$F_{B \to D^*}(1)$ from Lattice QCD

Andreas S. Kronfeld

Fermilab

with Shoji Hashimoto, Paul Mackenzie, Sinead Ryan, and Jim Simone

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Farewell to ACPMAPS

In 1987 Fermilab announced that it would build a supercomputer for lattice gauge theory, with the aim to carry out calculations of real interest to particle physics.

In 1991, it was upgraded to become, by one measure, the fastest computer in the world (that wasn't a secret). It had 600 processors—the Intel i860—each with the (for those days) mind-blowing clock speed of 40 MHz.

ACPMAPS decommissioned May 15, 2002.

α_s from $\bar{c}c$

m_q, m_s, m_c

f_{B_q}, f_{D_q}

$B \to \pi l \nu$

methods

Today's result was the last major project on ACPMAPS.

$B \to D^* l\nu$ and V_{cb}

The decay rate for $B \to D^* l\nu$ is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3}(w^2-1)^{1/2} m_{D^*}^3 (m_B - m_{D^*})^2 G(w) |V_{cb}|^2 |F_{B\to D^*}(w)|^2,$$

where $w = v \cdot v'$ and $G(1) = 1$. At zero recoil ($w = 1$) the HQS is more powerful—$F_{B\to D^*}(1)$ is close to 1—so this point is preferred.

So, $|V_{cb}|$ is determined by extrapolating data for

$$\frac{1}{(w^2-1)^{1/2}} \frac{d\Gamma}{dw}$$

to $w \to 1$. Then $F_{B\to D^*}(1)$ is taken from "theory".

To date models [Neubert] or a rigorous inequality + judgment [Bigi, Uraltsev, et al.] have been used to estimate $F_{B\to D^*}(1)$.

The Problem at Hand

The "form factor" $F_{B\to D^*}(w)$ is a linear combination of several form factors of the matrix elements $\langle D^*|V^\mu|B\rangle$ and $\langle D^*|A^\mu|B\rangle$.

At zero recoil all form factors but h_{A_1} are suppressed, so

$$F_{B\to D^*}(1) = h_{A_1}(1) = \langle D^*(v)|A^\mu|B(v)\rangle,$$

which should be "straightforward" to calculate in lattice QCD.

But a brute force calculation of this matrix element of $\langle D^*|A^\mu|B\rangle$ would not be interesting: similar matrix elements like $\langle \pi|V^\mu|B\rangle$ and $\langle 0|A^\mu|B\rangle$ have 15–20% errors (in the quenched approximation).

We have to involve heavy-quark symmetry from the outset: if we can focus on $F-1$, we have a chance of success, because a 20% error on $F-1$ is interesting: $0.2\times 0.1 = 0.02$. Hashimoto, ASK, Mackenzie, Ryan, Simone, hep-ph/0110253.

The Obstacles

There are three specific obstacles to overcome:

- statistical uncertainties
- normalization uncertainties: $A^\mu_{\mathrm{lat}} \doteq Z_A^{-1} A^\mu$
- how treat heavy quarks: $m_b a \not\ll 1$

The first two need computational insight;

[Hashimoto et al., hep-ph/9906376, hep-ph/0110253]

the last two theoretical insight.

[El-Khadra, ASK, Mackenzie, hep-lat/9604004; ASK, hep-lat/0002008]

[see also Harada *et al.*, hep-lat/0112044, hep-lat/0112045]

Anatomy of $h_{A_1}(1)$

At zero recoil heavy-quark symmetry implies

$$h_{A_1}(1) = \eta_A \left[1_{\text{Isgur-Wise}} + 0_{\text{Luke}} + \delta_{1/m^2} + \delta_{1/m^3} \right]$$

where η_A is a short-distance coefficient of the HQET, and the δ_{1/m^n} are (principally) long-distance matrix elements.

