# Theory of $D^0$ - $\overline{D^0}$ mixing

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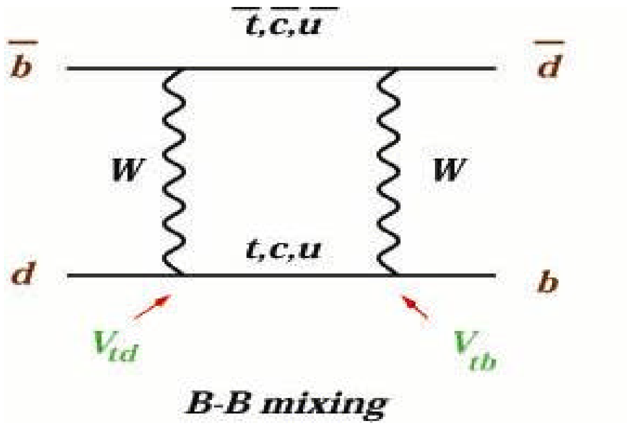
**Based on works with S. Bergmann, A.F. Falk, Y. Grossman, Z. Ligeti, Y. Nir**

# Introduction: why do we care?

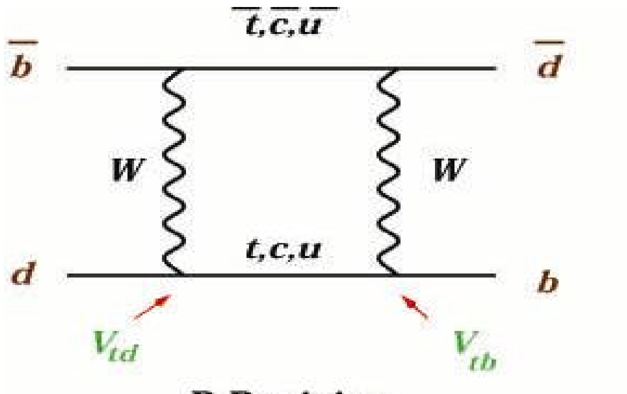



B-mixing:

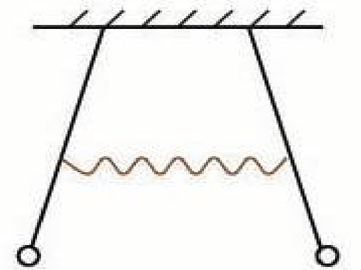
- $\Delta Q=2$: only at one loop in the Standard Model
- GIM mechanism: $rate \propto m_1^2 - m_2^2$
- sensitive to ultra-heavy particles in the loop

Expectation: *rate* is "large" in B system

# Introduction: why do we care?



B-B mixing


Coupled oscillators

B-mixing:

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$$\left|B(t)\right\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B^0}\right\rangle$$

Time-dependence: coupled Schrödinger equations

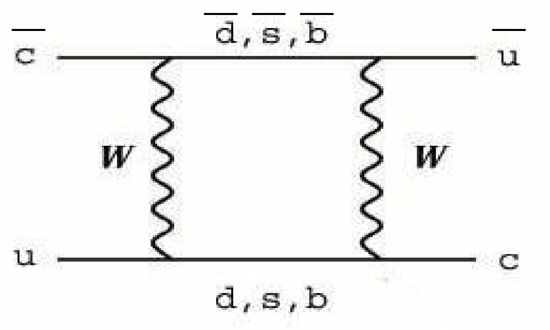
$$i\frac{\partial}{\partial t}\left|B(t)\right\rangle = \left(M - \frac{i}{2}\Gamma\right)\left|B(t)\right\rangle = \begin{bmatrix} A & p^2 \\ q^2 & A \end{bmatrix}\left|B(t)\right\rangle$$

