Precision Measurement of the Top Quark Mass in Dilepton Events at CDF

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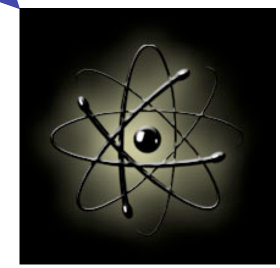
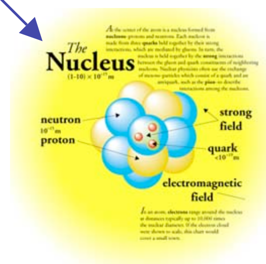
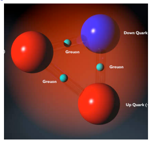
Probing the structure of our environment

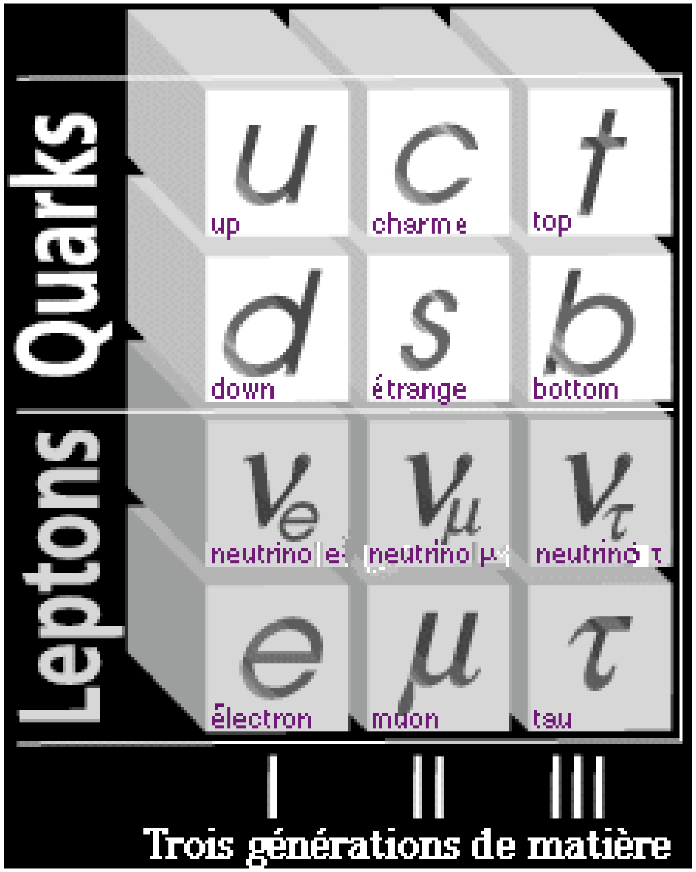






State of the Art

The Standard Model



State of the Art

Exercise 1.1.1.1.1a: Given locality, causality, Lorentz invariance, and known physical data since 1860, show that the Lagrangian describing all observed physical processes (sans gravity) can be written:

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + \ldots$$

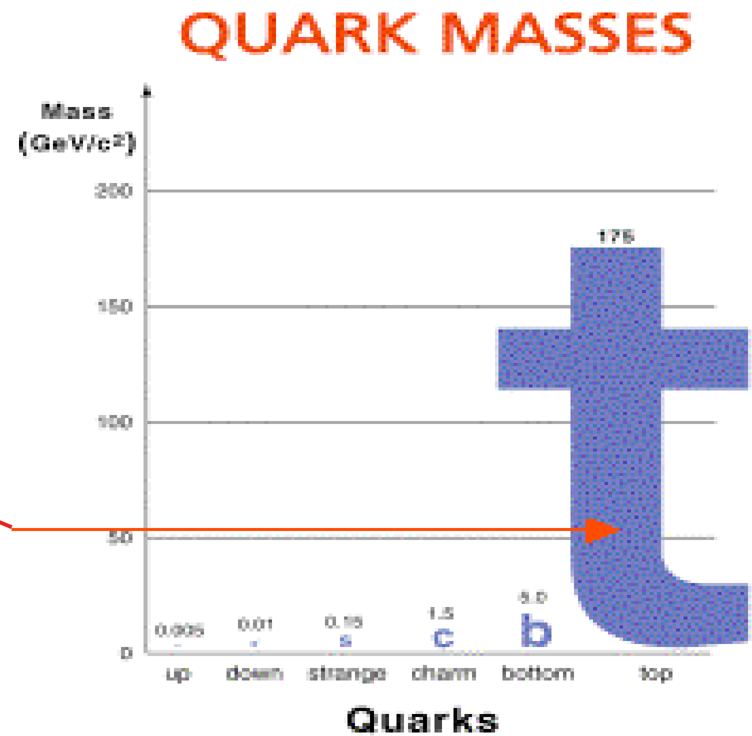
The Standard Model

The theoretical machinery is very effective and often beautiful…

…but its rather unlikely to be the simplest description.

Top Quark

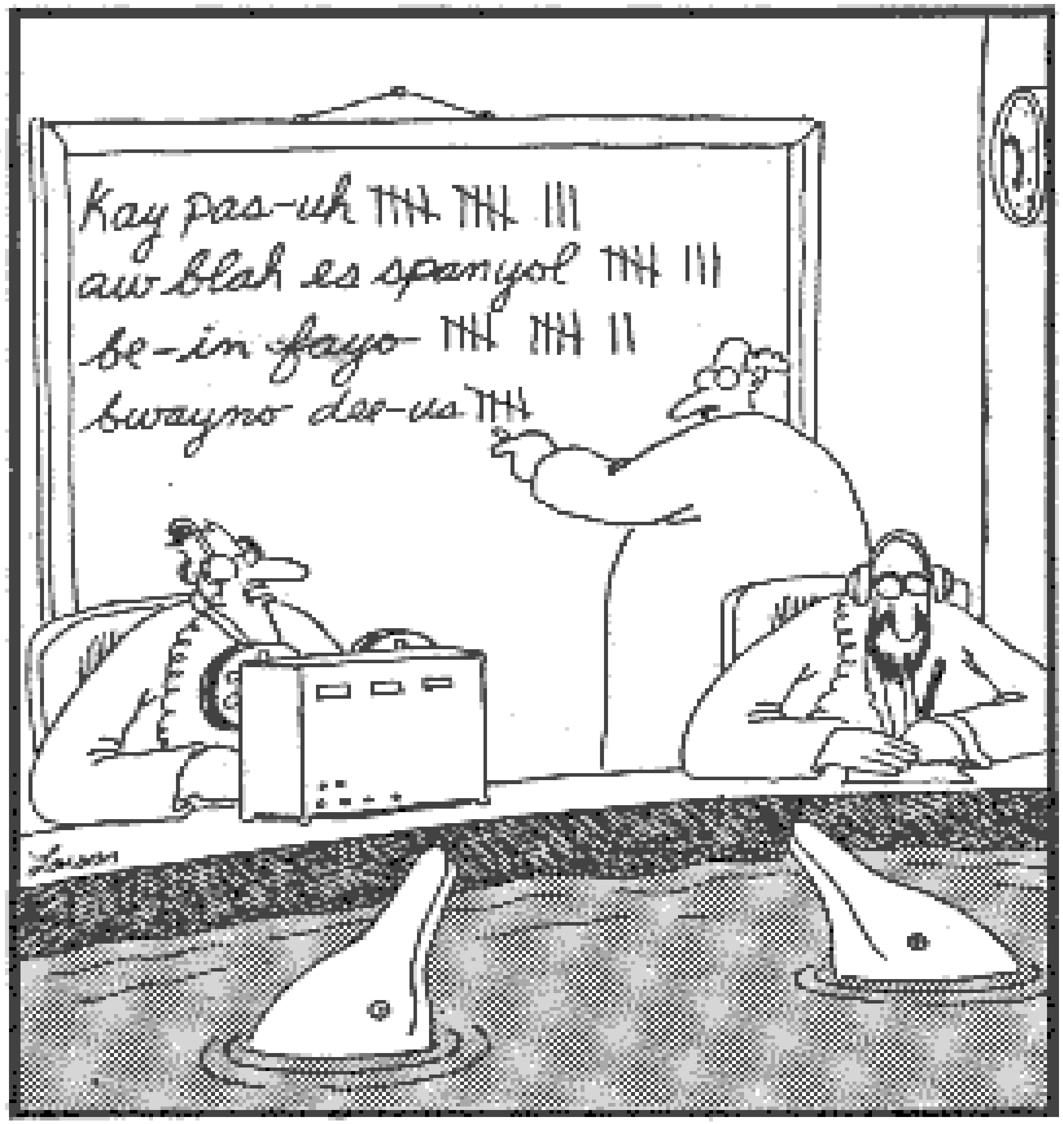
What are the clues?



