A Heterotic Standard Model

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Overview

String Theory

Introduction

The Calabi-Yau

Vector Bundles

A Heterotic Standard Model

Superpotential

Fin

joint work with Burt A. Ovrut, Tony Pantev, Yang Hui-He:

hep-th/0410055: Elliptic Calabi-Yau Threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson Lines
hep-th/0501070: A Heterotic Standard Model
hep-th/0502155: A Standard Model from the $E_8 \times E_8$ Heterotic Superstring
hep-th/0505041: Vector Bundle Extensions, Sheaf Cohomology, and the Heterotic Standard Model
hep-th/0509051: Heterotic Standard Model Moduli
hep-th/0510142: Moduli Dependent μ-Terms in a Heterotic Standard Model

Summary

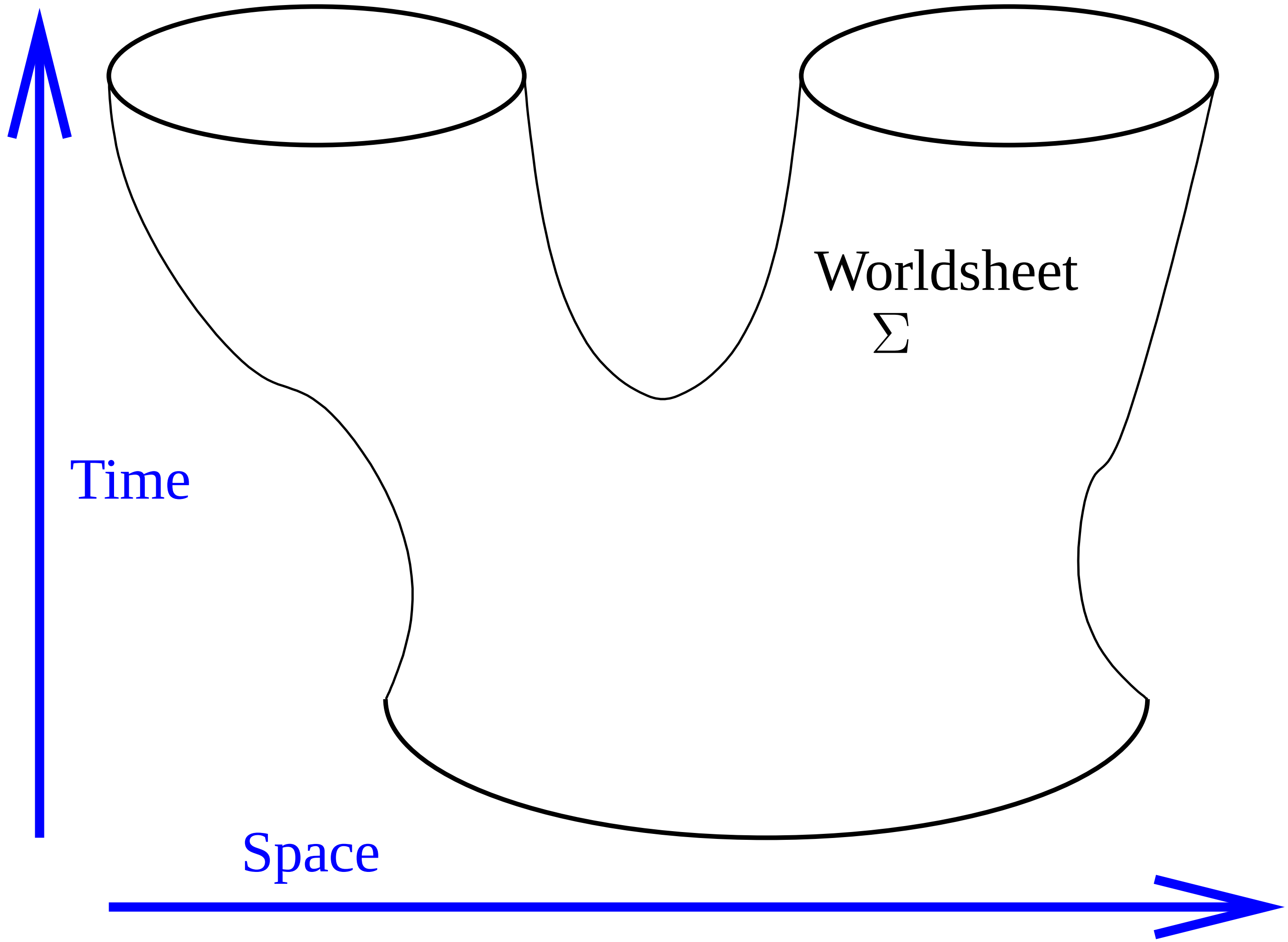
We found a compactification of the $E_8 \times E_8$ heterotic string with low-energy physics

