Determination of $|V_{ub}|$: Theoretical Issues

Michael Luke
University of Toronto
Outline

1. Introduction - why we care

   Exclusive: lattice
   • \( B \rightarrow o(\Omega) \)

2. Approaches:

   Inclusive: \( B \rightarrow X_u \) + cuts
   • \((q^2, m_X)\) plane (sometimes less is more...)
   • \( E \cdot \) endpoint

3. Summary
- the unitarity triangle provides a simple way to visualize
SM relations: \[
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0
\]

\[
\frac{1}{\sin \theta_c} \left| \frac{V_{ub}}{V_{cb}} \right|
\]
\[
\frac{1}{\sin \theta_c} \left| \frac{V_{td}}{V_{ts}} \right| \approx \frac{1}{\sin \theta_c} \left| \frac{V_{td}}{V_{ts}} \right|
\]

\( B_s-\bar{B_s} \) mixing (Tevatron)

\( CPV \) in \( B^0 \rightarrow \psi K_S \)

\( \Leftrightarrow \) already consistent at \( \sim 30\% \) level
\( \Leftrightarrow \) want to test at \( \leq 10\% \) level

| \( V_{cb} \), \( \sin 2\beta \), \( |V_{td}/V_{ts}| \) : “easy” (theory and experiment both tractable)
| \( V_{ub} \), \( \alpha \), \( \gamma \) : HARD - our ability to test CKM depends on the precision with which these can be measured
World Average '02: \[ \sin 2\beta = 0.780 \pm 0.077 \]: any deviation from SM will require precision measurements!
The problem (of course …): **HADRONIC PHYSICS**

\[ \frac{V_{ub}}{V_{cb}} = 0.090 \pm 0.025 \]

large model dependence

To believe small discrepancy = new physics, need model independent predictions

**Definition:** for the purposes of this talk,

MODEL DEPENDENT ⇔ theoretical uncertainty is **NOT** parametrically suppressed - theorists argue about \( O(1) \) effects

MODEL INDEPENDENT ⇔ theoretical uncertainty *is* parametrically suppressed (typically by \( \Lambda_{QCD}/m_b^n, \alpha_s(m_b^n) \)) - theorists argue about \( O(1) \times \) (small number)
**Exclusive Decays on lattice: \( B \to ^0 \int \), \( B \to \Omega \int \)**

\[
\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = f_+(E) \left[ p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(E) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu
\]

nonperturbative: calculate on lattice

vanishes for \( m=0 \)

**Lattice issues:**

- \( |p_\circ| \) small (currently <1 GeV) *(lattice spacing errors \( \sim a|p_\circ| \)*) - experimental data is best at low \( q^2 \)/large \( p_\circ \)
- chiral extrapolation to \( m_q \sim 0 \)
- **quenched** approximation (no light quark loops)
- \( B \to ^0 \int \) much easier than \( B \to \Omega \int \) *(\( \Omega \) width vanishes in quenched approximation; chiral extrapolation tricky in unquenched)*

Need unquenched calculation to be model-independent.

(NB unquenched calculations of \( f_B \) are now being performed)
\( \frac{d\Gamma}{dq^2} \) for \( B \rightarrow \phi \bar{K} \) at high \( q^2 \)/low \( p_\pi \)
\[ |V_{ub}|^2 = \frac{12\pi^2}{G_F^2 m_B} \frac{1}{T_B(p_{min}, p_{max})} \int_{p_{min}}^{p_{max}} dp \frac{d\Gamma_{B \rightarrow \pi^+\pi^-}}{dp} \]

\[ T_B(0.4 \text{ GeV, } 1.0 \text{ GeV}) = 0.55^{+0.15}_{-0.05}^{+0.09}_{-0.12}^{+0.09} \pm 0.06 \pm 0.09 \text{ GeV}^4 \pm \text{quenching} \]

- except for HQET matching, all uncertainties can be reduced with (finitely) more computer time (including unquenched calculation)

"... there do not appear to be any technical roadblocks to reducing the uncertainties ... to a few percent or better, over the course of the present round of experiments."
$|V_{ub}|$ from Inclusive Decays

\[ \frac{d\Gamma}{d(p.s.)} \sim \text{parton model} + \sum_n C_n \left( \frac{\Lambda_{QCD}}{m_b} \right)^n \]