Where does this mass
structure come from?
What does it reveal?
Why is the top so heavy?
Why is its Yukawa coupling ~ 1 ?

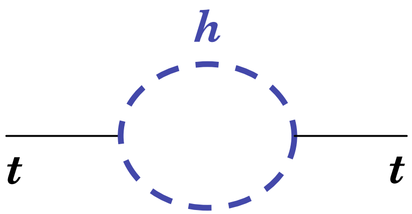
New Theories?

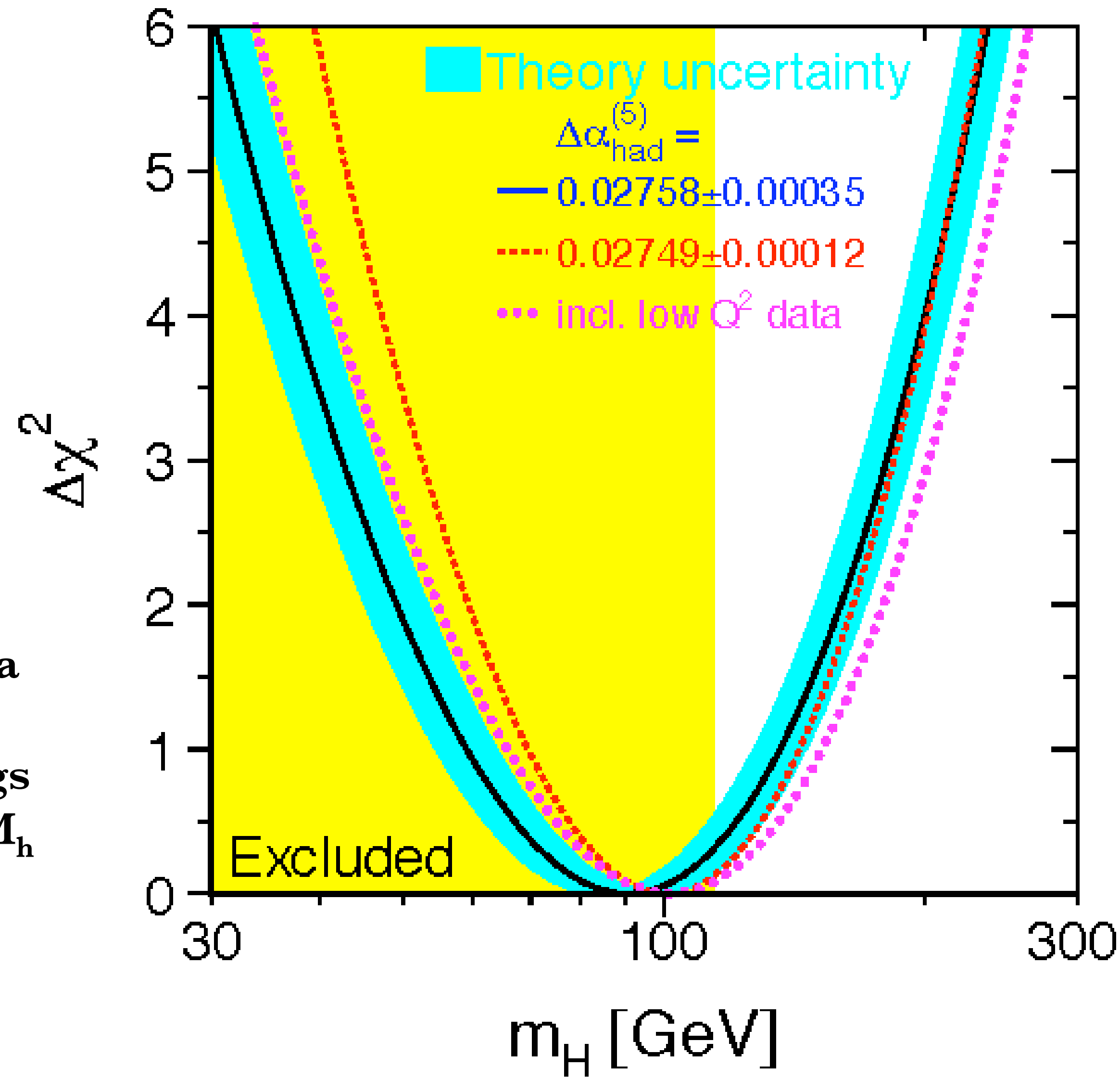
Is there some simple but foreign idea which could explain many of these mysteries?