- $N = 1$, $d = 4$ QFT.
- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.
- Exact MSSM matter spectrum.
- No vector-like pairs (except Higgs).
- No exotics, no triplets.
- $3 + 3 + 13$ moduli.
- Yukawa couplings and no Higgs μ-term.
- Gravity, axion-dilaton.

String Theory

Strings

Fundamental objects are “strings”



Lagrangian

Worldsheet fields:
Coordinates X and $2d$ metric g.

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$10d$ metric G, B-field B, and Dilaton Φ.

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$$S = \frac{1}{2\pi\alpha'} \int_\Sigma \mathrm{d}^2 z \sqrt{g} \times$$

$$\times \left[\Big(g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X) \Big) \partial_a X^\mu \partial_b X^\nu + \right.$$

$$\left. + \alpha' R \Phi(X) \right]$$

$E_8 \times E_8$ Heterotic String

Now add $(0,2)$ supersymmetry and quantize.

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Heterotic String

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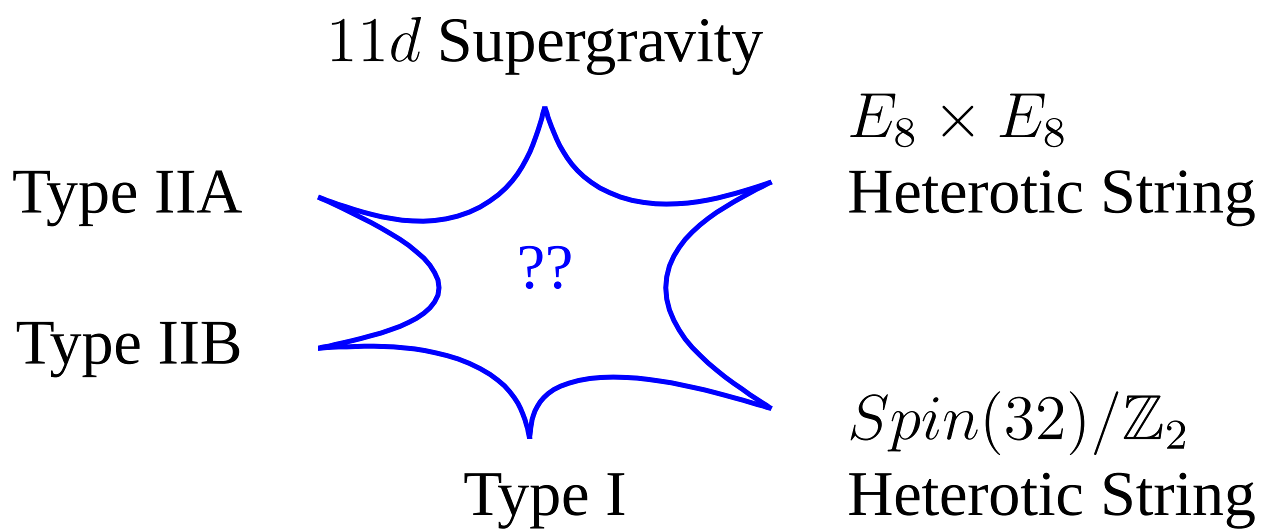

Focus on a certain way to add fermions: The $E_8 \times E_8$ heterotic string

Different string theories are connected by dualities

Results

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- $6d$ space X is small, scale $\sim 10^{15}$GeV.
- Want to preserve $N = 1$ supersymmetry $\Rightarrow X$ is Calabi-Yau.