(free quark decay) nonperturbative corrections

**Good:** relation between \( \sum_{X_u} \Gamma(B \rightarrow X_u \ell \bar{\nu}) \) & \(|V_{ub}| \) known to \(~5\%\)

**Bad:** can’t measure it!

\[ \Gamma(B \rightarrow X_c \ell \bar{\nu}) \sim 100 \times \Gamma(B \rightarrow X_u \ell \bar{\nu}) \]

\((~100 \times \text{background from charm!})\)

\(\therefore\) need to impose stringent cuts to eliminate charm background …

… and here the troubles begin ...
$B \rightarrow X_u \int \text{phase space}$
perturbative singularity (real + virtual gluons)

\[ m_X \sim m_B \left( m_B - m_b \right) \sim \Lambda_{QCD} m_B \]

real gluon emission

NO rate at parton level (purely nonperturbative)

\( m_X \ll m_D \)

\( b \rightarrow c \) allowed
PROBLEM: $\Lambda_{QCD} m_B$ and $m_D^2$ aren’t so different! ⇔ kinematic cut and singularity are perilously close …
What smooths out the singularity?

"Fermi motion" \( k \gamma \sim \Lambda_{\text{QCD}} \)

\[ k^+ = k^0 + k^3 \text{ (light cone momentum)} \]

\[ f(k^+) \sim \text{parton distribution function: nonperturbative!} \]

- \( f(k^+) \) must be modeled
- moment of \( f(k^+) \) are related to matrix elements of local operators (constrains models)
• singularity is smeared out by b quark light-cone distribution function $f(k_+)$
• rate is sensitive to details of $f(k_+)$ unless $m_X^2 >> \Lambda_{QCD} m_b$ (bad for $m_X < m_D$!) - introduces model dependence unless we know $f(k_+)$
The image contains a graph with the following details:

- The graph plots $q^2 (GeV^2)$ on the y-axis and $m_X^2 (GeV^2)$ on the x-axis.
- The graph indicates regions:
  - $s_0 > 0$: real gluon emission
  - $s_0 < 0$: NO rate at parton level (purely nonperturbative)
  - $s_0 = 0$: perturbative singularity (real+virtual gluons)
- The equation $m_X^2 = m_B (m_B - m_b) \sim \Lambda_{QCD} m_B$ is shown.
- The boundary $q^2 > (m_B - m_D)^2$ is highlighted in red.
- The graph is labeled with the text:
  - "b→c allowed"
  - "real gluon emission"
  - "NO rate at parton level (purely nonperturbative)"
  - "perturbative singularity (real+virtual gluons)"

The graph illustrates the relationship between $q^2$, $m_X^2$, and $s_0$, highlighting regions of perturbative and nonperturbative behavior in the context of QCD.
Lepton Invariant Mass Spectrum for $b \rightarrow u$ Decay

$\frac{1}{\Gamma} \frac{d\Gamma}{dq^2}$

$q^2 (GeV^2)$

$0.08$
$0.06$
$0.04$
$0.02$

lepton $q^2$ spectrum is insensitive to Fermi motion (usual OPE holds) - eliminates model dependence! (counterintuitive … LESS inclusive = BETTER behaved)
Pure $m_X$ cut:

- gets $\sim 80\%$ of $B \to X_u \gamma$ rate (for ideal cut $m_X < m_D$)

  BUT

- current theoretical predictions are strongly model dependent ($cf$ DELPHI determination
  $|V_{ub}| = (4.07 \pm 0.37 \pm 0.44 \pm 0.33) \times 10^{-3}$
  has cut $m_X < 1.6$ GeV)

- $f(k_\perp)$ can be extracted from photon spectrum in $B \to X_s \gamma$: error goes from formally $O(1)$ to $O(1/m_b)$ ... but size of higher twist terms is unknown ($30\%$?)
  (more on this later)

Pure $q^2$ cut:

- insensitive to $f(k_\perp)$ ... nonperturbative effects are subleading

- theory known to $O(1/m_b^3, \alpha_s^2)$
  (Czarnecki and Melnikov, PRL88:131801,2002)

