Determination of $|V_{ub}|$: Theoretical Issues

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Outline

1. Introduction - why we care

2. Approaches:

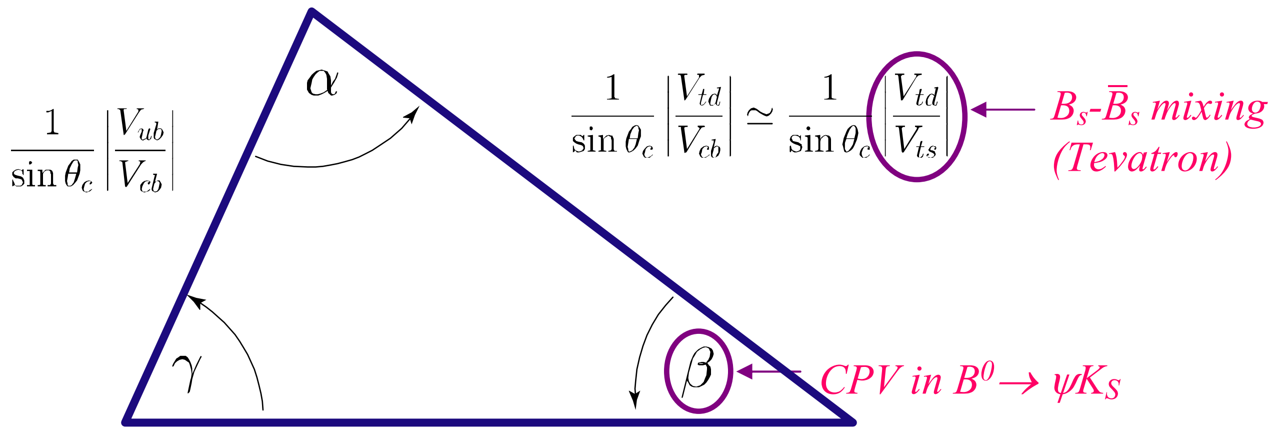
   Exclusive: lattice
   - *$B\to$°(Ω)`∫*

   Inclusive: $B\to \mathrm{X_u}$`∫ + *cuts*
   - *(q^2, m_X) plane (sometimes less is more…)*
   - *E` endpoint*

3. Summary

Introduction

- the unitarity triangle provides a simple way to visualize SM relations: $\boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$

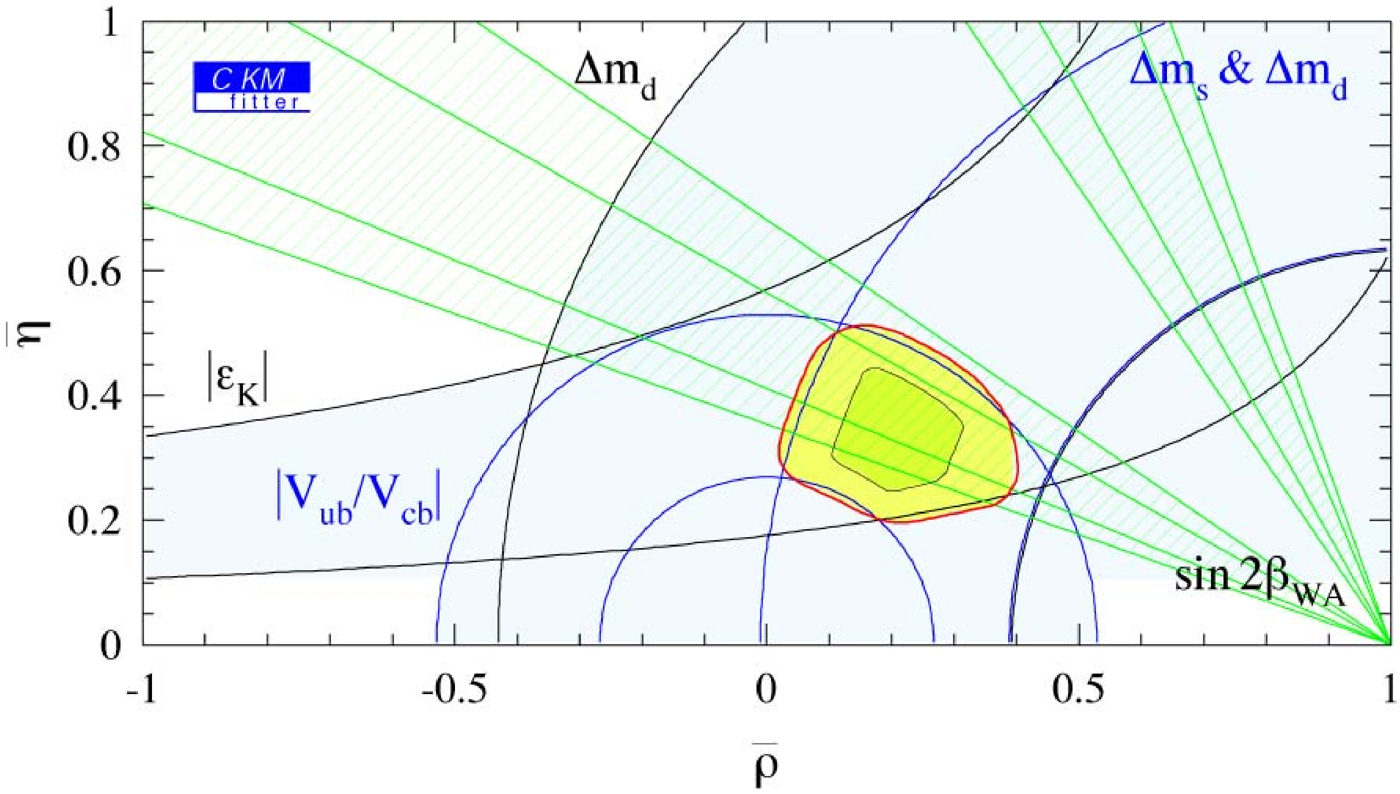


⇦ already consistent at ~30% level

⇦ want to test at ≤10% level

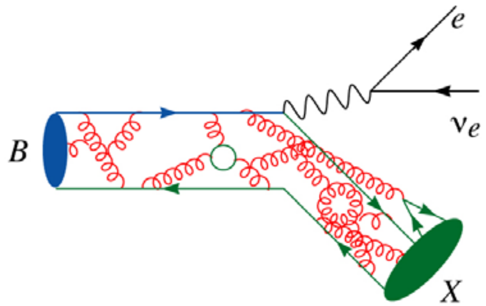
$|V_{cb}|$, sin 2β, $|V_{td}/V_{ts}|$: "easy" (theory and experiment both tractable)

$|V_{ub}|$, α, γ: **HARD** - our ability to test CKM depends on the precision with which these can be measured



World Average '02: *sin 2β=0.780±0.077: any deviation from SM will require precision measurements!*

The problem (of course …): **HADRONIC PHYSICS**



PDG: $\left|\frac{V_{ub}}{V_{cb}}\right| = 0.090 \pm 0.025$

large model dependence

To believe small discrepancy = new physics, need model independent predictions

Definition: for the purposes of this talk,

MODEL DEPENDENT ⇦ theoretical uncertainty is ***NOT*** parametrically suppressed - theorists argue about *O(1)* effects

MODEL INDEPENDENT ⇦ theoretical uncertainty ***is*** parametrically suppressed (typically by $(\Lambda_{QCD}/m_b)^n$, $\alpha_s(m_b)^n$) - theorists argue about $O(1)\times$(small number)