Calabi-Yau

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- A smooth algebraic variety with vanishing first Chern class.

Compactification I

So compactify on Calabi-Yau manifold X....

What does the low-energy $4d$ physicist see?

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- $\dim H^{2,1}(X)$ complex structure moduli,
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- Anomalous unless $X = T^6$.

Gauge Instanton

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- "Wilson lines": discrete subgroups $\mathbb{Z}_n \subset E_8 \times E_8'$.

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Note: E_8' is hidden sector.

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Introduction

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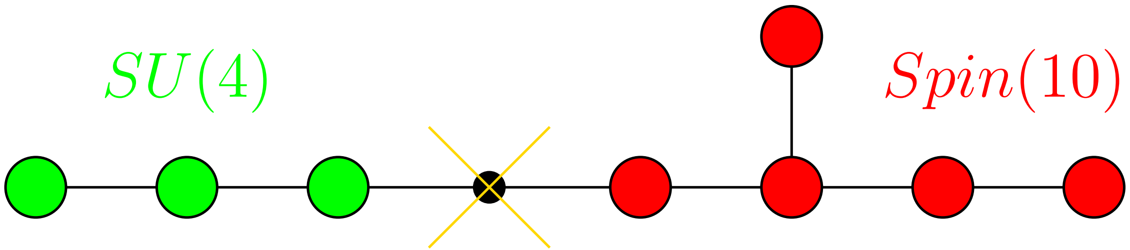
  $\Rightarrow$ proton decay suppressed.
- No exotic matter, no vector-like pairs.
- All of the ordinary matter fields (including right-handed Neutrino).

Gauge Instanton

Maximal regular subgroup $SU(4) \times Spin(10) \subset E_8$:



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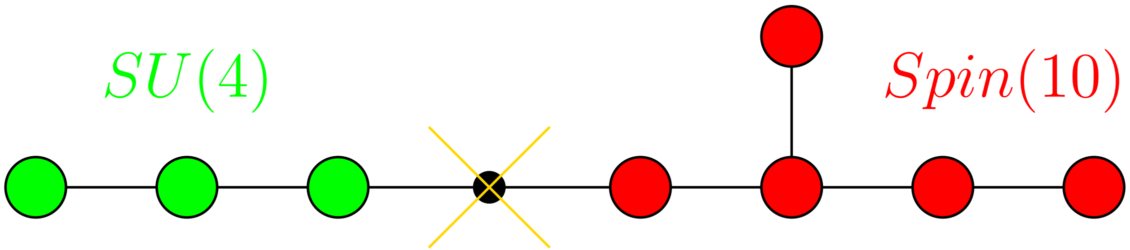

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The adjoint of E_8 (fermions in the $E_8 \times E_8$ heterotic string) decomposes as

$$\mathbf{248} = (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{15}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10})$$

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- matter fields charged in the $Spin(10)$,
- moduli.

More details on the next slide...

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- $\dim H^1(X, V \otimes V^\vee)$ vector bundle moduli.

Wilson Lines

Ancient Lore: $Spin(10)$ GUT with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines "works":

16 of $Spin(10)$: Breaks into one family of quarks and leptons including a right-handed Neutrino.

$\overline{\mathbf{16}}$ of $Spin(10)$: Anti-family.

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Compactification scale $\sim$ GUT scale
... but nice way to package representations.

Characters

$$G = \mathbb{Z}_3 \times \mathbb{Z}_3 = G_1 \times G_2$$

Fix generators g_1 and g_2.