Diagonalize: mass eigenstates $\neq$ flavor eigenstates

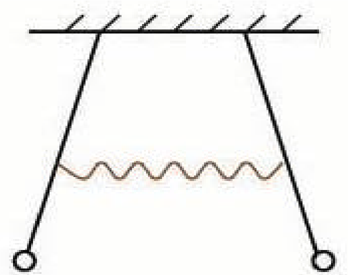
$$\left|B_{1,2}\right\rangle = p\left|B^0\right\rangle \pm q\left|\overline{B^0}\right\rangle$$

$$x = \frac{M_2 - M_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

# Introduction: why do we care?




D-D mixing



*Coupled oscillators*

D-mixing:

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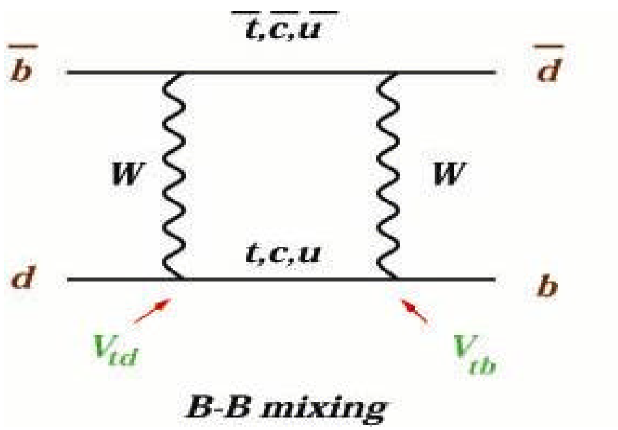
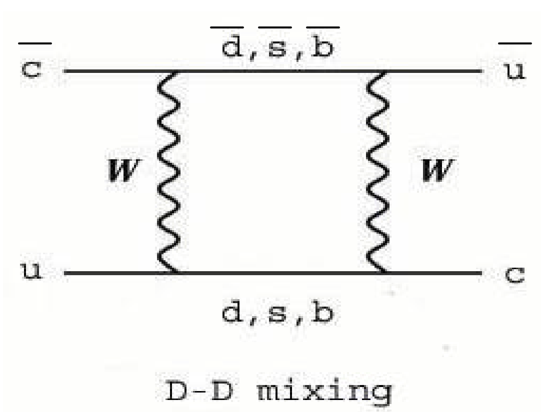


B-B mixing



D-D mixing

B-mixing:

- ΔQ=2: only at one loop in the Standard Model
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- sensitive to ultra-heavy particles in the loop

D-mixing:

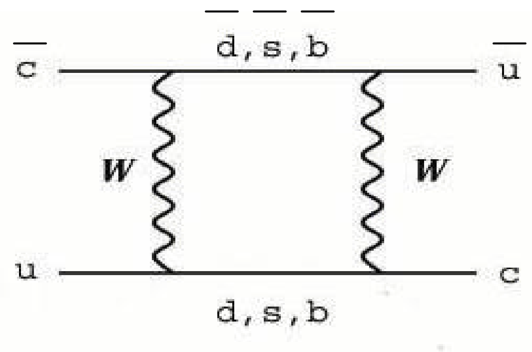
- the only probe of down-type quark dynamics
- GIM mechanism: no ultra-heavy quarks in the loop
- *b*-quark contribution $\propto V_{ub}\, m_b^2$ can be neglected

$$rate \propto f(m_s) - f(m_d) = 0 \quad (SU(3)_F \text{ limit})$$

- very sensitive to long-distance QCD, as $m_c \sim 1\, GeV$

Clean probe of New Physics?

# How would new physics affect mixing?




D-D mixing

D-D mass matrix:

Local operator ⇒ possible **New Physics!**

$$\left(M - \frac{i}{2}\Gamma\right)_{ij} = m_D^{(0)} d_{ij} + \frac{1}{2m_D}\left\langle D_i^0 \left| H_W^{\Delta C=2} \right| D_j^0 \right\rangle + \frac{1}{2m_D}\sum_I \frac{\left\langle D_i^0 \left| H_W^{\Delta C=1} \right| I \right\rangle \left\langle I \left| H_W^{\Delta C=1} \right| D_j^0 \right\rangle}{m_D^2 - m_I^2 + i\epsilon}$$