Exclusive Decays on lattice: $B\to$°`∫, $B\to\Omega$`∫

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(E)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2-m_\pi^2}{q^2}q^\mu\right] + \cancel{f_0(E)\frac{m_B^2-m_\pi^2}{q^2}q^\mu}$$

nonperturbative: calculate on lattice

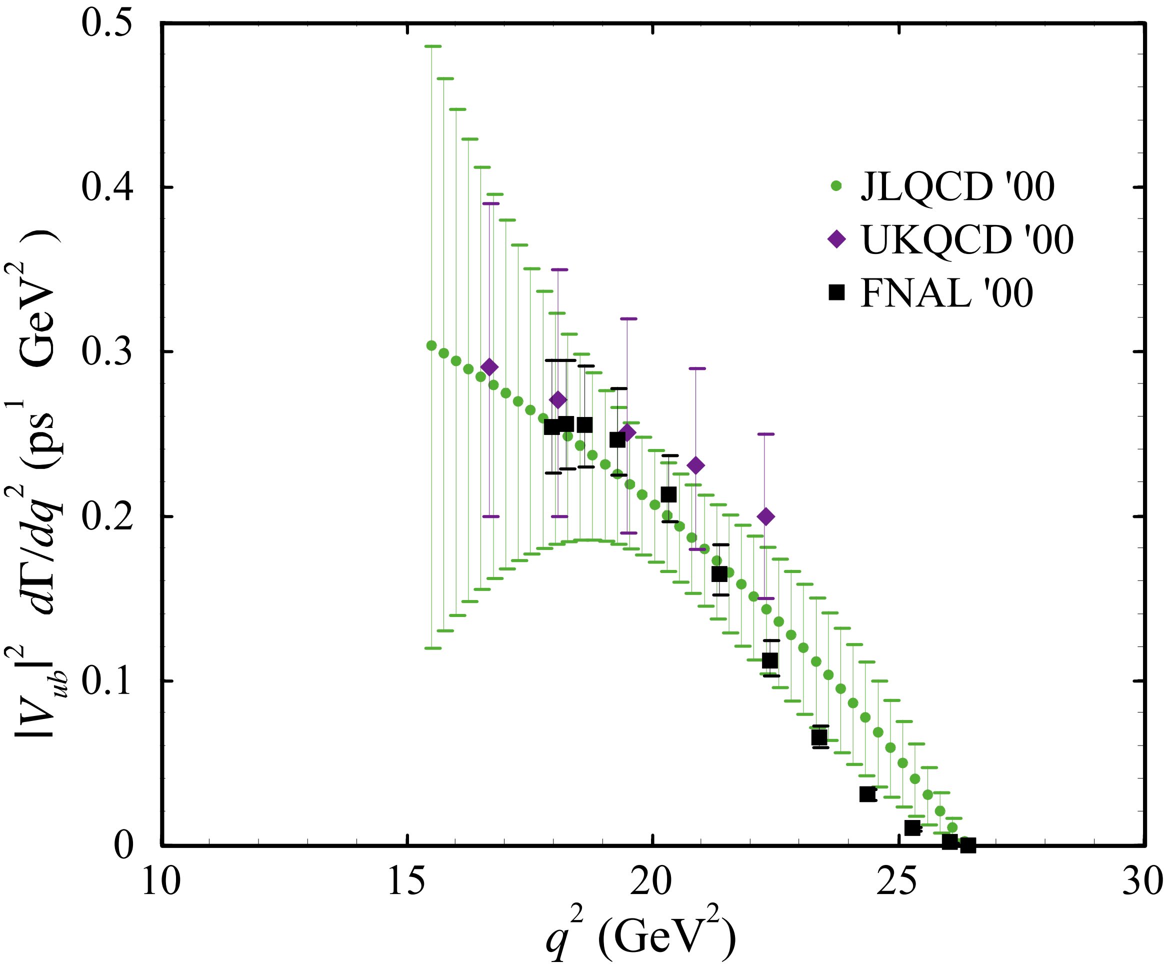
vanishes for m`=0

Lattice issues:

- *|p°| small (currently <1 GeV)* *(lattice spacing errors ~ a|**p°**|)* *- experimental data is best at low q^2/large p°*
- chiral extrapolation to $m_q \sim 0$
- quenched approximation (no light quark loops)
- $B\to$°`∫ much easier than $B\to\Omega$`∫ *(Ω width vanishes in quenched approximation; chiral extrapolation tricky in unquenched)*

> Need unquenched calculation to be model-independent.

(NB unquenched calculations of f_B are now being performed)



(from A. Kronfeld, hep-ph/0010074)

⇦ need to measure $d\Gamma/dq^2$ for $B\rightarrow$°`∫ at high q^2/low p_π

$$|V_{ub}|^2 = \frac{12\pi^2}{G_F^2 m_B} \frac{1}{T_B(p_{\min}, p_{\max})} \int_{p_{\min}}^{p_{\max}} dp \frac{d\Gamma_{B\to\pi\ell\nu}}{dp}$$

(A. El-Khadra et. al., PRD64, 014502)

$$T_B(0.4\text{ GeV},\ \ 1.0\text{ GeV}) = 0.55^{+.15}_{-.05}\ {}^{+.09}_{-.12}\ {}^{+.09}_{-.02} \pm .06 \pm .09\ \text{GeV}^4$$ ±quenching

statistical

chiral extrapolation

lattice spacing

matching

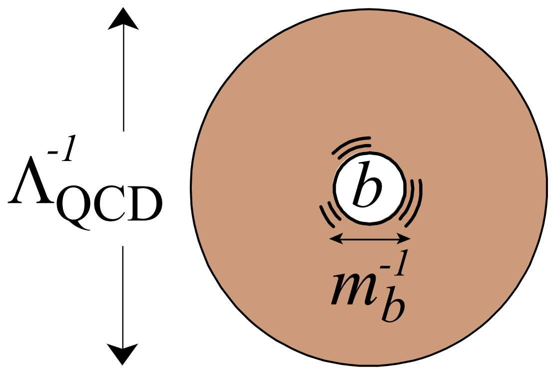
misc. (lattice units, …)

$\Delta V_{ub} \approx$15-18% + quenching error

- except for HQET matching, all uncertainties can be reduced with (finitely) more computer time (including unquenched calculation)

"... there do not appear to be any technical roadblocks to reducing the uncertainties ... to a few percent or better, over the course of the present round of experiments."

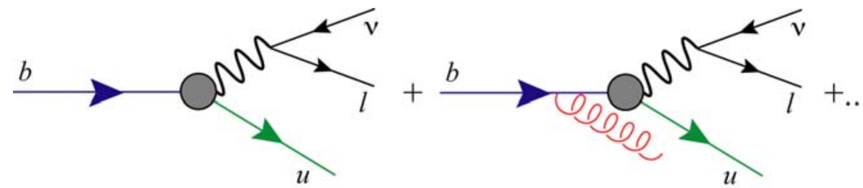
$|V_{ub}|$ from Inclusive Decays



$$\frac{d\Gamma}{d(p.s.)} \sim \text{parton model} + \underbrace{\sum_n C_n \left(\frac{\Lambda_{QCD}}{m_b}\right)^n}_{\text{nonperturbative corrections}}$$

(free quark decay)

nonperturbative corrections



Good: relation between $\sum_{X_u} \Gamma(B \to X_u \ell \bar{\nu})$ & $|V_{ub}|$ known to ~5%

Bad: can't measure it!

