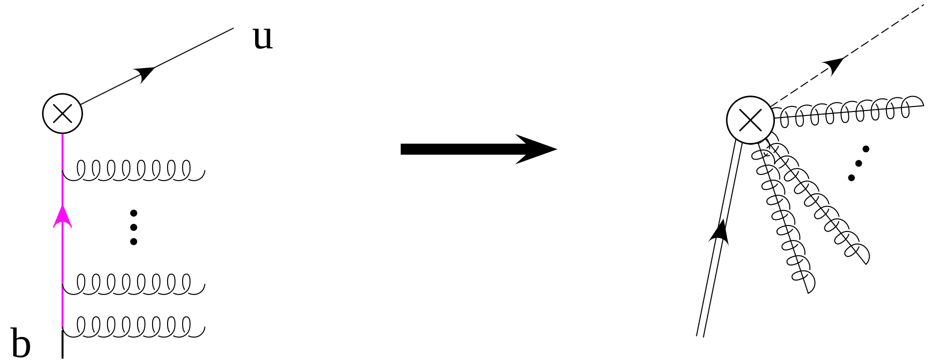

Consider matching for heavy-to-light current, $\bar{u}\,\Gamma\, b$

one-gluon: $\bar{\xi}_{n,p}\,(-g)\,\frac{\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\,\Gamma\, h_v$

Collinear Gauge Invariance

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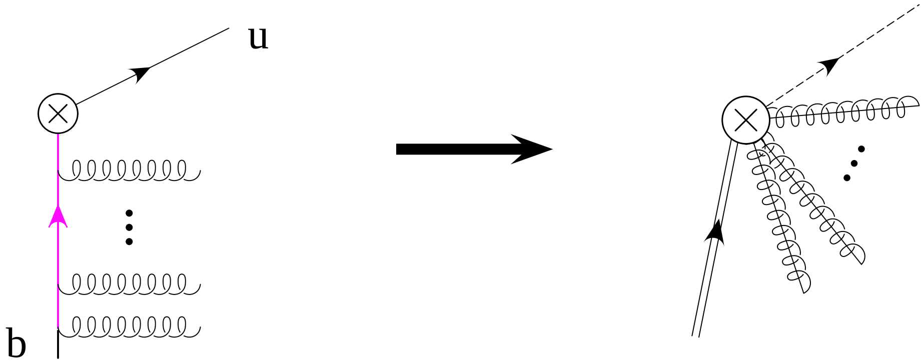

Consider matching for heavy-to-light current, $\bar{u}\,\Gamma\,b$

all orders: $\bar{\xi}_{n,p}\, W\, \Gamma\, h_v$, W a Wilson line

- form of the current is protected by Collinear Gauge Invariance

Collinear Gauge Invariance

integrate out
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$$\bar{n}\cdot A_{n,q} \sim 1$$
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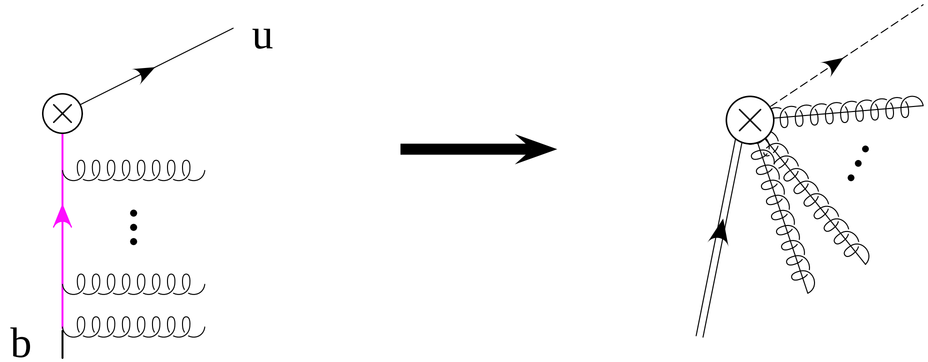