Characters (=1-d representations): Denote generators by χ_1 and χ_2, where ($\omega = e^{\frac{2\pi i}{3}}$)

$$\chi_1(g_1) = \omega \qquad \chi_1(g_2) = 1$$
$$\chi_2(g_1) = 1 \qquad \chi_2(g_2) = \omega\,.$$

All other characters are products of χ_1 and χ_2.

Wilson Line Breaking

$$Spin(10) \supset SU(3)\times SU(2)\times U(1)\times U(1)\times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\left\{ \begin{array}{c} \text{Standard Model} \\ \text{gauge group} \end{array} \right\} \times U(1)_{B-L} \times \{\text{Wilson lines}\}$$

$\mathbb{Z}_3 \times \mathbb{Z}_3$ is smallest Wilson line possible.

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$$Spin(10) \supset SU(3)\times SU(2)\times U(1)\times U(1)\times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\begin{aligned}\mathbf{16} = {} & \chi_1^2\chi_2(\mathbf{3},\mathbf{2},1,1) \oplus \chi_1^2(\mathbf{1},\mathbf{1},6,3) \oplus \\ & \oplus \chi_1^2\chi_2^2(\overline{\mathbf{3}},\mathbf{1},-4,-1) \oplus \chi_2^2(\overline{\mathbf{3}},\mathbf{1},2,-1) \oplus \\ & \oplus (\mathbf{1},\overline{\mathbf{2}},-3,-3) \oplus \chi_1(\mathbf{1},\mathbf{1},0,3)\end{aligned}$$

$$\begin{aligned}\mathbf{10} = {} & \chi_1(\mathbf{1},\mathbf{2},3,0) \oplus \chi_1\chi_2(\mathbf{3},\mathbf{1},-2,-2) \oplus \\ & \oplus \chi_1^2(\mathbf{1},\overline{\mathbf{2}},-3,0) \oplus \chi_1^2\chi_2^2(\overline{\mathbf{3}},\mathbf{1},2,2)\end{aligned}$$

Wilson Line Breaking

$$Spin(10) \supset SU(3) \times SU(2) \times U(1) \times U(1) \times \mathbb{Z}_3 \times \mathbb{Z}_3$$

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& \oplus \chi_1^2 \chi_2^2 (\overline{\mathbf{3}}, \mathbf{1}, -4, -1) \oplus \chi_2^2 (\overline{\mathbf{3}}, \mathbf{1}, 2, -1) \oplus \\
& \oplus (\mathbf{1}, \overline{\mathbf{2}}, -3, -3) \oplus \boxed{\chi_1 (\mathbf{1}, \mathbf{1}, 0, 3)} \\
\mathbf{10} = {} & \chi_1 (\mathbf{1}, \mathbf{2}, 3, 0) \oplus \chi_1 \chi_2 (\mathbf{3}, \mathbf{1}, -2, -2) \oplus \\
& \oplus \chi_1^2 (\mathbf{1}, \overline{\mathbf{2}}, -3, 0) \oplus \chi_1^2 \chi_2^2 (\overline{\mathbf{3}}, \mathbf{1}, 2, 2)
\end{aligned}$$

Right-handed Neutrino

Wish List

To make use of this group theory, we would like

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- With the “right” cohomology groups (low energy spectrum).

The Calabi-Yau

Covering Space

Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

Covering Space

Work with		Have in mind
Simply connected Calabi-Yau threefold $\widetilde{X}$ with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action	$=$	Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

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Work with		Have in mind
Simply connected Calabi-Yau threefold $\widetilde{X}$ with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action	$=$	Calabi-Yau threefold X with $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$
elliptically fibered (torus fibered with section)		torus fibered (assuming $\mathbb{Z}_3 \times \mathbb{Z}_3$ preserves fibration)

Fiber Product

Our Calabi-Yau is a double elliptic fibration

$$
\begin{array}{lccc}
\dim_{\mathbb{C}} = 3 : & & \widetilde{X} & \\
& \swarrow^{\pi_1} & & \searrow^{\pi_2} \\
\dim_{\mathbb{C}} = 2 : & B_1 & & B_2
\end{array}
$$

B_1 and B_2 are dP_9 surfaces.

$$\pi_1{}^{-1}\big(\{pt.\}\big) = T^2 , \quad \pi_2{}^{-1}\big(\{pt.\}\big) = T^2 .$$

Properties

- $\widetilde{X}$ is a simply connected Calabi-Yau threefold, $\pi_1(\widetilde{X}) = 0$.