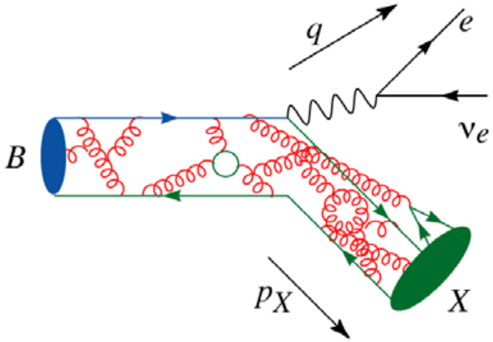
$$\Gamma(B \to X_c \ell \bar{\nu}) \sim 100 \times \Gamma(B \to X_u \ell \bar{\nu})$$

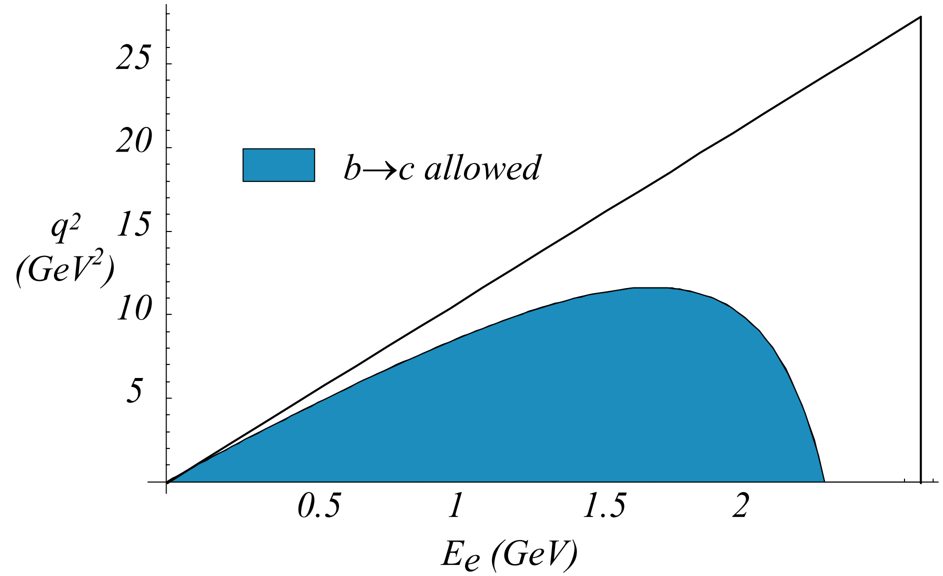
(~100 × background from charm!)

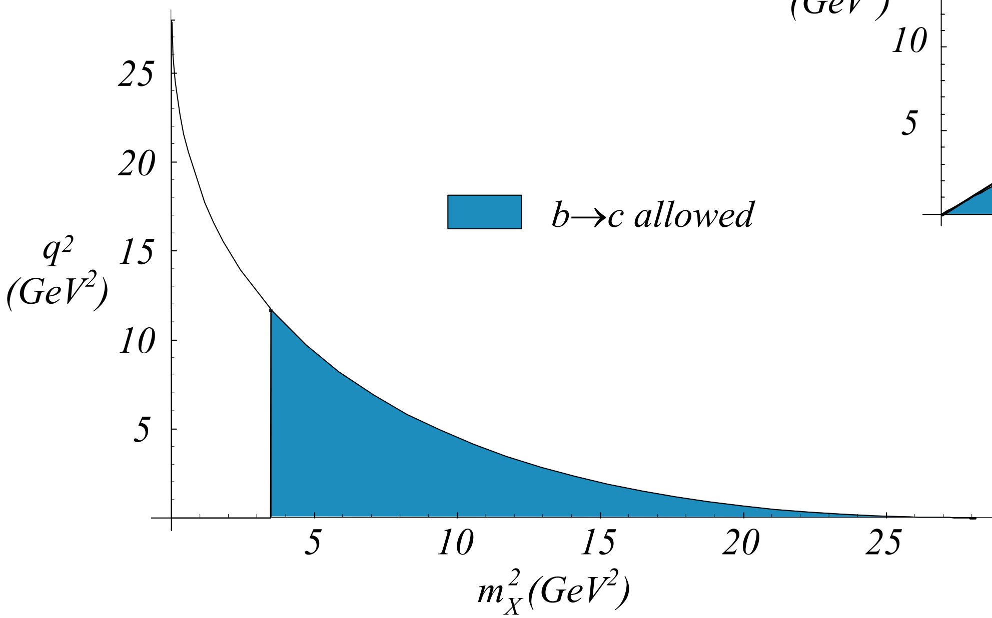
$\therefore$ need to impose stringent cuts to eliminate charm background …

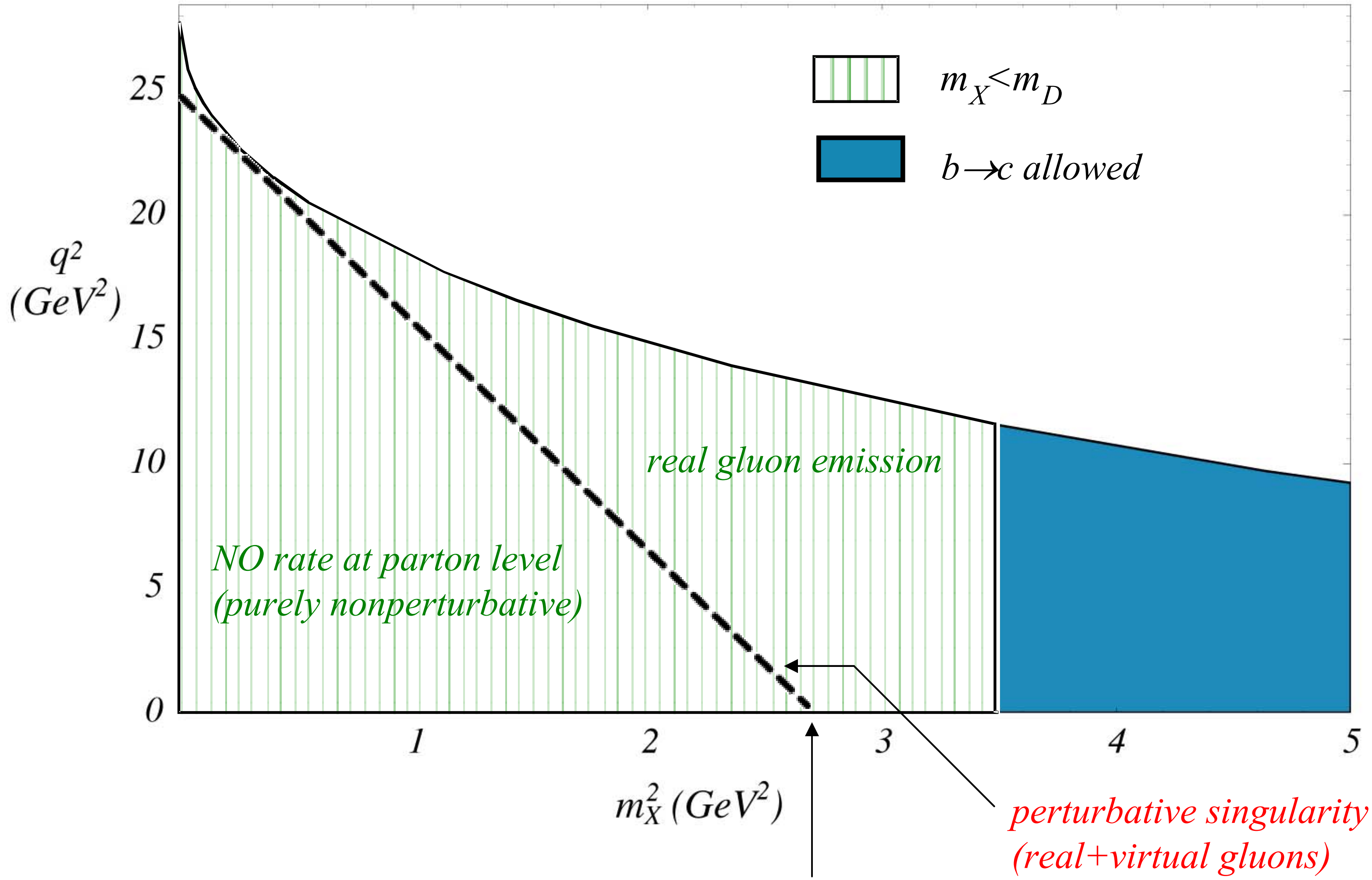
… and here the troubles begin ...