- leading & subleading renormalization group improvement known
  (Neubert & Becher, hep-ph/0105217)

  BUT

- nonperturbative corrections are large
  (reduced phase space $\Rightarrow \sim O(1/m_c^3)$, not $O(1/m_b^3)$) (Neubert, JHEP 0007:022 (2000))

- only $\sim 20\%$ of $B \to X_u \gamma$ rate
**Optimized Cuts**

(C. Bauer, Z. Ligeti and ML, hep-ph/0107074)

\[ q^2 > 8 \text{ GeV}^2, \]
\[ m_X < 1.7 \text{ GeV} \]

- \( b \rightarrow c \) allowed
- \( q^2 > 8 \text{ GeV}^2, \]
  \[ m_X < 1.7 \text{ GeV} \]

**Real gluon emission**

- \( s_0 < 0 \)
- \( s_0 > 0 \)

**NO rate at parton level (purely nonperturbative)**

\[ m_X^2 = m_B (m_B - m_b) \sim \Lambda_{QCD} m_B \]

**Perturbative singularity (real+virtual gluons)**

\[ s_0 = 0 \]
**Effects of Fermi motion**

Simple model:

\[
f(k_+) = \frac{32}{\pi^2 \Lambda} \left(1 - \frac{k_+}{\Lambda}\right)^2 e^{-\frac{4}{3}\left(1 - \frac{k_+}{\Lambda}\right)^2}
\]

(Use model to estimate sensitivity to Fermi motion, NOT to get final result! ... extract \( f(k_+) \) from \( B \rightarrow X_s \gamma \))

- Do not need to know structure function well to have negligible uncertainty on \( |V_{ub}| \)
Strategy:

- combine lepton and hadron invariant mass cuts: larger rate, smaller errors than pure $q^2$ cut

- make cut on $m_X$ as large as possible, keeping the background from $B \to$ charm under control (depends on detector resolution, modeling of $B \to (D, D^*)$)

- make $q^2$ cut as low as possible, keeping the contribution from Fermi motion and perturbative uncertainties small
Additional uncertainties for optimized cuts:

- $m_b$ - rate is proportional to $m_b^5$ … kinematic cuts also depend on $m_b$, so cut rate is MORE sensitive - need precise value of $m_b$! (qu: is ±80 (±30) MeV error on $m_b$ realistic? .. probably not yet)

- perturbative corrections - known to $O(\alpha_s^2\beta_0)$

- weak annihilation (WA) - a potential problem for ALL inclusive determinations which include large $q^2$ region  (M. Voloshin, hep-ph/0106040)

\[
O\left(16\pi^2 \times \frac{\Lambda_{QCD}^3}{m_b^3} \times (\text{factorization violation})\right) \sim 0.03 \left(\frac{f_B}{0.2 \text{ GeV}}\right) \left(\frac{B_2 - B_1}{0.1}\right) 
\]

~3% (?? guess!) contribution to rate at $q^2 = m_b^2$

⇒ relative size of effect gets worse the more severe the cut
⇒ no reliable estimate of size - can test by comparing charged and neutral $B$’s
### Representative cuts:

(a) $q^2 > 6 \text{ GeV}^2$, $m_X < m_D$  \quad 46\% \text{ of rate}

(b) $q^2 > 8 \text{ GeV}^2$, $m_X < 1.7 \text{ GeV}$  \quad 33\% \text{ of rate}

(c) $q^2 > 11 \text{ GeV}^2$, $m_X < 1.5 \text{ GeV}$  \quad 18\% \text{ of rate}

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Size (in $V_{ub}$)</th>
<th>Improvement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_b$</td>
<td>$\pm 80 \text{ MeV}$: 7%, 8%, 10%</td>
<td>RG improved $\chi$ sum rules, moments of $B$ decay spectra, lattice</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>2%, 3%, 7%</td>
<td>full two-loop calculation</td>
</tr>
<tr>
<td>$1/m_b^3$ (weak annihilation)</td>
<td>3%, 4%, 8%</td>
<td>compare $B^\pm$, $B^0$ compare S.L. width of $D^0$, $D_S$, lattice</td>
</tr>
</tbody>
</table>
E · endpoint

Charge Lepton Energy Spectrum for $b \rightarrow u$ Decay

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dE_l} (GeV^{-1}) \]