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- Group actions on B_1, B_2 lift to $\widetilde{X}$.

Group Actions

We classified all $\mathbb{Z}_3 \times \mathbb{Z}_3$ actions on the dP_9 surfaces. $\Rightarrow$ all $\mathbb{Z}_3 \times \mathbb{Z}_3$ actions on $\widetilde{X}$.

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- One $3+3$ parameter family with a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action.
- $2+2$ and $1+1$ parameter families with fixed points.
- If $\widetilde{X}$ has no $\mathbb{Z}_3 \times \mathbb{Z}_3$ fixed points then

$$X \stackrel{\text{def}}{=} \widetilde{X} \Big/ (\mathbb{Z}_3 \times \mathbb{Z}_3)$$

is smooth, otherwise it has singularities.

Torus Orbifolds

Orbifold $T^6 = T^2 \times T^2 \times T^2$ by group action

$$\big(z_1, z_2, z_3\big) \mapsto \big(-z_1, z_2, -z_3\big)$$

$$\big(z_1, z_2, z_3\big) \mapsto \left(z_1, -z_2, \frac{1}{2} - z_3\right)$$

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- $3 + 3$ parameter family of orbifolds.
- $16 + 16$ parameter resolve singularities.

Moduli Space

$$\mathcal{M}(\widetilde{X})_{\text{Kähler}} \times \mathcal{M}(\widetilde{X})_{\text{complex}}$$

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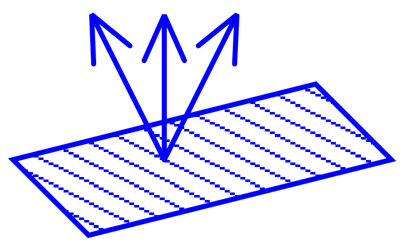
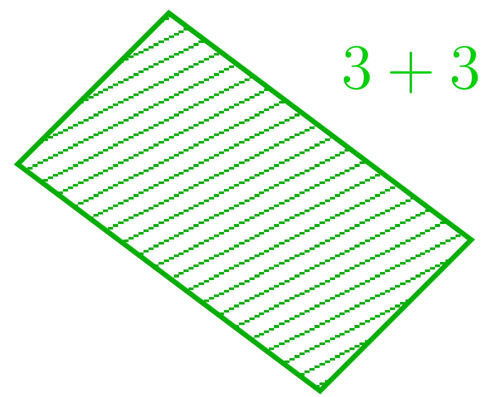
$16 + 16$ Blow-up directions

Free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action

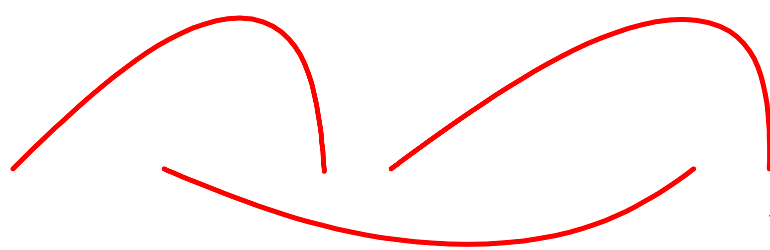




$3 + 3$-dim

$3 + 3$ parameter family

$T^6 / (\mathbb{Z}_3 \times \mathbb{Z}_3)$



$1 + 1$ and $2 + 2$-dim with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fixed points

Invariant Cohomology

In the following, only consider the $3+3$ parameter family with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action.

$$\Rightarrow \qquad H^{p,q}\big(X\big) = H^{p,q}\big(\widetilde{X}\big)^G$$

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Hodge diamond
$$h^{p,q}(X) = \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix}$$

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$h^{1,1}(X)$ = 3-dim. lattice of line bundles.