"Matthews… we're getting another one of those strange 'aw blah es span yol' sounds"

Higgs





Higgs connection

Radiative correction to M_t via Higgs loop

Heavy top means heavy Higgs

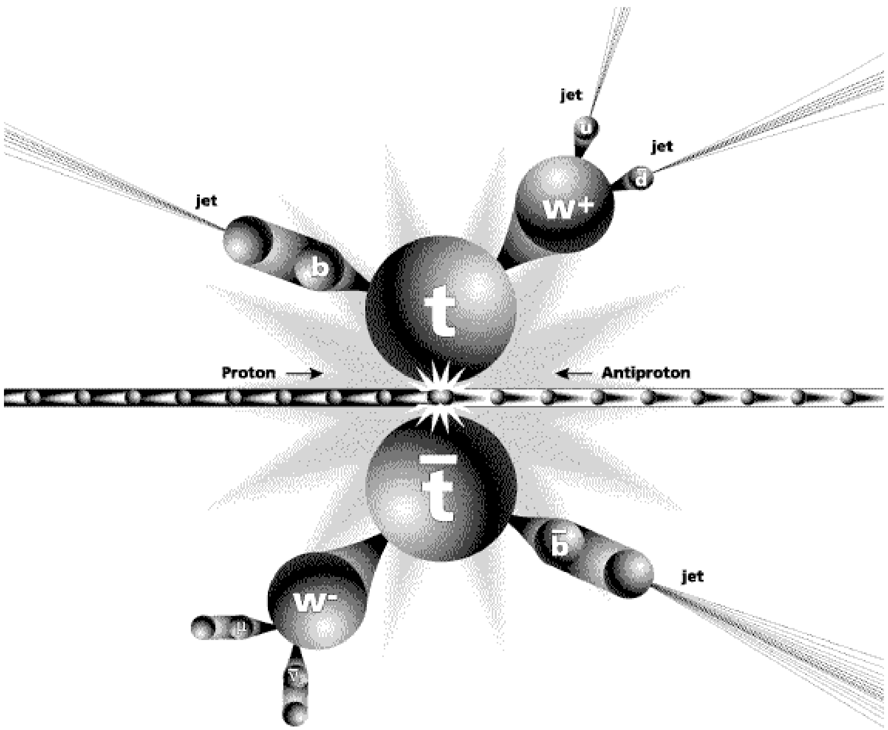
M_t provides constraints on M_h

Outline

- Top quark pair production
  - Decay
  - Detector signature
- Dilepton mass measurement problem
  - Template strategy
  - Matrix-element strategy
- Measurements
  - Current results
  - Combination of measurements