Consider matching for heavy-to-light current, $\bar{u}\,\Gamma\,b$

all orders: $\bar{\xi}_{n,p}\,W\,\Gamma\,h_v$, W a Wilson line

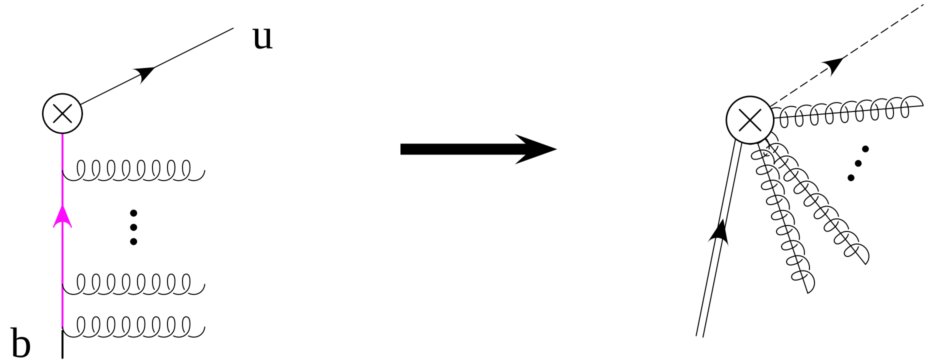
- form of the current is protected by Collinear Gauge Invariance
- also constrains form of hard loop corrections:

$$C(\bar{\mathcal{P}})\,\bar{\xi}_{n,p}\,W\,\Gamma\,h_v\,,\qquad \bar{\mathcal{P}}$$ a label operator

Collinear Gauge Invariance

integrate out
offshell quarks

$$\bar{n}\cdot A_{n,q} \sim 1$$
$$\bar{n}\cdot q_i \sim 1$$



Consider matching for heavy-to-light current, $\bar{u}\,\Gamma\, b$

all orders: $\bar{\xi}_{n,p}\, W\, \Gamma\, h_v$, W a Wilson line

- form of the current is protected by Collinear Gauge Invariance
- and the form of the collinear Lagrangian:

$$\mathcal{L} = \bar{\xi}_{n,p'} \left\{ n\cdot iD^{us} + g n\cdot A_{n,q} + \left(\not{\mathcal{P}}_\perp + g \not{A}^\perp_{n,q} \right) W \frac{1}{\bar{\mathcal{P}}} W^\dagger \left(\not{\mathcal{P}}_\perp + g \not{A}^\perp_{n,q'} \right) \right\} \frac{\not{\bar{n}}}{2}\, \xi_{n,p}$$

Factorization

DIS $F_1(x,Q^2) = \frac{1}{x}\int_x^1 d\xi\ H(\xi/x, Q, \mu)\ f_{i/p}(\xi,\mu)$

$[Q^2 \gg \Lambda^2_{QCD}]$

$B \to D\pi$ $\langle D\pi|\bar{d}u\bar{c}b|B\rangle = N F^{B\to D}(0)\ \int_0^1 d\xi\ T(\xi, Q, \mu)\ \phi_\pi(\xi,\mu)$

$[Q = \{m_Q, E_\pi\} \gg \Lambda_{QCD}]$

Politzer-Wise, B.B.N.S.

Factorization

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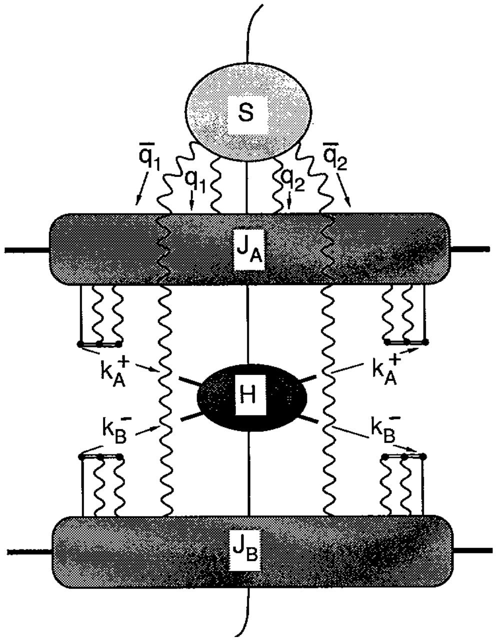
Politzer-Wise, B.B.N.S.

Traditional proofs: analyze diagrams



Collins,Soper,Sterman

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$[Q = \{m_Q, E_\pi\} \gg \Lambda_{QCD}]$ Politzer-Wise, B.B.N.S.