Vector Bundles

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Every line bundle is of the form

- $\mathcal{O}_{\widetilde{X}}(x_1\tau_1 + x_2\tau_2 + x_3\phi)$, $x_1, x_2, x_3 \in \mathbb{Z}$.
- $\mathcal{O}_{B_i}(y_1 t_i + y_2 f_i)$, $y_1, y_2 \in \mathbb{Z}$.

Equivariant Line Bundles

Recall: $X = \widetilde{X}/G \,, \quad G \stackrel{\text{def}}{=} \mathbb{Z}_3 \times \mathbb{Z}_3.$

Line bundles on $X = \widetilde{X}/G$

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G-equivariant line bundles on $\widetilde{X}$	=	Line bundles on $X = \widetilde{X}/G$

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An equivariant line bundle is a line bundle $\mathcal{L}$ together with a group action $\gamma : G \times \mathcal{L} \to \mathcal{L}$

$\mathcal{L}$ equivariant line bundle

$\Rightarrow \chi\mathcal{L}$ another equivariant line bundle.

The Serre Construction

A way to construct many rank 2 vector bundles on a surface (here: B_1 and B_2).

- Take two line bundles $\mathcal{L}_1$, $\mathcal{L}_2$.
- An ideal sheaf I (sheaf of functions vanishing at some fixed points).
- Define $\mathcal{S}$ as an extension

$$0 \longrightarrow \mathcal{L}_1 \longrightarrow \mathcal{S} \longrightarrow \mathcal{L}_2 \otimes I \longrightarrow 0$$

- Cayley-Bacharach property $\Rightarrow$ generic extension is a rank 2 vector bundle.

Constructing Vector Bundles

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

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Constructing Vector Bundles

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Operations:

- Tensor product of bundles.
- ~~Sums of bundles.~~ Not good enough.
- Extensions of bundles.

A Heterotic Standard Model

Serre Construction

Define rank 2 bundles $\mathcal{W}_i$ on B_i

$$0 \longrightarrow \chi_1 \mathcal{O}_{B_1}(-f_1) \longrightarrow \mathcal{W}_1 \longrightarrow \chi_1^2 \mathcal{O}_{B_1}(f_1) \otimes I_3 \longrightarrow 0$$

$$0 \longrightarrow \chi_2^2 \mathcal{O}_{B_2}(-f_2) \longrightarrow \mathcal{W}_2 \longrightarrow \chi_2 \mathcal{O}_{B_2}(f_2) \otimes I_6 \longrightarrow 0$$

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$$0 \to \chi_2^2 \mathcal{O}_{B_2}(-f_2) \to \mathcal{W}_2 \to \chi_2 \mathcal{O}_{B_2}(f_2) \otimes I_6 \to 0$$

Note: $\mathcal{W}_1$ and $\mathcal{W}_2$ are rank 2 vector bundles on the base dP_9.

The Gauge Bundle

Define these two rank 2 vector bundles on $\widetilde{X}$

$$\mathcal{V}_1 \stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \otimes \pi_1^*(\mathcal{W}_1)$$
$$\mathcal{V}_2 \stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes \pi_2^*(\mathcal{W}_2)$$

We define the rank 4 bundle $\mathcal{V}$ finally as a generic extension

$$0 \longrightarrow \mathcal{V}_1 \longrightarrow \mathcal{V} \longrightarrow \mathcal{V}_2 \longrightarrow 0$$

Spectrum

The massless spectrum

$= \text{zero modes of } \not{D}_{E_8}$

$= H^1 \text{ cohomology of the adjoint bundle } \mathcal{E}_8^{\mathcal{V}/G}.$

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$$H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}/G}\Big) = \\ = H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}}/G\Big)$$

Spectrum

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$= H^1$ cohomology of the adjoint bundle $\mathcal{E}_8^{\mathcal{V}/G}$.