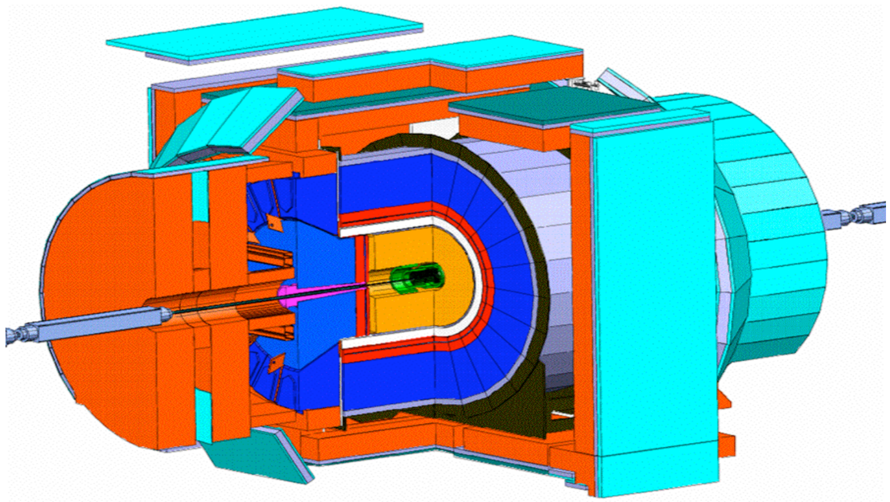
Top Quark Production

The FermiLab Tevatron is the only facility with enough energy to produce and observe top quarks