In SCET:

- replaced by power counting and operator analysis
- eg. decoupling of ultrasoft gluons by field redefinitions

$$\xi_{n,p} \;=\; Y_n\, \xi^{(0)}_{n,p}\,, \qquad A_{n,q} = Y_n\, A^{(0)}_{n,q}\, Y^\dagger_n$$

where $Y_n = \mathrm{P}\exp\left[ig\int_{-\infty}^x ds\, n\cdot A^{us}(ns)\right]$

- integrating out $p^2 \simeq Q^2\lambda$ modes gives soft Wilson line, S_n

Applications

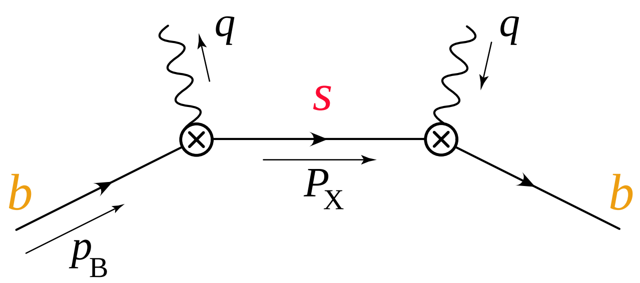
Hard Scattering: Factorization proofs for "classic" processes are simplified

B.F.P.R.S.

- $\gamma^*\gamma \to \pi$, $\gamma^*\pi \to \pi$
- $e^- p \to e^- X$, $p\bar{p} \to X\ell^+\ell^-$, $e^- p \to \gamma^{(*)} p'$

$B \to X_s \gamma$ near endpoint

Optical Thm: $\Gamma \sim \mathrm{Im} \int d^4x \, e^{-iq\cdot x} \langle B | T\{J^\dagger_\mu(x) J^\mu(0)\} | B \rangle$



	$P_X^2 = m_B(m_B - 2E_\gamma)$
standard OPE	$\sim m_B^2$
endpoint region	$\sim m_B \Lambda_{QCD}$
resonance region	$\sim \Lambda_{QCD}^2$

For endpoint: $E_\gamma \gtrsim 2.2\,\mathrm{GeV}$, X_s collinear, B usoft, $\lambda = \sqrt{\frac{\Lambda_{QCD}}{m_B}}$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu) \int_{2E_\gamma - m_b}^{\bar\Lambda} dk^+ \, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma, \mu)$$

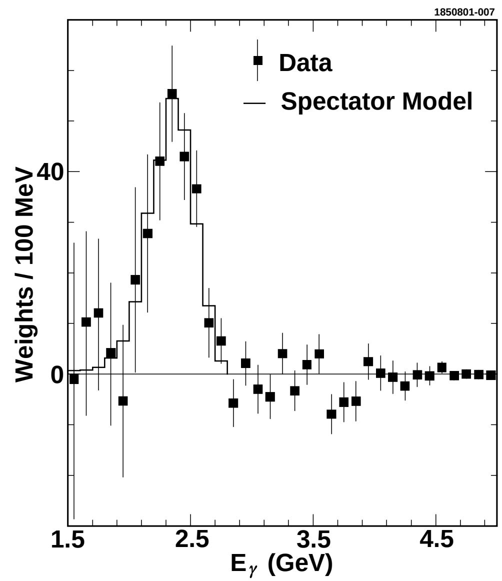
Korchemsky, Sterman ('94)

SCET gives direct all orders proof of this result (B.P.S.)

$B \to X_s\gamma$ near endpoint

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu)\int_{2E_\gamma - m_b}^{\bar\Lambda} dk^+\, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma,\mu)$$

CLEO '01



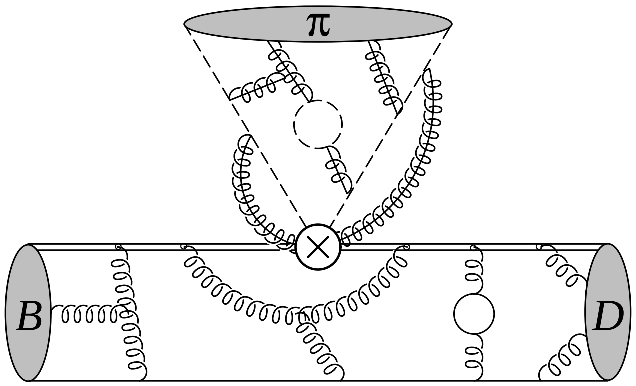
Useful for extracting $\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2}$ using $B \to X_s\gamma$ and $B \to X_u e\nu$ spectrums

Neubert, Bigi et al.,
Leibovich, Low, Rothstein

Factorization for $B \to D\pi$

History:

Naive Factorization:	$\langle O \rangle = N F^{B\to D} f_\pi$
Bjorken ('89)	Color Transparency
Dugan & Grinstein ('91)	LEET, $\mathcal{L} = \bar{\xi}\, in\cdot D\, \xi$
Generalized Factorization:	$\langle O \rangle = N F^{B\to D} \int dx\, T(x) \phi_\pi(x)\, f_\pi$
Politzer & Wise ('91)	Proposal
Beneke et al. ('00)	Two loop proof
Bauer, Pirjol, I.S. ('01)	All orders proof



$Q = m_b, m_c, E_\pi \gg \Lambda_{QCD}$

B, D are soft, π collinear

Other Applications

Reparameterization Invariance: small changes in n, $\bar{n}$

Chay and Kim

Manohar, Mehen, Pirjol, Stewart

- constrains power suppressed collinear operators

Sudakov Logarithms in SCET:

- solving RGE's sums Sudakov double logarithms

e.g.: $B \to X_s \gamma$ and $B \to X_u e \nu$ B.F.L. and B.F.P.S.

$\Upsilon \to X \gamma$ Bauer,Chiang,Fleming,Leibovich,Low

– end of SCET part –

Large E vs. Large N_c

Proof Applies to: $\bar{B}^0 \to D^+\pi^-$, $B^- \to D^0\pi^-$,

(and $\pi \to \rho$),
(and $D \to D^*$),
(and $D \to D^{**}$)

- agreement is at the 0-30% level
- these decays also factor for large N_c

Large E vs. Large N_c

Large N_c Factorization

- disfavored by $D \to K^* \pi$ modes, e.g. large strong phase
- in general gives $\langle D^* X | \bar{d} u \bar{c} b | B \rangle = \langle D^* X' | \bar{c} b | B \rangle \langle X'' | \bar{d} u | 0 \rangle$
- $B \to D^* \omega \pi^-$ and $B \to D^* \pi^+ \pi^- \pi^- \pi^0$ (without X') good for small and large m_X^2

Ligeti, Luke, Wise

Large E vs. Large N_c

Large N_c Factorization

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- in general gives $\langle D^* X|\bar{d}u\bar{c}b|B\rangle = \langle D^* X'|\bar{c}b|B\rangle\langle X''|\bar{d}u|0\rangle$
- $B \to D^*\omega\pi^-$ and $B \to D^*\pi^+\pi^-\pi^-\pi^0$ (without X') good for small and large m_X^2

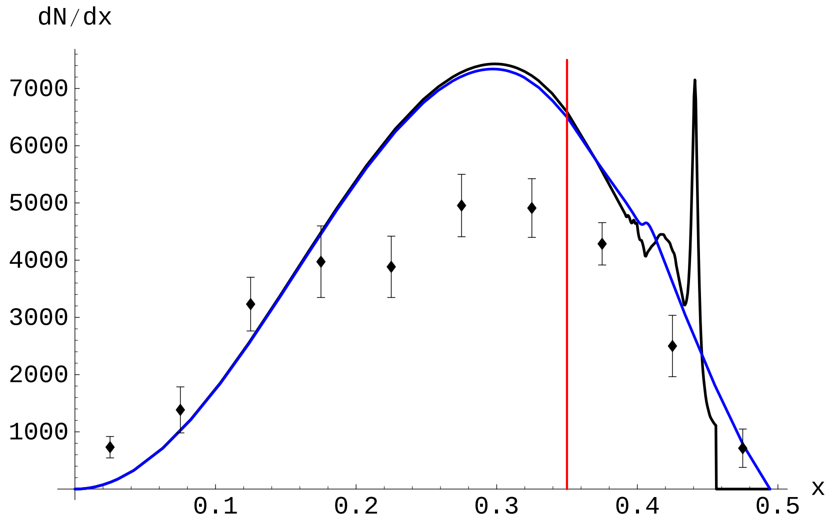
Ligeti, Luke, Wise

What about inclusive decays?

Bauer,Grinstein,Pirjol,Stewart (in progress)

see also Calmet, Mannel, Schwarze

no X' in SV limit, $m_b, m_c \gg m_b - m_c \gg \Lambda_{\rm QCD}$



$B \to D^{*0} X$ (CLEO '97)

agrees ok for large E

model dependence from $\bar{c}s$

$B \to D^{*0} X_u$ would be cleaner

Conclusions

- Factorization emerges from an effective theory, proofs are simplified
- Universal, same steps for many inclusive/exclusive processes
- New framework for investigating power corrections
- Factorization in $B \to \pi e \nu$, $B \to \pi\pi$, work in progress