The structure of the $1/m_Q^n$ corrections is

$$\begin{aligned}
\delta_{1/m^2} &= -\frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{(2m_c)(2m_b)} - \frac{\ell_P}{(2m_b)^2} \\
\delta_{1/m^3} &= -\frac{\ell_V^{(3)}}{(2m_c)^3} + \frac{\ell_A^{(3)}\Sigma + \ell_D^{(3)}\Delta}{(2m_c)(2m_b)} - \frac{\ell_P^{(3)}}{(2m_b)^3}
\end{aligned}$$

where $\Sigma = 1/(2m_c) + 1/(2m_b)$ & $\Delta = 1/(2m_c) - 1/(2m_b)$.

Calculating η_A and the ℓs

η_A is short distance, so perturbation theory should be adequate.

To calculate the ℓs, one must make sure that their renormalization scheme ends up the same as the one used for η_A.

Lattice gauge theory with Wilson fermions has the same Isgur-Wise symmetries as continuum QCD, for all $m_Q a$. It therefore admits a description with HQET, provided $m_Q \gg \Lambda$. [hep-lat/9604004, hep-lat/0002008, hep-lat/0112044, hep-lat/0112045]

So, one needs some quantities with small statistical and normalization errors, and whose HQE contains the ℓs. Then, calculate the short-distance part ($m_Q a$ enters here) in perturbation theory, extract the ℓs from a fit, and reconstitute $h_{A_1}(1)$.

Suitable Observables

From hep-ph/9906376 (the $B \to D$ form factor) we know that certain ratios have the desired low level of uncertainty

$$\frac{\langle D|\bar{c}\gamma^4 b|B\rangle\langle B|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle B|\bar{b}\gamma^4 b|B\rangle} = \left\{\eta_V^{\text{lat}}\left[1-\ell_P\Delta^2-\ell_P^{(3)}\Delta^2\Sigma\right]\right\}^2$$

$$\frac{\langle D^*|\bar{c}\gamma^4 b|B^*\rangle\langle B^*|\bar{b}\gamma^4 c|D^*\rangle}{\langle D^*|\bar{c}\gamma^4 c|D^*\rangle\langle B^*|\bar{b}\gamma^4 b|B^*\rangle} = \left\{\eta_V^{\text{lat}}\left[1-\ell_V\Delta^2-\ell_V^{(3)}\Delta^2\Sigma\right]\right\}^2$$

$$\frac{\langle D^*|\bar{c}\gamma^j\gamma_5 b|B\rangle\langle B^*|\bar{b}\gamma^j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma^j\gamma_5 c|D\rangle\langle B^*|\bar{b}\gamma^j\gamma_5 b|B\rangle} = \left\{\check{\eta}_A^{\text{lat}}\left[1-\ell_A\Delta^2-\ell_A^{(3)}\Delta^2\Sigma\right]\right\}^2$$

For lattice gauge theory, these follow from hep-lat/0002008, leaning heavily on Falk and Neubert [PRD47 (1993) 2965].

η^{lat}s computed to one loop + BLM [Harada, Hashimoto, ASK, Onogi, hep-lat/0112045].

The Debt Owed to Luke's Theorem

We need the $1/m_Q^2$ corrections to the double ratios, but the lattice action and currents do not normalize all such terms correctly. HQET reveals several sources of such contributions [Falk&Neubert; Mannel; ASK]:

double insertions of $L_{\mathrm{HQET}}^{(1)}$ — are correctly normalized (to order α_s)

single insertions of $L_{\mathrm{HQET}}^{(1)}$ into matrix elements of $j_{\mathrm{HQET}}^{(1)}$
vanish at zero recoil by a generalization of Luke's theorem

single insertions of $L_{\mathrm{HQET}}^{(2)}$ — vanish by Luke's (or Ademollo-Gatto) theorem

matrix elements of $j_{\mathrm{HQET}}^{(2)}$ — are essential, but double ratios help

Matrix elements of $j^{(2)}_{\rm HQET}$ enter the double ratios in the following way:

$$\frac{[1-\lambda(X_b/m_b^2 - 1/m_c m_b + X_c/m_c^2)]^2}{[1-\lambda(2X_c-1)/m_c^2][1-\lambda(2X_b-1)/m_b^2]} = 1 - \lambda\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2,$$

where λ is proportional to λ_1 or λ_2, and X_Q/m_Q^2 indicates incorrect normalization. The other terms are "correctly normalized"—in practice, only at the tree level.