$B \to X_u$ `∫ ***phase space***

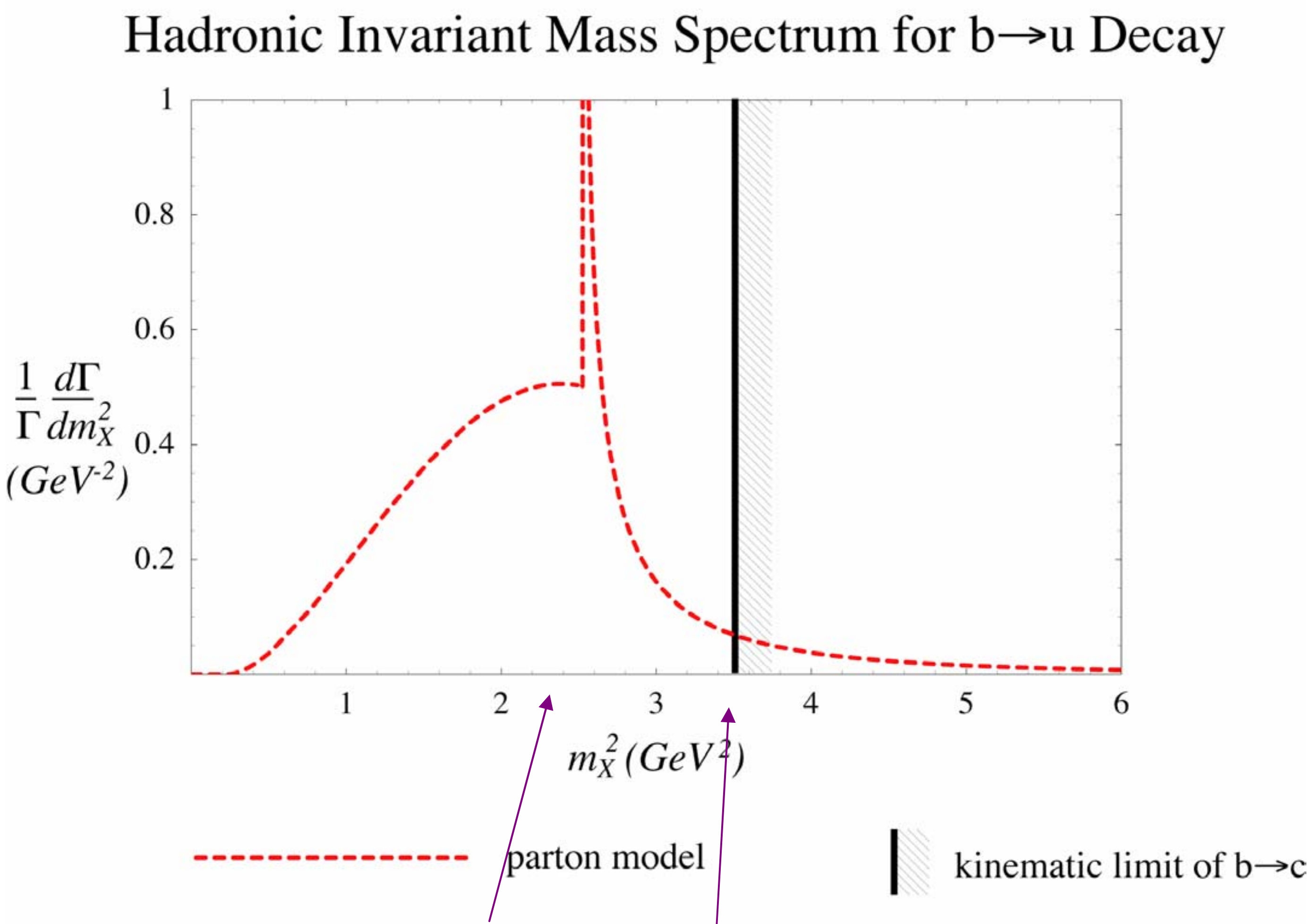








$$m_X^2 = m_B(m_B - m_b) \sim \Lambda_{QCD} m_B$$

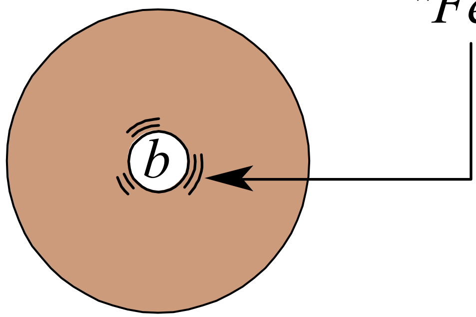


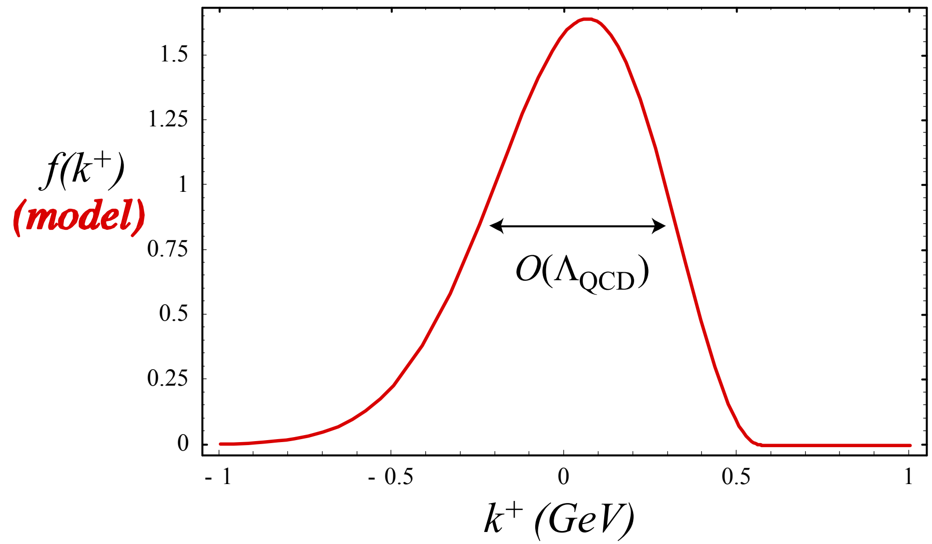
PROBLEM: $\Lambda_{QCD} m_B$ and $m_D{}^2$ aren't so different!

⇦ kinematic cut and singularity are perilously close …

What smooths out the singularity?