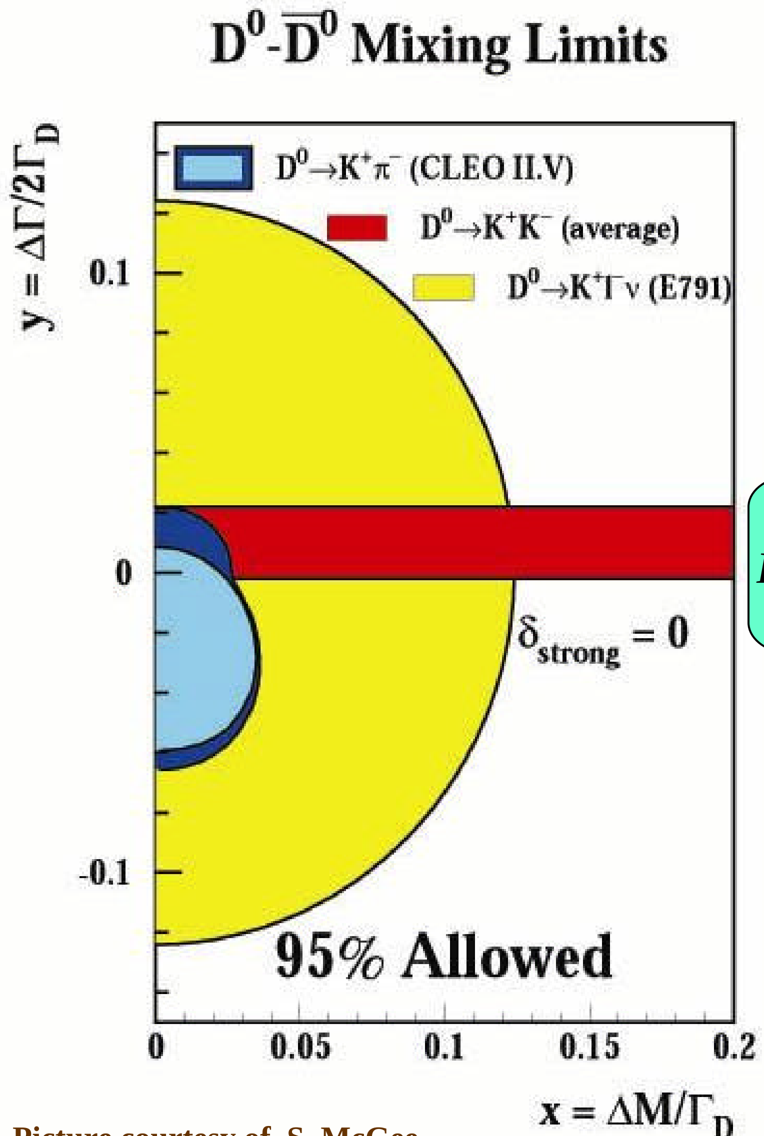
Real intermediate states, affect both x and y ⇒ **Standard Model**

1. $\boxed{x >> y}$ : signal for New Physics?
   $\boxed{x \approx y}$ : Standard Model?

2. CP violation in mixing/decay

*With b-quark contribution neglected*: only **2** generations contribute ⇒ **real 2x2 Cabibbo matrix**

# Experimental constraints




Picture courtesy of S. McGee

1. Time-dependent $D^0(t) \to K^+p^-$ analysis

$$\Gamma\left[D^0(t) \to K^+p^-\right] = e^{-\Gamma t}\left|A_{K^+p^-}\right|^2$$

$$\times\left[R + \sqrt{R}\,R_m\left(y'\cos f - x'\sin f\right)\Gamma t + \frac{R_m^2}{4}\left(y^2 + x^2\right)(\Gamma t)^2\right]$$

$$R = \left|\frac{A_{K^+p^-}}{\overline{A}_{K^+p^-}}\right|^2$$

$$R_m e^{if} = \frac{q}{p}$$

$if\ D^0 \leftrightarrow \overline{D^0}$:

$R_m \leftrightarrow R_m^{-1}$

$x' \leftrightarrow -x'$

2. Lifetime difference analysis

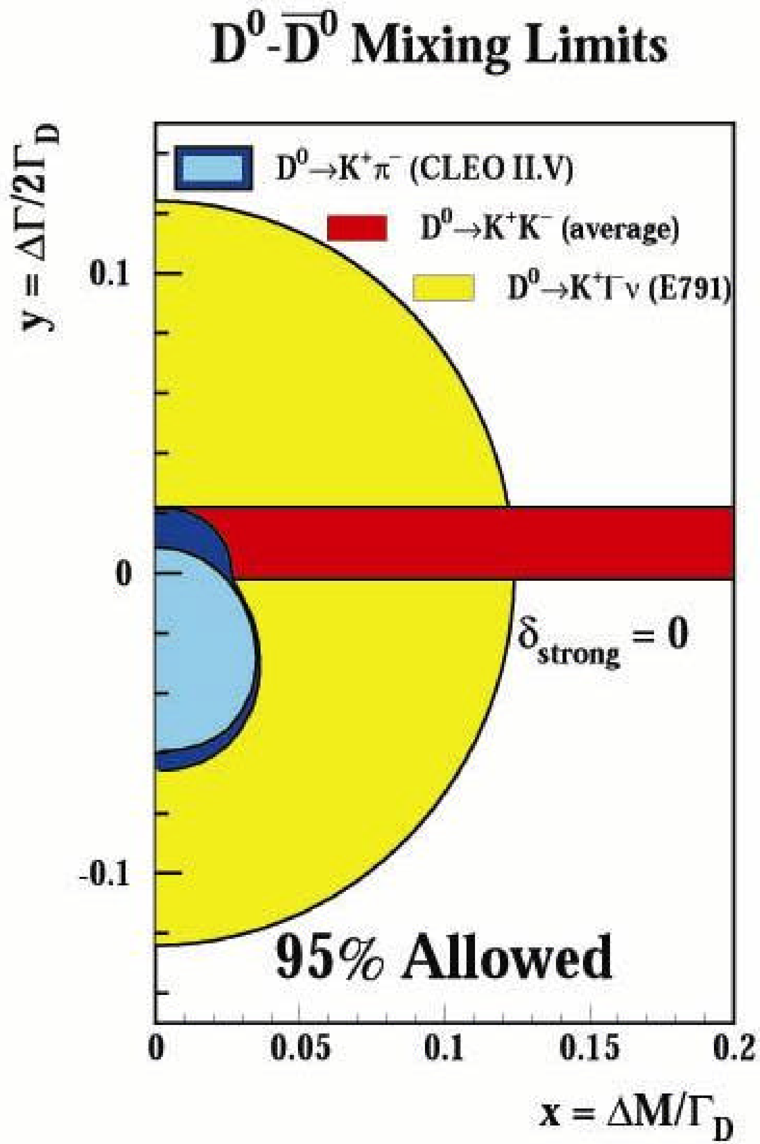
$$y_{CP} = \frac{t\left(D \to p^+K^-\right)}{t\left(D \to K^+K^-\right)} - 1 = y\cos f - x\sin f\,\frac{A_m}{2}$$