The correct normalization of $1/m_c m_b$ is built into the current we used.

The cancellation of the others is an essential feature of the double ratios.

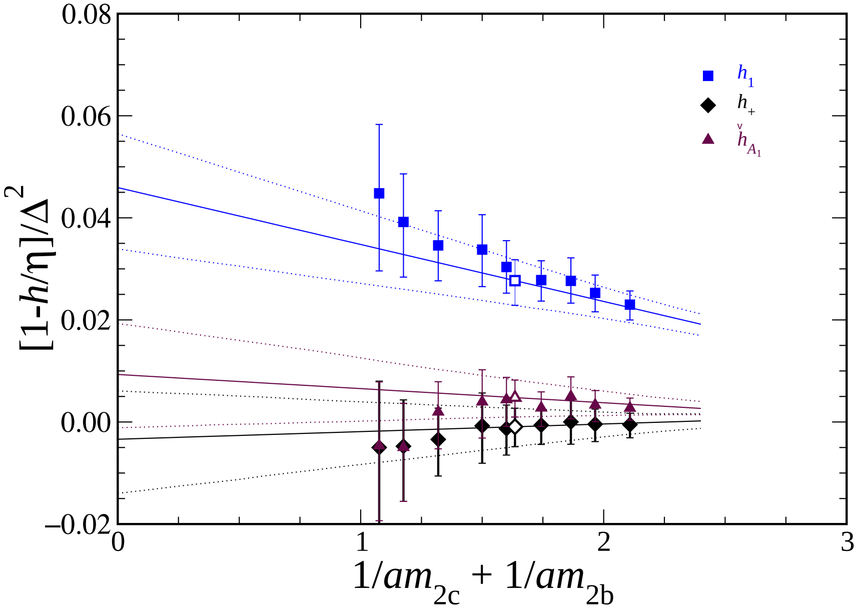
The double ratios do suffer from uncertainties of order $\alpha_s(\bar{\Lambda}/m_Q)^2$. These, and other matching uncertainties, of order α_s^2 and $(\bar{\Lambda}/m_Q)^3$, are put into the error budget.

Error Budget

uncertainty	h_{A_1}		$1 - h_{A_1}$ (%)	
statistics and fitting	$+0.0238$	-0.0173	$+27$	-20
adjusting m_c and m_b	$+0.0066$	-0.0068	$+8$	-8
α_s^2	±0.0082		± 9	
$\alpha_s(\bar{\Lambda}/2m_Q)^2$	±0.0114		±13	
$(\bar{\Lambda})^3/(2m_Q)^3$	±0.0017		± 2	
a dependence	$+0.0032$	-0.0141	$+4$	-16
chiral (m_q)	$+0.0000$	-0.0163	$+0$	-19
quenching	$+0.0061$	-0.0143	$+7$	-16
total systematic	$+0.0171$	-0.0302	$+20$	-35
total (stat $\oplus$ syst)	$+0.0293$	-0.0349	$+34$	-40

The row labeled "total systematic" does not include uncertainty from fitting, which is lumped with the statistical error.
The statistical error is that after chiral extrapolation.