"*Fermi motion*" $k^{š} \sim \Lambda_{QCD}$

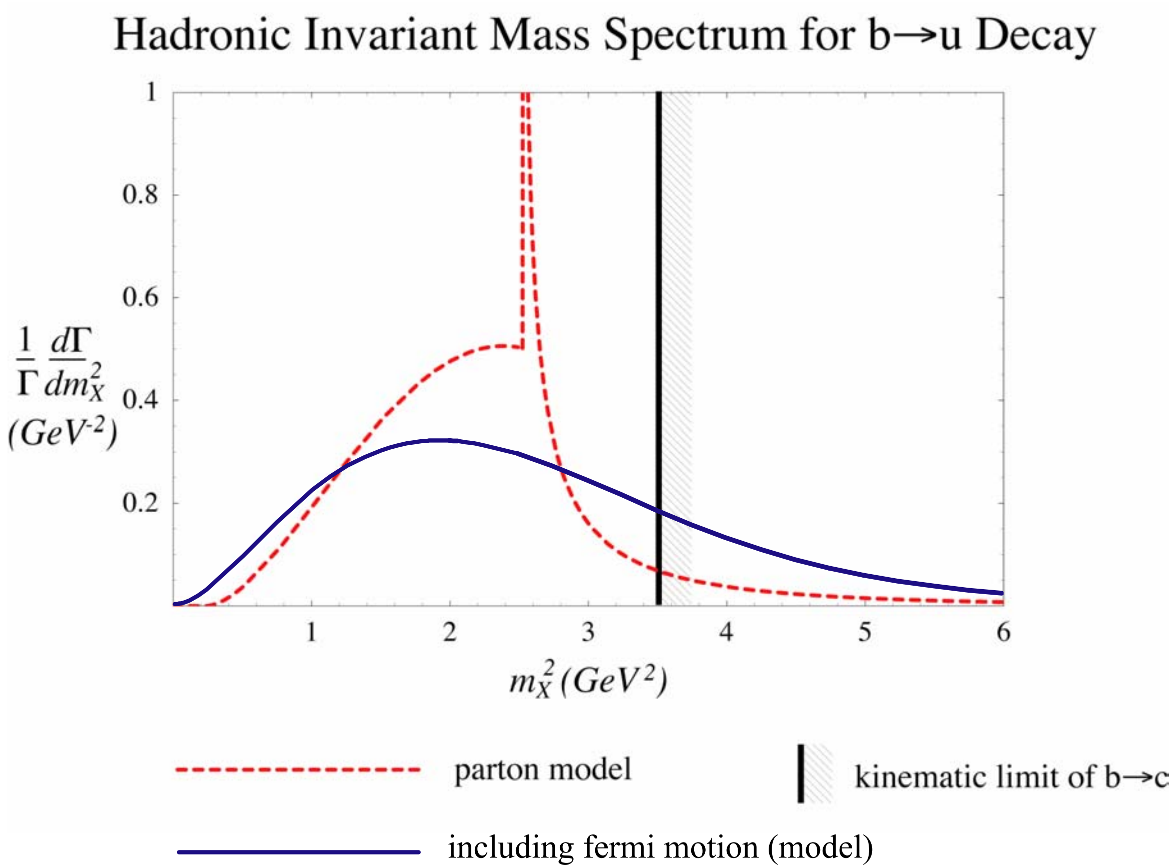




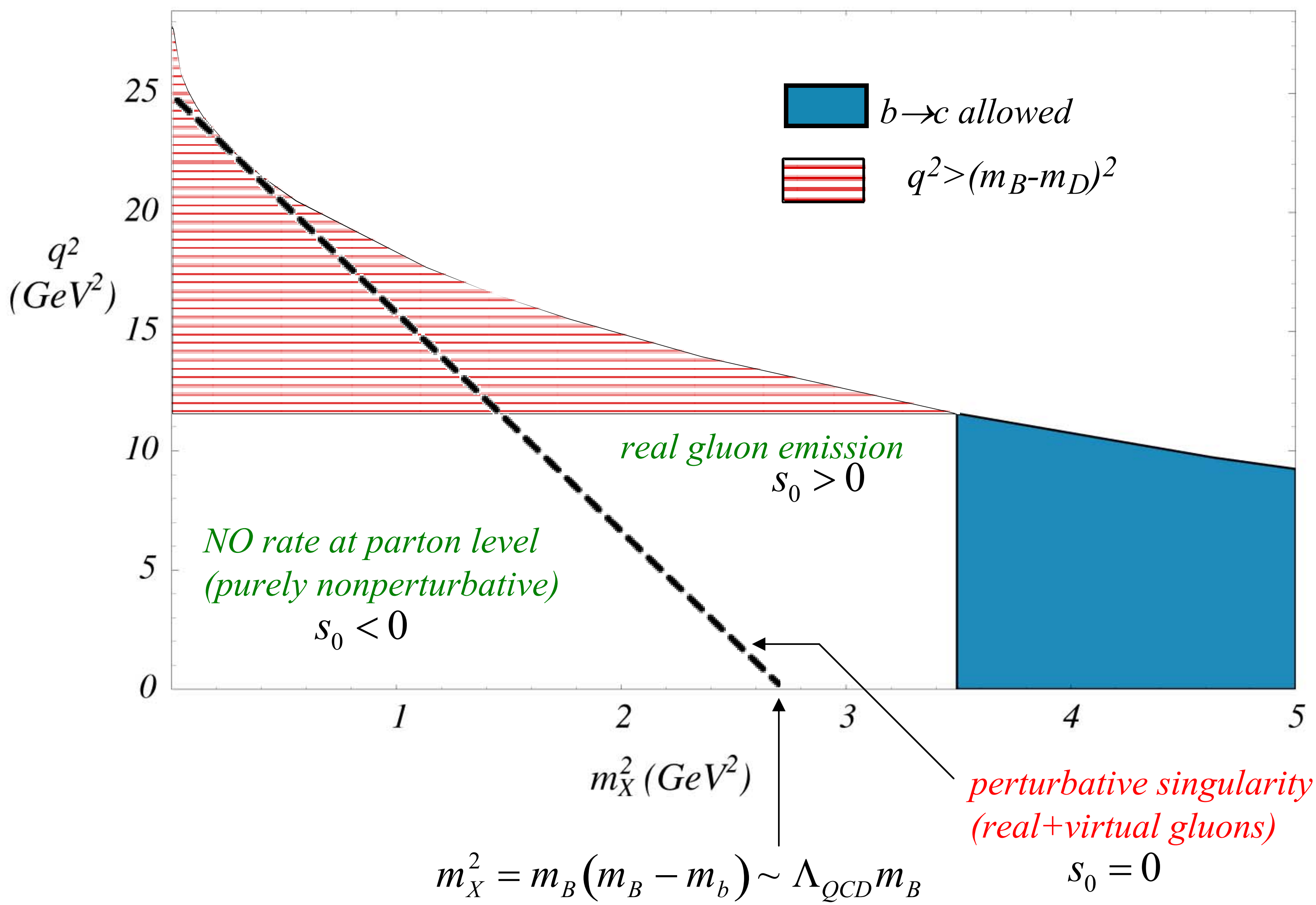
$k^+ = k^0 + k^3$ *(light cone momentum)*

$f(k^+)$ ~ parton distribution function: nonperturbative!

- $f(k^+)$ must be modeled
- moment of $f(k^+)$ are related to matrix elements of local operators (constrains models)