3. Semileptonic analysis

$$rate \propto x^2 + y^2$$

Quadratic in x,y: not so sensitive

# Experimental constraints




Several groups have measured $y_{CP}$

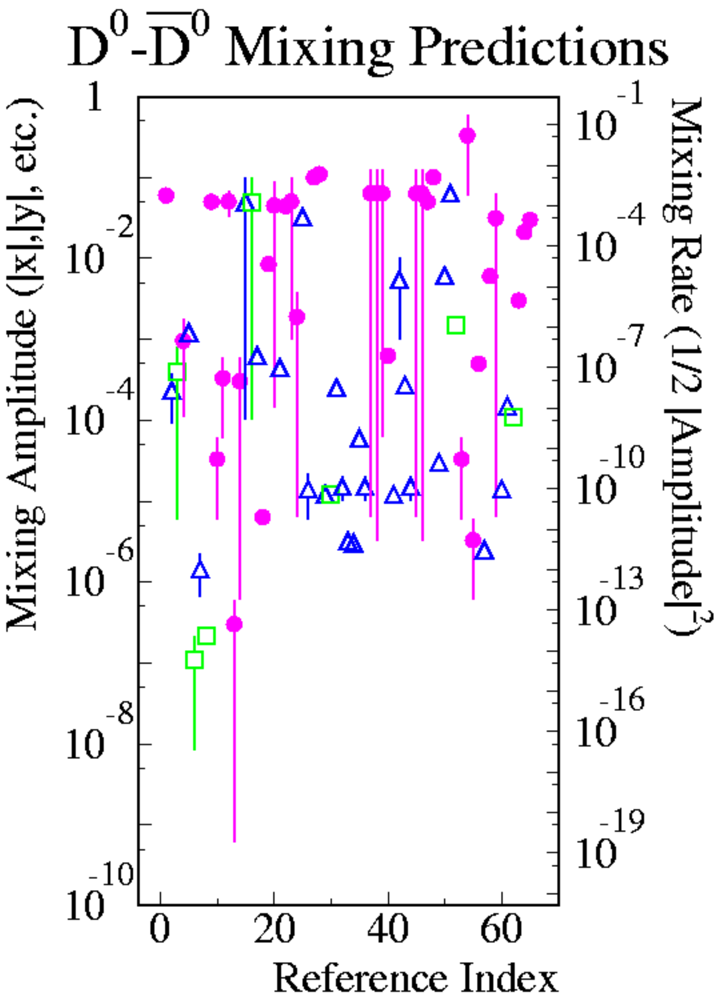
| Experiment | Value |
|---|---|
| FOCUS (2000) | (3.42±1.39±0.74)% |
| E791(2001) | (0.8±2.9±1.0)% |
| CLEO (2002) | (-1.2±2.5±1.4)% |
| Belle (2002) | $(-0.5\pm1.0\,^{+0.7}_{-0.8})\%$ |
| BaBar (2002) | $(1.4\pm1.0\,^{+0.6}_{-0.7})\%$ |

World average: (1.0±0.7)%

G. Raz

What are the expectations for x and y?

# Theoretical estimates

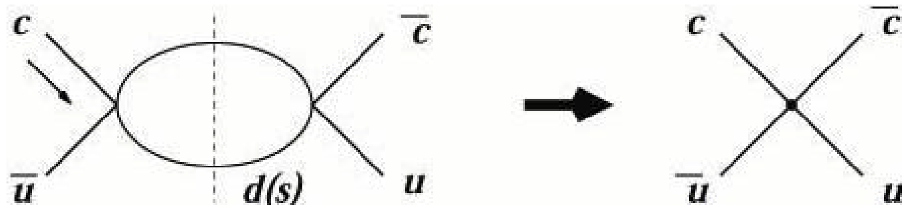



- Theoretical predictions are all over the board
- Can y ~ 1% be convincingly accommodated?
- Is it possible to have y >> x?
- Does it still mean that y ~ x?

# Theoretical estimates I

A. Short distance gives a tiny contribution, consider y as an example

$m_c$ is quite large !!!




$$y = \frac{1}{m_D \Gamma} \langle D^0 | ? | \overline{D}^0 \rangle$$

… as can be seen form the straightforward computation…

$$y_{sd} = \frac{N_C + 1}{2p\ N_C \Gamma} X_D \boxed{\frac{\left(m_s^2 - m_d^2\right)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2}} \Big[ C_2^2 + 2 C_1 C_2 + C_1^2 N_C$$
$$- \frac{2\left(2N_C - 1\right)}{1 + N_C} \frac{B_D'}{B_D} \frac{M_D^2 C_2^2}{\left(m_c + m_u\right)^2} \left( 1 + \left( N_C \frac{C_1^2}{C_2^2} + 2 \frac{C_1}{C_2} \right) \frac{2 - N_C}{2N_C - 1} \right)$$

with $\langle D^0 | \bar{u} \Gamma_m c \ \bar{u} \Gamma^m c | D^0 \rangle = \frac{1 + N_C}{N_C} \frac{4 F_D^2 m_D^2}{2 m_D} B_D, \ etc.$

4 unknown matrix elements

similar for x (trust me!)