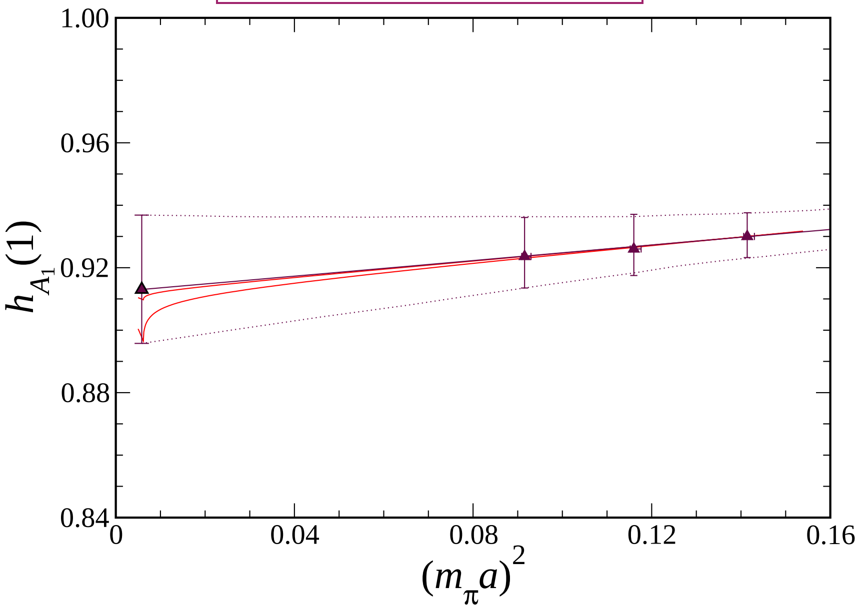
Fitting



Here we see an example of the fit to the heavy-quark masses.

As expected, ℓ_V is the largest effect. Including $\ell_V^{(3)}$ improves the statistical error.

Chiral (m_q)



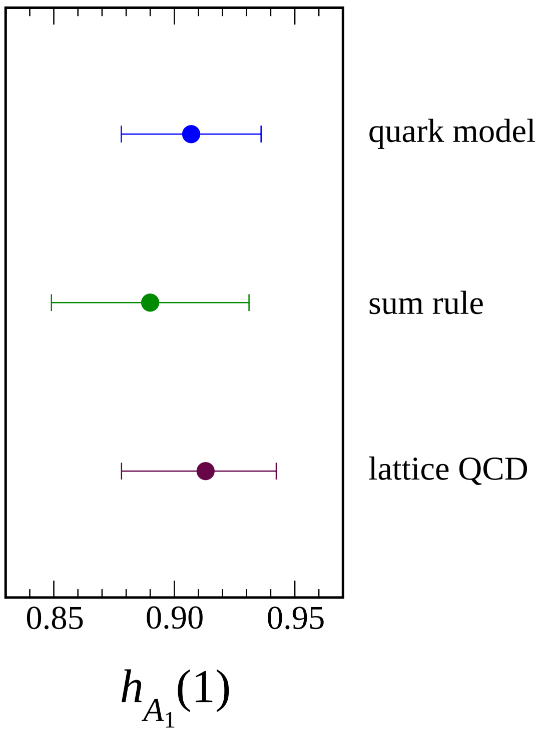
In addition to a linear dependence on m_π^2, there should be a pion loop contribution.

The former increases the statistical error; the omission of the latter is considered to be a systematic error.

Result and Comparison

After putting everything back together again, we find [hep-ph/0110253]

$$F_{B\to D^*}(1) = 0.913 \; \underset{\text{stat}}{{}^{+0.024}_{-0.017}} \; \underset{\text{match}}{\pm 0.016} \; \underset{a}{{}^{+0.003}_{-0.014}} \; \underset{m_q}{{}^{+0.000}_{-0.016}} \; \underset{\text{quench}}{{}^{+0.006}_{-0.014}}$$



The defects (as I see them) are as follows:

The quark model omits some dynamics (more than quenching). Not clear that QM gives HQET mx element in $\overline{\mathrm{MS}}$ scheme.

Sum rule has incalculable contribution from excitations with $(M - m_{D^*})^2 < \mu^2$.

Present lattice result has quenched approximation (but in error budget).

Flat or Gaussian?

If you are interested in a CKM fit, you may want an idea of what's likely.