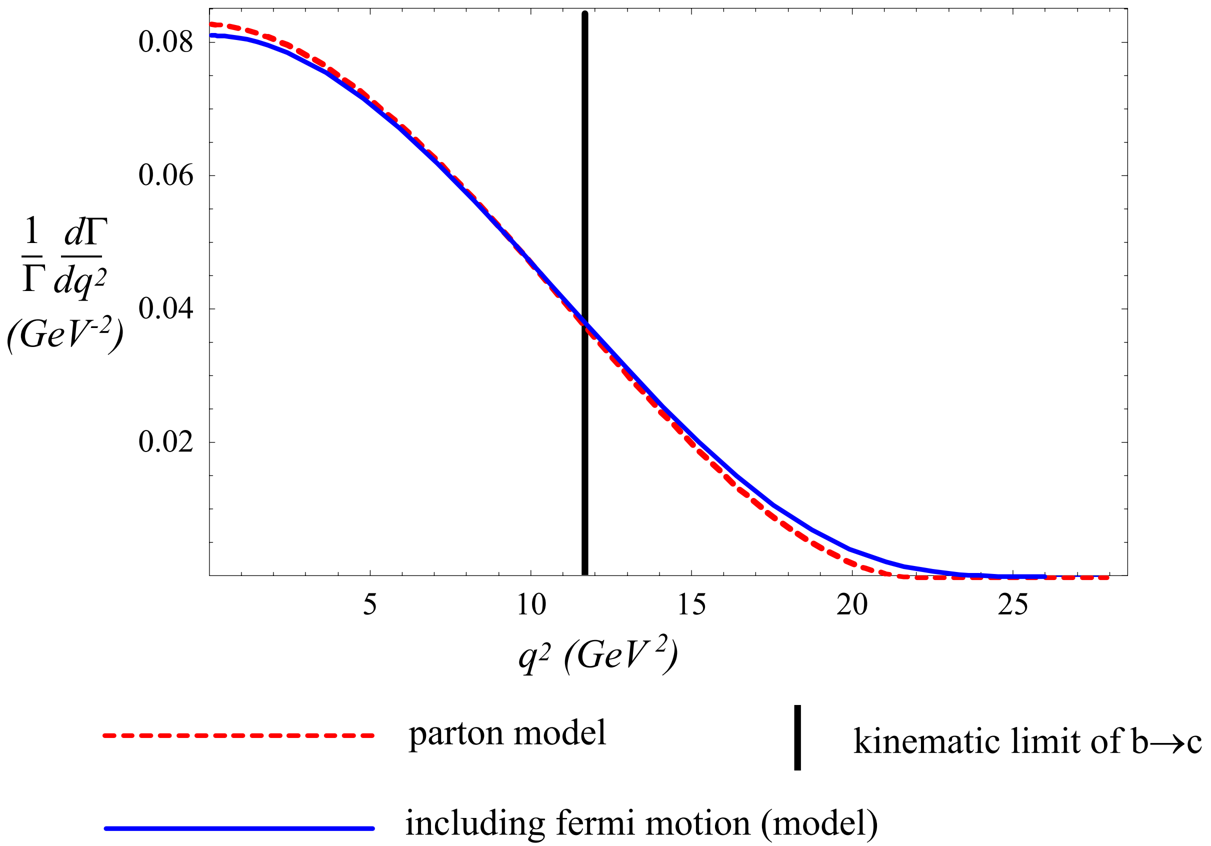


- singularity is smeared out by b quark light-cone distribution function $f(k_+)$
- rate is sensitive to details of $f(k_+)$ unless $m_X^2 >> \Lambda_{QCD} m_b$ (bad for $m_X < m_D$!) - introduces model dependence unless we know $f(k_+)$



b→c allowed
$q^2>(m_B-m_D)^2$
q^2 (GeV^2)
0
5
10
15
20
25
1
2
3
4
5
m_X^2 (GeV^2)
real gluon emission
$s_0>0$
NO rate at parton level
(purely nonperturbative)
$s_0<0$
perturbative singularity
(real+virtual gluons)
$s_0=0$
$m_X^2=m_B(m_B-m_b)\sim\Lambda_{QCD}m_B$

Lepton Invariant Mass Spectrum for b→u Decay



⇦ lepton q^2 spectrum is insensitive to Fermi motion (usual OPE holds) - eliminates model dependence! (counterintuitive … LESS inclusive = BETTER behaved)

(V. Barger et. al., PLB251 (1990) 629; A. Falk, Z. Ligeti and M. Wise, PLB406 (1997) 225; I. Bigi, R.D. Dikeman and N. Uraltsev, E.P.J C4 (1998) 453)

Pure m_X cut:

- gets ~80% of $B \to X_u$ `∫ rate (for ideal cut $m_X < m_D$)

BUT

- current theoretical predictions are strongly model dependent (*cf* DELPHI determination

$$|V_{ub}| = (4.07 \pm 0.37 \pm 0.44 \pm 0.33) \times 10^{-3}$$

has cut $m_X < 1.6$ GeV)

- *f(k₊)* can be extracted from photon spectrum in $B \to X_s \gamma$: error goes from formally $O(1)$ to $O(1/m_b)$ ... but size of higher twist terms is unknown (30%?) *(more on this later)*

(C. Bauer, Z. Ligeti and ML, PLB479 (2000) 395)

Pure q^2 cut:

- insensitive to $f(k_+)$ ... nonperturbative effects are subleading

- theory known to $O(1/m_b^3, \alpha_s^2)$

(Czarnecki and Melnikov, PRL88:131801,2002)

- leading & subleading renormalization group improvement known

(Neubert & Becher, hep-ph/0105217)

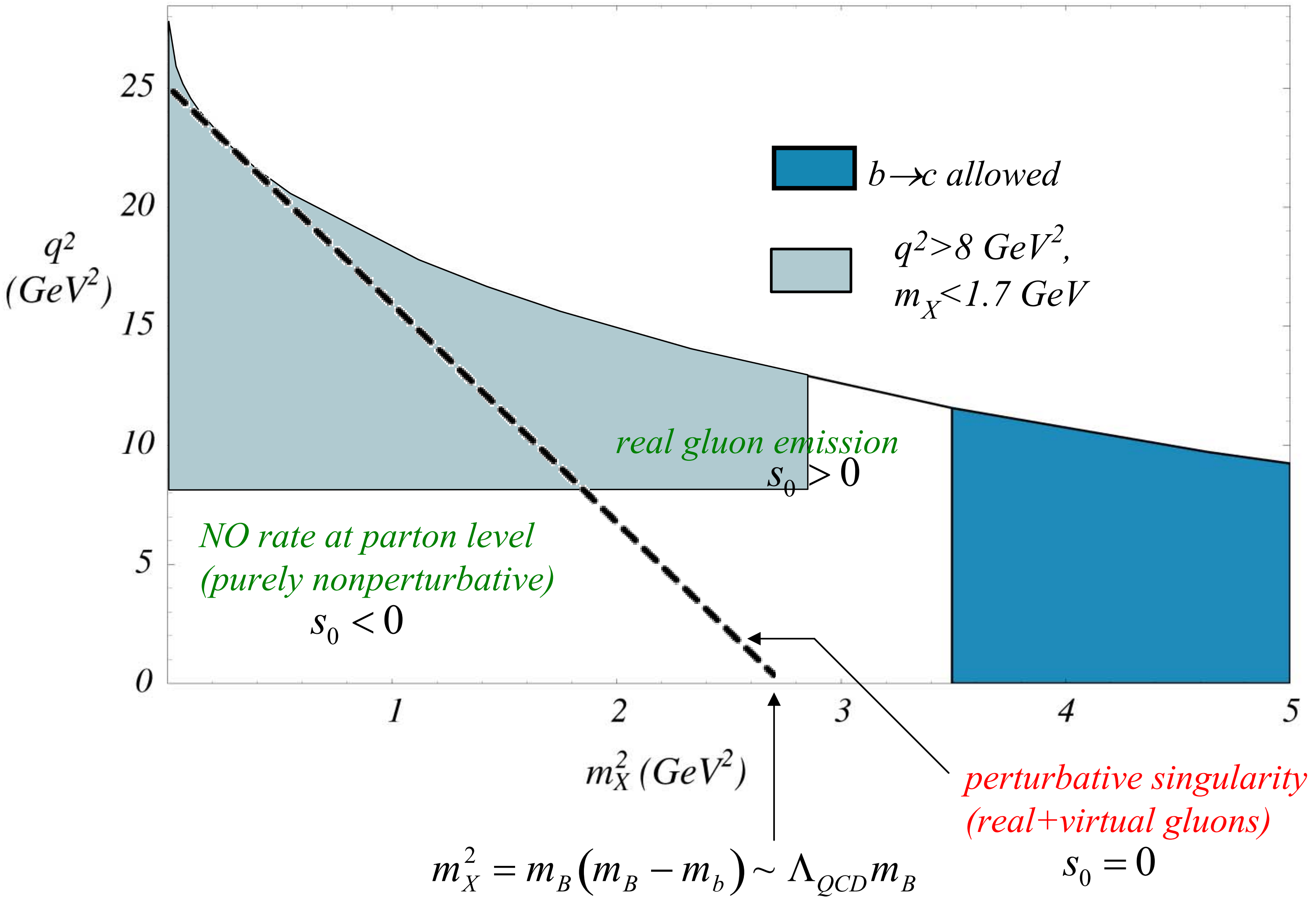
BUT

- nonperturbative corrections are large (reduced phase space $\Rightarrow$ ~$O(1/m_c^3)$, not $O(1/m_b^3)$) (Neubert, JHEP 0007:022 (2000))