# Theoretical estimates I

A. Short distance + “subleading corrections” (in $1/m_c$ expansion):

$$y_{sd}^{(6)} \propto \frac{\left(m_s^2 - m_d^2\right)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2} m_{had}^{-2} \propto m_s^6 \Lambda^{-6}$$

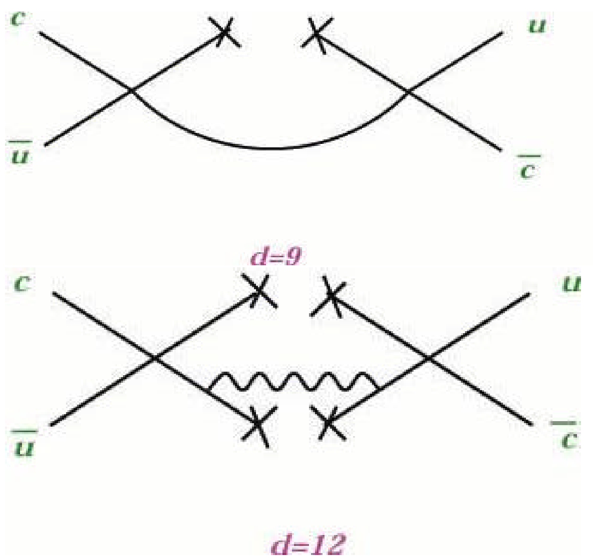
$$x_{sd}^{(6)} \propto \frac{\left(m_s^2 - m_d^2\right)^2}{m_c^2} m_{had}^{-2} \propto m_s^4 \Lambda^{-4}$$

4 unknown matrix elements

…subleading effects?

$$y_{sd}^{(9)} \propto m_s^3 \Lambda^{-3}$$

$$x_{sd}^{(9)} \propto m_s^3 \Lambda^{-3}$$




15 unknown matrix elements

Georgi, …
Bigi, Uraltsev

$$y_{sd}^{(12)} \propto b_0 \alpha_s^2(m) m_s^2 \Lambda^{-2}$$

$$x_{sd}^{(12)} \propto \alpha_s(m) m_s^2 \Lambda^{-2}$$

Twenty-something unknown matrix elements

Leading contribution!!!

Guestimate: $x \sim y \sim 10^{-3}$ ?

Resume: model-independent computation with model-dependent result

# Theoretical estimates II

$m_c$ is NOT large !!!

B. Long distance might give a large result? Let's see…

$$y = \frac{1}{2\Gamma}\sum_n r_n \left[ \left\langle D^0 \left| H_W^{\Delta C=1} \right| n \right\rangle \left\langle n \left| H_W^{\Delta C=1} \right| \overline{D}^0 \right\rangle + \left\langle \overline{D}^0 \left| H_W^{\Delta C=1} \right| n \right\rangle \left\langle n \left| H_W^{\Delta C=1} \right| D^0 \right\rangle \right]$$

… with n being all states to which $D^0$ and $\overline{D}^0$ can decay. Consider ππ, πK, KK intermediate states as an example…

$$y_2 = Br\left(D^0 \to K^+K^-\right) + Br\left(D^0 \to p^+p^-\right)$$
$$- 2\cos d \sqrt{Br\left(D^0 \to K^+p^-\right)Br\left(D^0 \to p^+K^-\right)}$$

cancellation expected!

If every Br is known up to *O(1%)* ⇒ the result is expected to be *O(1%)*!