- Rate above $B \rightarrow \text{charm endpoint}$ extremely sensitive to Fermi motion (numerically, model dependence is stronger than for $\Gamma(m_X < m_D)$)
- (lowering the cut (how well is charm background understood?) reduces sensitivity to $f(k_+)$)

Michael Luke, University of Toronto  FPCP '02, May 18, 2002  23
- as with $m_X$ spectrum, $f(k_\perp)$ dependence may be eliminated by relating $d\Gamma/dE_e$ to photon spectrum in $B \rightarrow X_s \gamma$ (Neubert, PRD49 (1994) 4623)

Recent progress:

(1) relation between spectra worked out to NLO accuracy (subleading Sudakov logs resummed)  
(Leibovich, Low, Rothstein, PRD61 (2000) 053006)

(2) contribution of operators other than $O_7$ included (large)  (Neubert, hep-ph/0104280)

(3) $O(1/m_b)$ (higher twist) corrections relating $f(k_\perp)$ in $B \rightarrow X_s \gamma$ to $B \rightarrow X_u \gamma$ parametrized (decay Hamiltonians have different Dirac structure!)  (Leibovich, Ligeti, Wise, hep-ph/0205148; Bauer, ML and Mannel, hep-ph/0205150)
The effects of subleading “shape functions” are surprisingly large, ...

\[ \frac{d\Gamma}{dy} \propto 2\theta(1-y) - \frac{\lambda_1}{3m_b^2}\delta(1-y) - \frac{\rho_1}{9m_b^3}\delta''(1-y) + \ldots \]

leading twist terms …

\[ - \frac{\lambda_1}{3m_b^2}\delta(1-y) - \frac{11\lambda_2}{m_b^2}\delta(1-y) + \ldots \]

subleading twist terms … sum to \(f(k_+\))

(corresponding coefficient in \(B \to X_s\gamma\) is 3)

\[ \delta(E_c) \]

\( \Rightarrow \) Simple Model: subleading effects are \(O(20\%)\) (in \(V_{ub}\)) for \(E_{cut} = 2.3\) GeV, decrease with \(E_{cut}\)
Additional Caveats:

(a) weak annihilation is concentrated at endpoint of $E_e$ spectrum $\sim 3\%$
correction to $B \rightarrow X$ rate $\Leftrightarrow \sim 30\%$ correction to rate in endpoint region $\Leftrightarrow$
$\sim 15\%$ uncertainty in $|V_{ub}|$ (another example of a higher twist effect) (see also

(b) very restricted phase space - duality problems?

CLEO ‘01:

$|V_{ub}| = (4.08 \pm 0.34 \pm 0.44$
$\pm 0.16 \pm 0.24) \times 10^{-3}$

Estimate of theoretical uncertainty (WA not included)
Summary

Theory and experiment have now evolved to the level that a model-independent, precision (10% level) determination of $|V_{ub}|$ is possible (!)

Exclusive Decays: Lattice

- $B \rightarrow \phi \gamma$ for low $|p_\phi| < 1\text{ GeV}$: need unquenched calculation of form factor. Other systematics appear under control ... when??

Inclusive Decays: OPE/twist expansion

- need to design cuts that exclude $b \rightarrow c$ without introducing large uncertainties

Theoretical reliability:

$$(q^2, m_X) \text{ cut} > q^2 \text{ cut} > m_X \text{ cut} > E_e \text{ cut}$$

(experimental difficulty is in (roughly) the opposite order ...)

Michael Luke, University of Toronto         FPCP '02, May 18, 2002
Experimental measurements can help beat down the theoretical errors:

(a) better determination of $m_b$ (moments of $B$ decay distributions)

(b) test size of WA (weak annihilation) effects - compare $D^0$ & $D_S$ S.L. widths, extract $|V_{ub}|$ from $B^\pm$ and $B^0$ separately

(c) improve measurement of $B \rightarrow X_s \gamma$ photon spectrum - get $f(k_\perp)$ - $1/m_b$ corrections??

(d) (most important) measure $|V_{ub}|$ in as many CLEAN ways as possible - different techniques have different sources of uncertainty (c.f. inclusive and exclusive determinations of $|V_{cb}|$)