- only ~20% of $B \to X_u$ `∫ rate

Optimized Cuts

(C. Bauer, Z. Ligeti and ML, hep-ph/0107074)

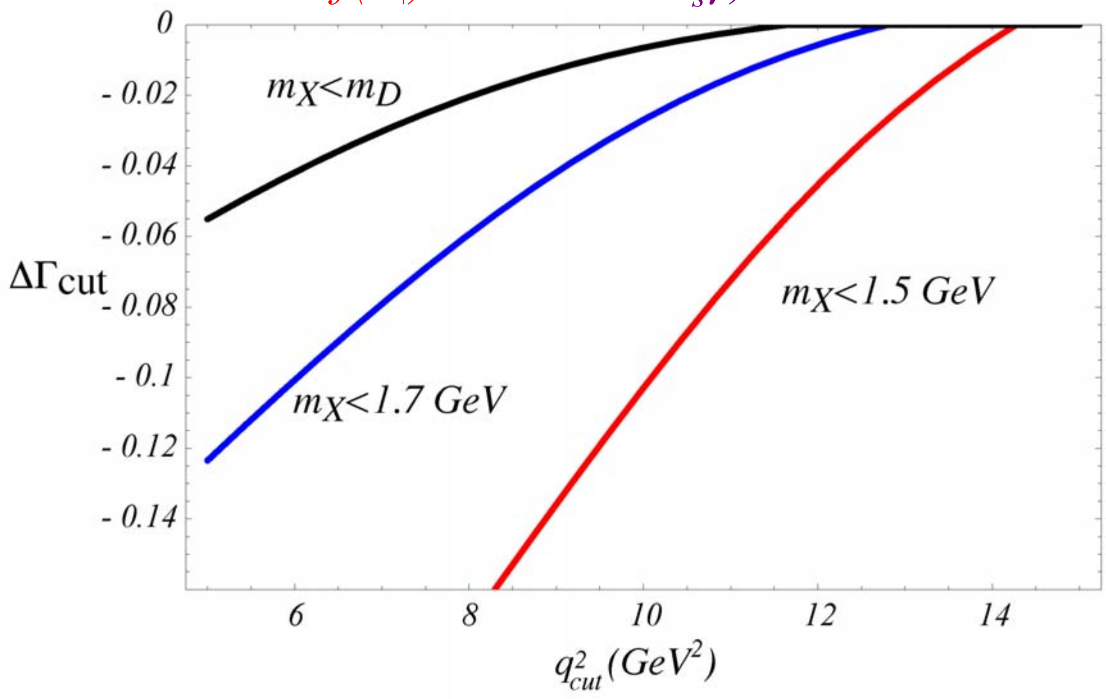


Effects of Fermi motion

Simple model:

$$f(k_+) = \frac{32}{\pi^2 \Lambda}\left(1 - \frac{k_+}{\Lambda}\right)^2 e^{-\frac{4}{\pi}\left(1-\frac{k_+}{\Lambda}\right)^2}$$

(use model to estimate sensitivity to Fermi motion, NOT to get final result! ... extract $f(k_+)$ from $B \to X_s \gamma$)



- do not need to know structure function well to have negligible uncertainty on $|V_{ub}|$

Strategy:

- combine lepton and hadron invariant mass cuts: larger rate, smaller errors than pure q^2 cut

- make cut on m_X as large as possible, keeping the background from $B \to$charm under control (depends on detector resolution, modeling of $B \to (D, D^*)$ `∫)

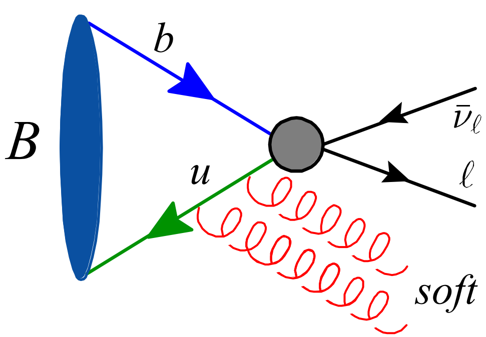
- make q^2 cut as low as possible, keeping the contribution from Fermi motion and perturbative uncertainties small

Additional uncertainties for optimized cuts:

• m_b - rate is proportional to m_b^5 … kinematic cuts also depend on m_b, so cut rate is MORE sensitive - need precise value of m_b! *(qu: is ±80 (±30) MeV error on m_b realistic? .. probably not yet)*

• perturbative corrections - known to $O(\alpha_s^2\beta_0)$

• weak annihilation (WA) - a potential problem for ALL inclusive determinations which include large q^2 region *(M. Voloshin, hep-ph/0106040)*



$$O\left(16\pi^2 \times \frac{\Lambda_{QCD}^3}{m_b^3} \times (\textit{factorization violation})\right) \sim 0.03\left(\frac{f_B}{0.2\ \text{GeV}}\right)\left(\frac{B_2 - B_1}{0.1}\right)$$

~3% (?? guess!) contribution to rate at $q^2 = m_b^2$

⇦ relative size of effect gets worse the more severe the cut

⇦ no reliable estimate of size - can test by comparing charged and neutral B's

Representative cuts:

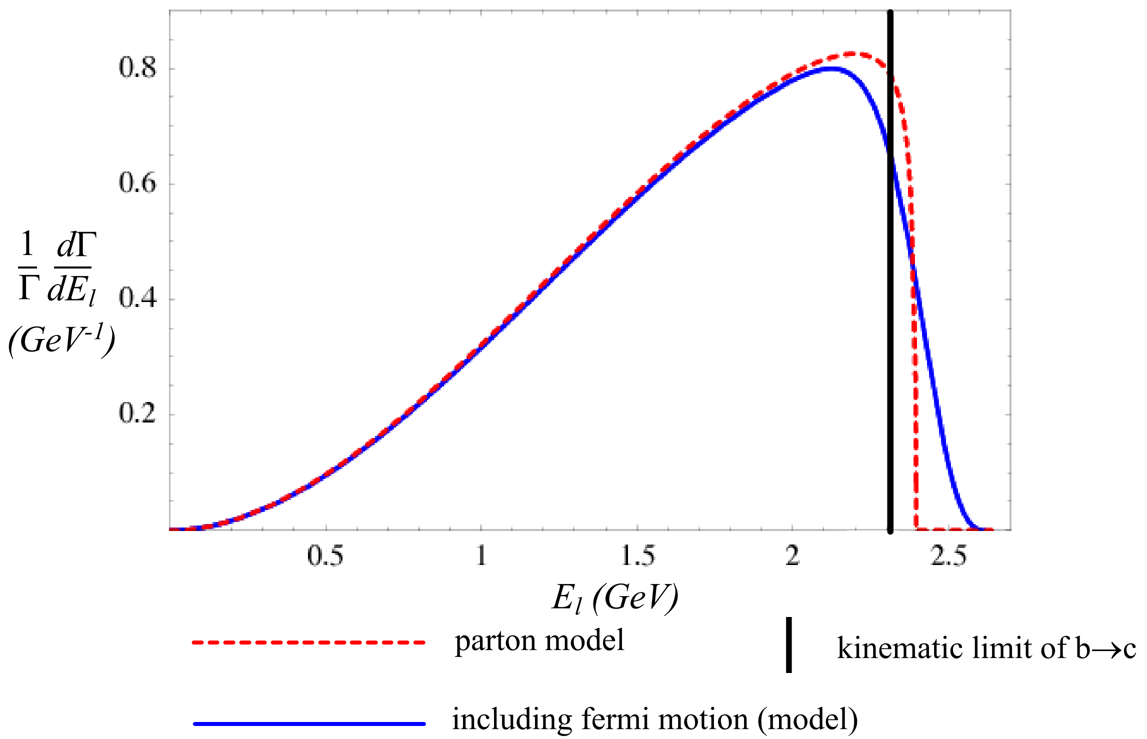
(a) $q^2>6$ GeV^2, $m_X<m_D$ 46% of rate