The result here is a series of large numbers with alternating signs, SU(3) forces 0

x = ? Extremely hard…

need to restructure the calculation…

Resume: model-dependent computation with model-dependent result

## Questions:

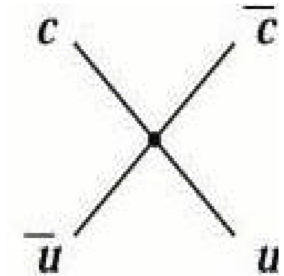
1. Can any model-independent statements be made for $x$ or $y$ ?

What is the order of SU(3) breaking?
i.e. if $x, y \propto m_s^n$ what is n?

2. Can one claim that $y \sim 1\%$ is natural?

# Theoretical expectations

At which order in $SU(3)_F$ breaking does the effect occur? Group theory?




$$\left\langle D^0 \right| H_W \, H_W \left| \overline{D}^0 \right\rangle \Rightarrow \left\langle 0 \right| D \, H_W \, H_W \, D \left| 0 \right\rangle$$

*is a singlet with* $D \to D_i$ *that belongs to* ***3*** *of* $SU(3)_F$ *(one light quark)*

The $\Delta C=1$ part of $H_W$ is $\left(\bar{q}_i c\right)\left(\bar{q}_j q_k\right)$, $i.e.\ 3 \times \bar{3} \times \bar{3} = \overline{15} + 6 + \bar{3} + \bar{3} \Rightarrow H_k^{ij}$

$$\begin{aligned} O_{\overline{15}} = & (\bar{s}d)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) \\ & - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s) \end{aligned}$$

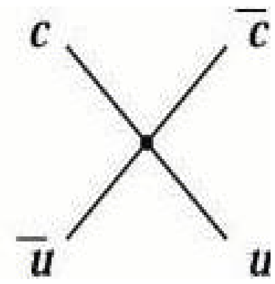
$\Rightarrow$

$$\begin{aligned} O_6 = & (\bar{s}d)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) \\ & - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s) \end{aligned}$$

Introduce SU(3) breaking via the quark mass operator $M_j^i = diag\,(m_u, m_d, m_s)$

All nonzero matrix elements built of $D_i, H_k^{ij}, M_j^i$ must be SU(3) singlets

# Theoretical expectations




*note that $D_i D_j$ is symmetric* $\Rightarrow$ *belongs to **6** of $SU(3)_F$*

$$\langle D^0 | H_W \, H_W | \bar{D}^0 \rangle \Rightarrow \langle 0 | D \, H_W \, H_W \, D | 0 \rangle$$

Explicitly,

$$DD \Rightarrow D_6$$
$$H_W H_W \Rightarrow O_{\overline{60}} + O_{42} + O_{15'}$$

1. No $\bar{6}$ in the decomposition of $H_W H_W \Rightarrow$ no SU(3) singlet can be formed

$\Rightarrow$ D mixing is prohibited by SU(3) symmetry

2. Consider a single insertion of $M^i_j \Rightarrow D_6 M$ transforms as $6 \times 8 = 24 + \overline{15} + 6 + \bar{3} \Rightarrow$ still no SU(3) singlet can be formed

$\Rightarrow$ NO D mixing at first order in SU(3) breaking

3. Consider double insertion of $M \Rightarrow DDM \; : \; 6 \times (8 \times 8)_S = \boxed{(60 + \overline{42}} + 24 + \overline{15} + \overline{15} + 6) + (24 + \overline{15} + 6 + \bar{3}) + 6$

$\Rightarrow$ D mixing occurs only at the second order in SU(3) breaking

**A.F., Y.G., Z.L., and A.A.P.**

# Theoretical expectations

• Does it always work? SU(3) breaking must enter perturbatively…

$$A_i = A_{SU(3)} + d_i$$

• Known counter-examples:

1. Very narrow light quark resonance with $m_R \sim m_D$

$$x, y \sim \frac{g_{DR}^2}{m_D^2 - m_R^2} \sim \frac{g_{DR}^2}{m_D^2 - m_0^2 - 2 m_0 d_R}$$

Most probably don't exists…

see E.Golowich and A.A.P.