CDF at Run2

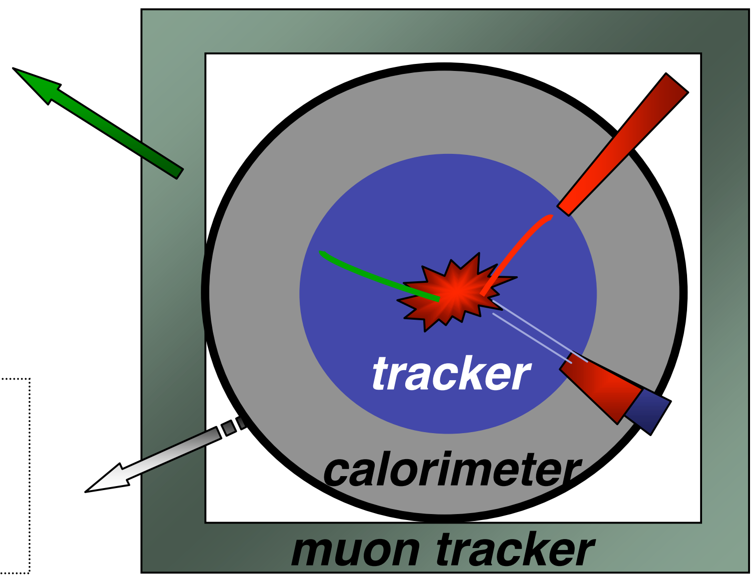
CDF



Cartoon Detector

muon
Muon track
Central track

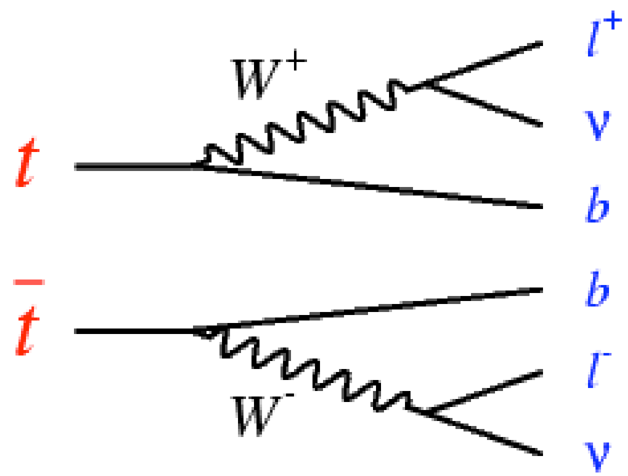
electron
EM shower
Central track

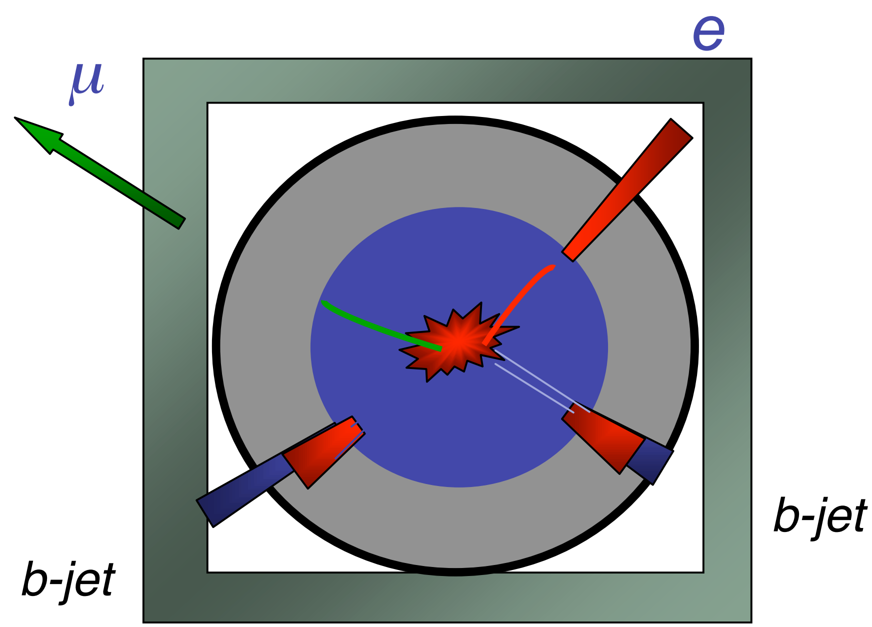


$\not\!\!E_T$
Neutrinos,
new physics

Jets
EM and hadronic
showers

Dilepton Signature





Dileptons are clean

More leptons!
Fewer jets
Smaller backgrounds

Small statistics

bb+ee/eμ/μμ: 5%
bb+e/μ+qq : 30%
bb+qq+qq : 44%

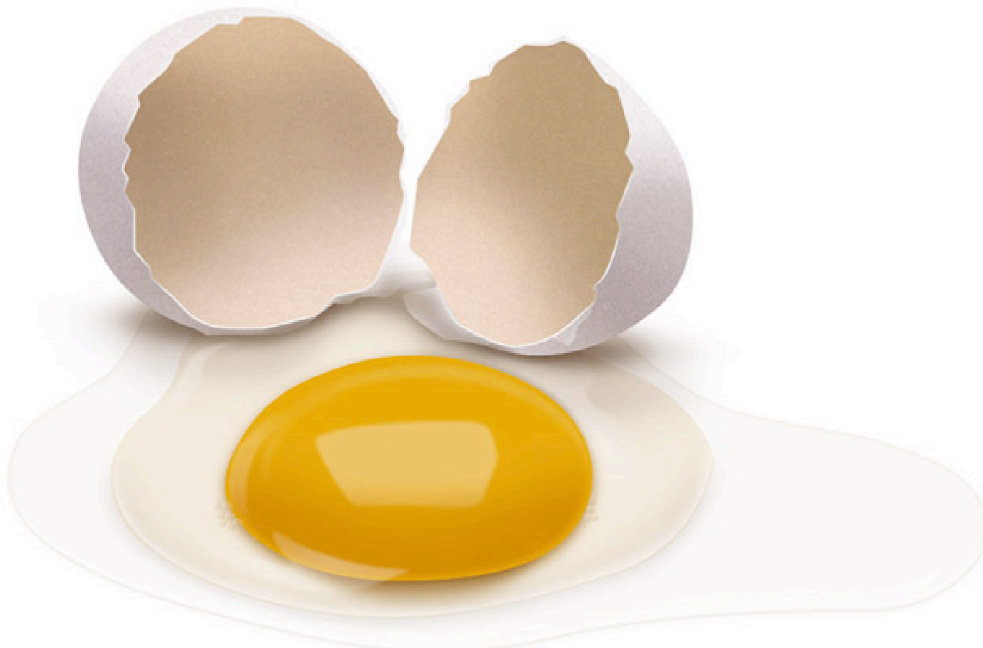
Particle ID and measurement

Charged leptons are well-measured
Jets measure quark energies
Neutrinos are undetected

How to measure mass

How do we know how much mass the top had?

We can't just put the pieces back together again