Work with | **Have in mind**

$$\boxed{H^1\Big(\widetilde{X},\ \mathcal{E}_8^{\mathcal{V}}\Big)^G} \quad = \quad \boxed{\begin{aligned} H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}/G}\Big) &= \\ = H^1\Big(X,\ \mathcal{E}_8^{\mathcal{V}}/G\Big) \end{aligned}}$$

Gauge Group Breaking

$$\mathbf{248} = (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{15}, \mathbf{1}) \oplus$$
$$\oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10})$$

$$\mathbf{10} = \chi_2(\mathbf{1}, \mathbf{2}, 3, 0) \oplus \chi_1^2\chi_2(\mathbf{3}, \mathbf{1}, -2, -2) \oplus$$
$$\oplus \chi_2^2(\mathbf{1}, \overline{\mathbf{2}}, -3, 0) \oplus \chi_1\chi_2^2(\overline{\mathbf{3}}, \mathbf{1}, 2, 2)$$

Correspondingly, the fermions split as...

Vector Bundle Breaking

$$\mathcal{E}_8^V = \Big(\mathcal{O}_{\widetilde{X}} \otimes \theta(\mathbf{45})\Big) \oplus \Big(\operatorname{Ad}(\mathcal{V}) \otimes \theta(\mathbf{1})\Big) \oplus$$
$$\oplus \Big(\mathcal{V} \otimes \theta(\mathbf{16})\Big) \oplus \Big(\mathcal{V}^\vee \otimes \theta(\overline{\mathbf{16}})\Big) \oplus \Big(\wedge^2 \mathcal{V} \otimes \theta(\mathbf{10})\Big)$$

where $\theta(\cdots)$ is the trivial bundle.

$$\theta(\mathbf{10}) = \Big[\chi_2 \theta\big(\mathbf{1}, \mathbf{2}, 3, 0\big)\Big] \oplus \Big[\chi_1^2 \chi_2 \theta\big(\mathbf{3}, \mathbf{1}, -2, -2\big)\Big] \oplus$$
$$\oplus \Big[\chi_2^2 \theta\big(\mathbf{1}, \overline{\mathbf{2}}, -3, 0\big)\Big] \oplus \Big[\chi_1 \chi_2^2 \theta\big(\overline{\mathbf{3}}, \mathbf{1}, 2, 2\big)\Big]$$

The Higgs Sector

For example, focus on the fields in the **10**:

$$
\begin{aligned}
H^1\Big(\widetilde{X},\, \mathcal{E}_8^V\Big)^G = &\ \text{(lots of other fields)} \oplus \\
&\oplus \Big[\chi_2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\mathbf{1}, \mathbf{2}, 3, 0) \oplus \\
&\oplus \Big[\chi_1^2\chi_2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\mathbf{3}, \mathbf{1}, -2, -2) \oplus \\
&\oplus \Big[\chi_2^2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\mathbf{1}, \overline{\mathbf{2}}, -3, 0) \oplus \\
&\oplus \Big[\chi_1\chi_2^2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\Big]^G \otimes (\overline{\mathbf{3}}, \mathbf{1}, 2, 2)\,.
\end{aligned}
$$

Cohomology

The necessary cohomology groups for $\mathcal{V}$ are

$$
\begin{aligned}
H^1\left(\widetilde{X}, \mathcal{V}\right) &= 3\,\mathrm{Reg}(G) \\
H^1\left(\widetilde{X}, \mathcal{V}^\vee\right) &= 0 \\
H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) &= H^1\left(\widetilde{X}, \mathcal{V}_1 \otimes \mathcal{V}_2\right) = \\
&= \chi_1 \chi_2 \oplus \chi_1^2 \chi_2^2 \oplus \chi_2 \oplus \chi_2^2
\end{aligned}
$$

Doublet-Triplet Splitting

$$H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right) = \chi_1\chi_2 \oplus \chi_1^2\chi_2^2 \oplus \chi_2 \oplus \chi_2^2$$

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$$1 = \left[\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2\mathcal{V}\right)\right]^G \qquad \text{up Higgs}$$

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Doublets and triplets are completely split.