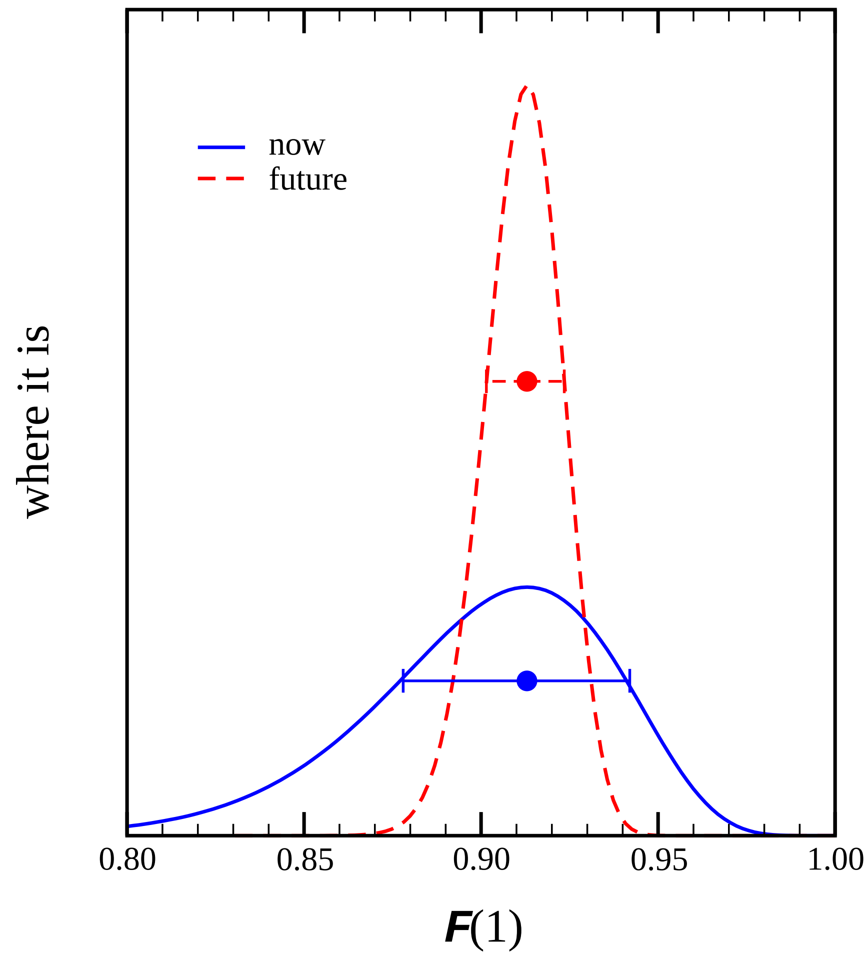
The statistical part is straightforward and close to Gaussian.

The systematics entail work and insight, but they are not guesses.

The distribution is not flat: 0.90–0.91 is most likely; there must be a tail.

$$P(x) = Nx^7e^{-7x}, \quad x = \frac{1 - F(1)}{0.087}$$

captures these features. In the future we could reducing the uncertainty by $\div 3$.



Future Computing

For B physics it is important to remove the quenched approximation (more so than to reduce the lattice spacing much further). To do so, we need more computing. It sounds expensive, but it isn't.

Fermilab Theory Group & Computing Division, the MILC Collaboration, and Cornell are building a cluster of PCs. Supported by SciDAC.

80 nodes at left, with Myrinet switch

Scale to hundreds, then to thousands.

Replace $1/3$ yearly.

Similar ideas at JLab/MIT, Wuppertal,. . . .

`http://theory.fnal.gov/pcqcd/`

Backup Slide

Scheme Dependence

If HQET is renormalized with an explicit subtraction point

$$\begin{aligned}
\eta_A^{\text{lat}}(c) &= \eta_A^{\text{lat}}(0) + c\alpha_s\mu^2\left(\Delta^2 + \frac{8/3}{(2m_c)(2m_b)}\right) \\
\eta_V^{\text{lat}}(c) &= \eta_V^{\text{lat}}(0) + c\alpha_s\mu^2\Delta^2 \\
\check{\eta}_A^{\text{lat}}(c) &= \check{\eta}_A^{\text{lat}}(0) - c\alpha_s\mu^2\Delta^2/3
\end{aligned}$$

where, *e.g.*, $c = 4/3\pi$ (BSUV) . With $\eta_V^{\text{lat}}(c)$ and $\check{\eta}_A^{\text{lat}}(c)$ the fit parameters change:

$$\begin{aligned}
\ell_V(c) &= \ell_V(0) + c\alpha_s\mu^2 \\
\ell_A(c) &= \ell_A(0) - c\alpha_s\mu^2/3 \\
\ell_P(c) &= \ell_P(0) + c\alpha_s\mu^2
\end{aligned}$$

When reconstituting $h_{A_1}(1)$ from $\eta_A(c)$ and $\ell_{V,A,P}(c)$, the $c\alpha_s\mu^2$ terms cancel.