Information has been lost

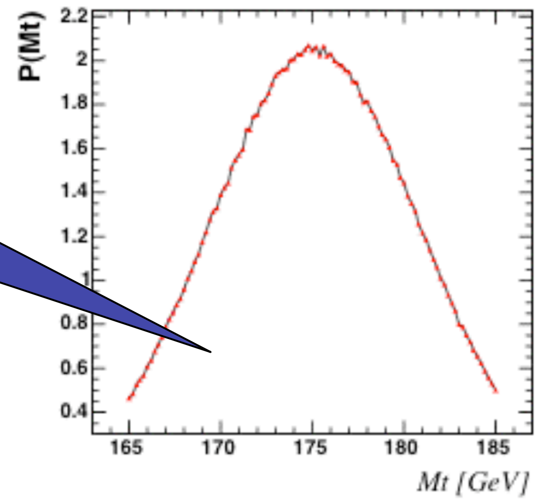
-Neutrinos have escaped undetected

-B-quarks have hadronized, showered, been clustered into *jets*

Solution

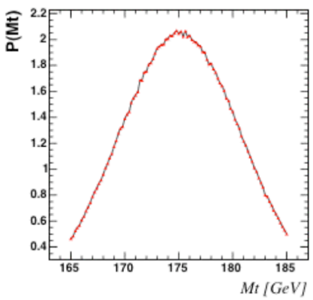
Integrate over unknown quantities.

Each event has a ***curve,*** *rather than a single* ***mass value***



Construct probability curve.

Probability calculation



Template strategy

Construct parameterized probability; fit parameters to examples from simulated events

This sweeps difficult effects into the parameters.

Matrix-element strategy

Calculate probability directly from matrix-element

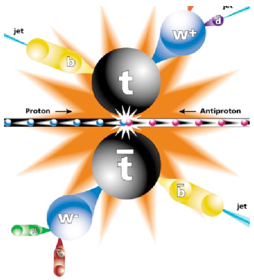
Make simplifying approximations

Measure any necessary corrections using simulated events.