Summary

Our Heterotic Standard Model has

- Three families of quarks and leptons.
- Zero anti-families.
- 1 Higgs–Higgs conjugate pair (exact MSSM matter spectrum).
- No vector-like pairs (except Higgs).
- No triplets.
- No exotics.
- $3 + 3 + 13$ moduli.
- Hidden sector.

More Details

Hidden Sector:

- $Spin(12)$ gauge group.
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The heterotic anomaly cancels:

$$c_2\left(T\widetilde{X}\right) - c_2\left(\widetilde{\mathcal{V}}\right) - c_2\left(\widetilde{\mathcal{H}}\right) = 0$$

Superpotential

The Lagrangian

We do not know the Kähler potential. What can we learn from the superpotential W?

- Higgs μ-terms $\phi H \bar{H}$
- Yukawa couplings $Q_i H \bar{Q}_i + Q_i \bar{H} \bar{Q}_i$

Field	Name
ϕ	Vector bundle moduli
H	Higgs
$\bar{H}$	Higgs-conjugate
Q_i	Quarks & leptons of the i-th family
$\bar{Q}_i$	Anti-Q_i

Superpotential

The cubic terms in the superpotential are

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$$H^1\left(\widetilde{X}, \mathcal{V}\otimes\mathcal{V}^\vee\right)\otimes H^1\left(\widetilde{X}, \wedge^2\mathcal{V}\right)\otimes H^1\left(\widetilde{X}, \wedge^2\mathcal{V}^\vee\right) \longrightarrow H^3\left(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\right) = \mathbb{C}$$

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- Yukawa couplings

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Leray Degrees

Bundle-valued cohomology =
cohomology of bundle-valued differential forms.

Leray Degrees

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Split cohomology according to base & fiber degrees:

Field	Cohomology	Fiber 1	Base	Fiber 2
$Q_1, \bar{Q}_1$	$H^1(\widetilde{X}, \mathcal{V})$	1	0	0
$Q_2, Q_3, \bar{Q}_2, \bar{Q}_3$	$H^1(\widetilde{X}, \mathcal{V})$	0	0	1
$H, \bar{H}$	$H^1(\widetilde{X}, \wedge^2\mathcal{V})$	0	1	0
$\phi_1, \ldots, \phi_3$	$H^1(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee)$	1	0	0
$\phi_4, \ldots, \phi_{13}$	$H^1(\widetilde{X}, \mathcal{V} \otimes \mathcal{V}^\vee)$	0	0	1

Selection Rule

Cubic terms must have degree 1 in each of the three directions.

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Higgs μ-terms $\phi_i H \bar{H}$ are forbidden.

Yukawa couplings

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$Q_2, Q_3, \bar{Q}_2, \bar{Q}_3$	0	0	1
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Yukawa couplings are allowed!

$$W = \sum_{i=2,3} \left(\lambda_i Q_1 H \bar{Q}_i + \bar{\lambda}_i Q_i \bar{H} \bar{Q}_1 \right)$$

Fin

Summary

We found a compactification of the $E_8 \times E_8$ heterotic string with low-energy physics

- $N = 1$, $d = 4$ QFT.
- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.
- Exact MSSM matter spectrum.
- No vector-like pairs (except Higgs).
- No exotics, no triplets.
- $3 + 3 + 13$ moduli.
- Yukawa couplings and no Higgs μ-term.
- Gravity, axion-dilaton.

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- But: apparently $\#\{\text{Higgs}\} - \#\{\mathbf{3}\} = O(1)$.
- Selection rules often forbid mass terms for vector-like pairs.
- Even if they do not, only reliable up to intermediate scale.