2. Part of the multiplet is kinematically forbidden

Example: both $D^0 \to 4p \;\; and \;\; D^0 \to 4K$ are from the same multiplet, but the latter is kinematically forbidden

see A.F., Y.G., Z.L., and A.A.P.

# Theoretical expectations

- Two major sources of SU(3) breaking

1. phase space

$$m_K \neq m_p \neq m_h \ldots$$

2a. matrix elements (absolute value)

$$f_K \neq f_p \ldots$$

2b. matrix elements (phases aka FSI)

$$\mathrm{Im}\,\frac{A\left(D^0 \to K^+ p^-\right)}{A(\overline{D}^0 \to K^+ p^-)} \neq 0$$

Take into account only the first source (computable)!

# SU(3) and phase space

- "Repackage" the analysis: look at the complete multiplet contribution

$$y = \sum_{F_R} y_{F,R} Br\left(D^0 \to F_R\right) \sim \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma\left(D^0 \to n\right)$$

y for each SU(3) multiplet

Each is **0** in SU(3)

- Does it help? If only phase space is taken into account: **no** **(mild)** model dependence

$$y_{F,R} = \frac{\sum_{n \in F_R} \langle \overline{D}^0 | H_W | n \rangle r_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle r_n \langle n | H_W | D^0 \rangle}$$

if CP is conserved

$$= \frac{\sum_{n \in F_R} \langle \overline{D}^0 | H_W | n \rangle r_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \to n)}$$

Can consistently compute !

# Results

| Final state representation | | $y_{F,R}/s_1^2$ | $y_{F,R}$ (%) |
|---|---|---|---|
| $PP$ | 8 | −0.0038 | −0.018 |
| | 27 | −0.00071 | −0.0034 |
| $PV$ | $8_S$ | 0.031 | 0.15 |
| | $8_A$ | 0.032 | 0.15 |
| | 10 | 0.020 | 0.10 |
| | $\overline{10}$ | 0.016 | 0.08 |
| | 27 | 0.040 | 0.19 |
| $(VV)_{s\text{-wave}}$ | 8 | −0.081 | −0.39 |
| | 27 | −0.061 | −0.30 |
| $(VV)_{p\text{-wave}}$ | 8 | −0.10 | −0.48 |
| | 27 | −0.14 | −0.70 |
| $(VV)_{d\text{-wave}}$ | 8 | 0.51 | 2.5 |
| | 27 | 0.57 | 2.8 |

| Final state representation | | $y_{F,R}/s_1^2$ | $y_{F,R}$ (%) |
|---|---|---|---|
| $(3P)_{s\text{-wave}}$ | 8 | −0.48 | −2.3 |
| | 27 | −0.11 | −0.54 |
| $(3P)_{p\text{-wave}}$ | 8 | −1.13 | −5.5 |
| | 27 | −0.07 | −0.36 |
| $(3P)_{\text{form-factor}}$ | 8 | −0.44 | −2.1 |
| | 27 | −0.13 | −0.64 |
| $4P$ | 8 | 3.3 | 16 |
| | 27 | 2.2 | 9.2 |
| | 27′ | 1.9 | 11 |

- Product is naturally O(1%)
- No (symmetry-enforced) cancellations
- Does NOT occur for x

| Final state | fraction |
|---|---|
| $PP$ | 5% |
| $PV$ | 10% |
| $(VV)_{s\text{-wave}}$ | 5% |
| $(VV)_{d\text{-wave}}$ | 5% |
| $3P$ | 5% |
| $4P$ | 10% |

naturally implies that *y*~*1%* and *x* < *y* !

# Conclusions

- x,y=0 in the SU(3) limit (as $V_{ub}$ is very small)
- it is a second order effect
- it is quite possible that $y \sim 1\%$ with $x<y$

if true, search for New Physics
is complicated

- expect new data from BaBar/Belle/CLEO/CLEOc/CDF(?)
- currently: $x \approx (\pm 2.8 \pm 2.5)\%, \quad y \approx (\pm 0.9 \pm 3.6)\%$ (allowing NP)
- CP-violation in mixing is a "smoking gun" signal for New Physics