(b) $q^2>8$ GeV^2, $m_X<1.7$ GeV 33% of rate

(c) $q^2>11$ GeV^2, $m_X< 1.5$ GeV 18% of rate

Uncertainty	Size (in V_{ub})	Improvement?
Δm_b	± 80 MeV: 7%, 8%, 10% ± 30 MeV: 3%, 3%, 4%	RG improved Υ sum rules, moments of B decay spectra, lattice
α_s	2%, 3%, 7%	full two-loop calculation
$1/m_b^3$ (weak annihilation)	3%, 4%, 8%	compare $B^\pm$, B^0 compare S.L. width of D^0, D_S, lattice

Charge Lepton Energy Spectrum for b→u Decay



-rate above $B \to$ charm endpoint extremely sensitive to Fermi motion
(numerically, model dependence is stronger than for $\Gamma(m_X<m_D)$)
(lowering the cut (how well is charm background understood?) reduces sensitivity to $f(k_+)$)

- as with m_X spectrum, *f*(k_+) dependence may be eliminated by relating $d\Gamma/dE_e$ to photon spectrum in $B \to X_s\gamma$ (Neubert, PRD49 (1994) 4623)

Recent progress:

(1) relation between spectra worked out to NLO accuracy (subleading Sudakov logs resummed)
(Leibovich, Low, Rothstein, PRD61 (2000) 053006)

(2) contribution of operators other than O_7 included (large) (Neubert, hep-ph/0104280)

(3) O($1/m_b$) (higher twist) corrections relating *f*(k_+) in $B \to X_s\gamma$ to $B \to \mathrm{X_u}$ `∫ parametrized (decay Hamiltonians have different Dirac structure!) (Leibovich, Ligeti, Wise, hep-ph/0205148; Bauer, ML and Mannel, hep-ph/0205150)

The effects of subleading "shape functions" are surprisingly large ….

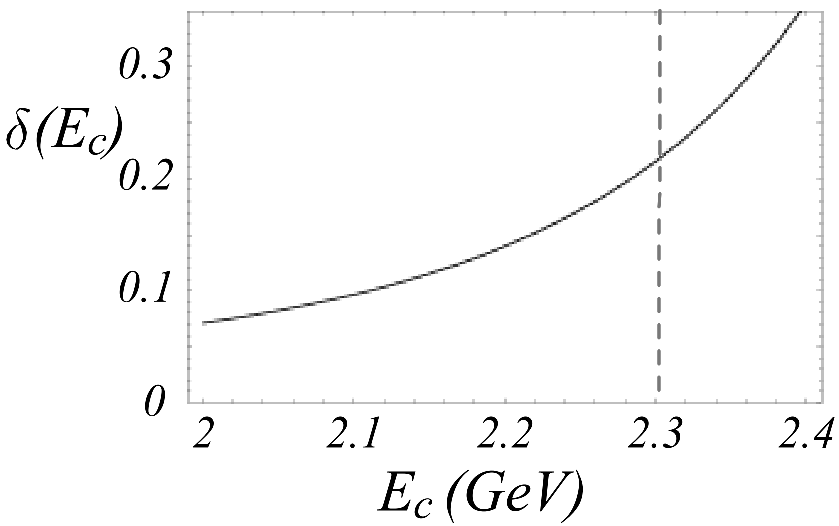
$$\frac{d\Gamma}{dy} \propto 2\theta(1-y) - \frac{\lambda_1}{3m_b^2}\delta'(1-y) - \frac{\rho_1}{9m_b^3}\delta''(1-y) + \ldots$$

leading twist terms … sum to $f(k_+)$

$$-\frac{\lambda_1}{3m_b^2}\delta(1-y) - \frac{11\lambda_2}{m_b^2}\delta(1-y) + \ldots$$

subleading twist terms … sum to new distribution functions

(corresponding coefficient in $B \to X_s \gamma$ is 3)



⇦ Simple Model: subleading effects are O(20%) (in V_{ub}) for $E_{cut}=2.3$ GeV, decrease with E_{cut}

Additional Caveats:

(a) weak annihilation is concentrated at endpoint of E_e spectrum ~3% correction to $B \rightarrow$ X`∫ rate ⇦ ~30% correction to rate in endpoint region ⇦ ~15% uncertainty in $|V_{ub}|$ *(another example of a higher twist effect)* *(see also* Leibovich, Ligeti, Wise, hep-ph/0205148*)*

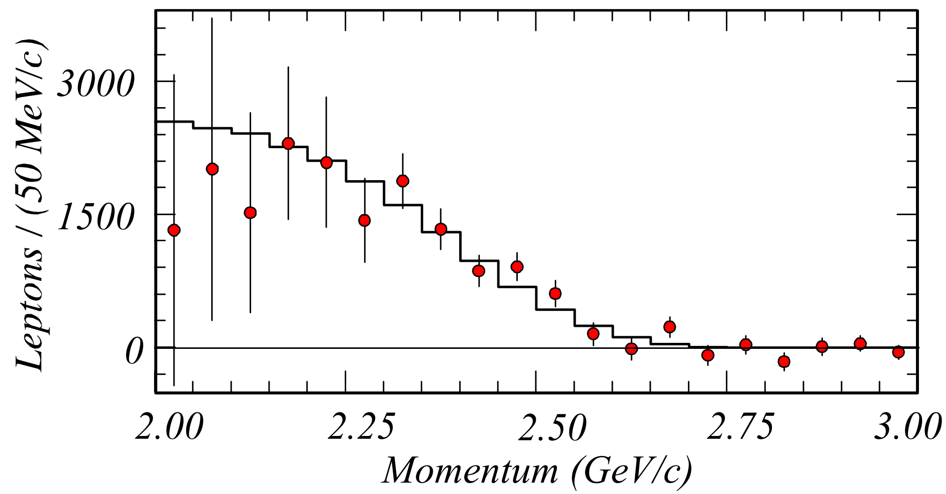
(b) very restricted phase space - duality problems?

CLEO '01:

exp't

$$\boxed{\left|V_{ub}\right| = (4.08 \pm 0.34 \pm 0.44 \pm 0.16 \pm 0.24) \times 10^{-3}}$$

Estimate of theoretical uncertainty (WA not included)



Summary

Theory and experiment have now evolved to the level that a model-independent, precision (10% level) determination of $|V_{ub}|$ is possible (!)

Exclusive Decays: Lattice

• $B \rightarrow$°`∫ for low $|p_{\circ}| < 1\ GeV$: need unquenched calculation of form factor. Other systematics appear under control ... when??

Inclusive Decays: OPE/twist expansion

• need to design cuts that exclude $b \rightarrow c$ without introducing large uncertainties

Theoretical reliability:

$$\boxed{(q^2, m_X)\ cut > q^2\ \text{cut} > m_X\ \text{cut} > E_e\ \text{cut}}$$

(experimental difficulty is in (roughly) the opposite order ...)

Experimental measurements can help beat down the theoretical errors:

(a) better determination of m_b (moments of B decay distributions)

(b) test size of WA (weak annihilation) effects - compare D^0 & D_S S.L. widths, extract $|V_{ub}|$ from $B^{\pm}$ and B^0 separately

(c) improve measurement of $B \to X_s \gamma$ photon spectrum - get $f(k_+)$ - $1/m_b$ corrections??

(d) (most important) measure $|V_{ub}|$ in as many CLEAN ways as possible - different techniques have different sources of uncertainty (*c.f.* inclusive and exclusive determinations of $|V_{cb}|$)