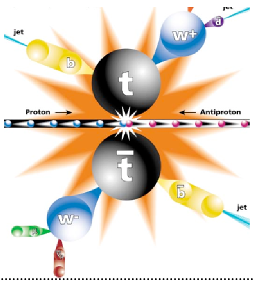
Template Method

$$P(\textit{partons} \mid M_t) \quad x \quad P(\textit{measurement} \mid \textit{partons}) = P(\textit{measurement} \mid M_t)$$

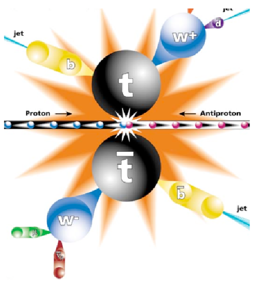
M_t = 165



M_t = 175



M_t = 185



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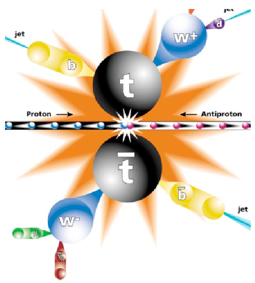

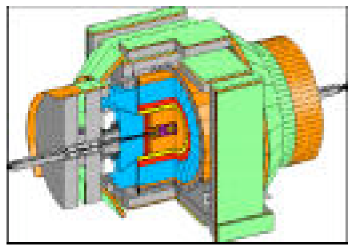

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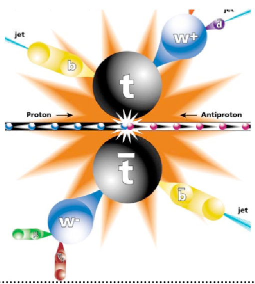

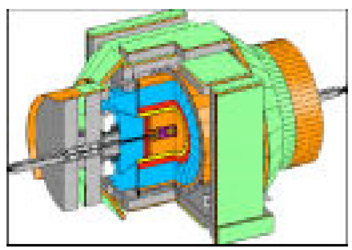

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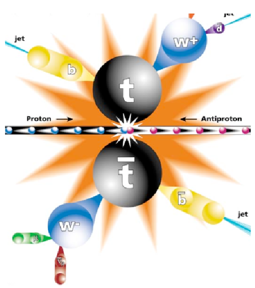

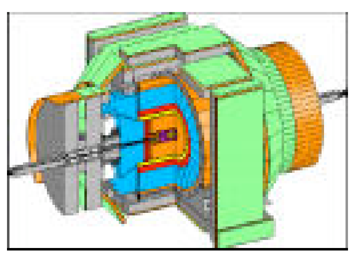

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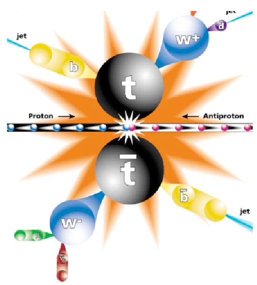

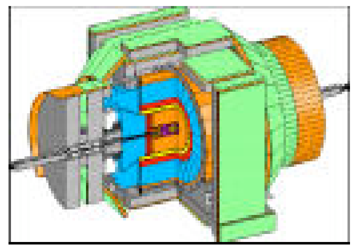

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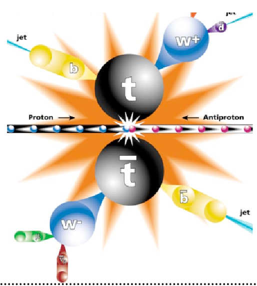

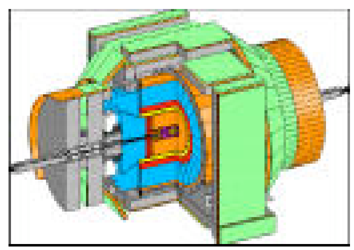

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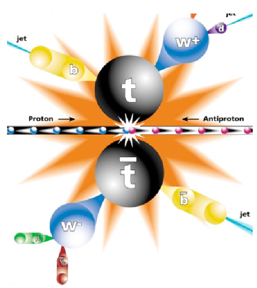
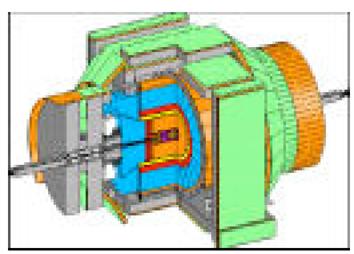
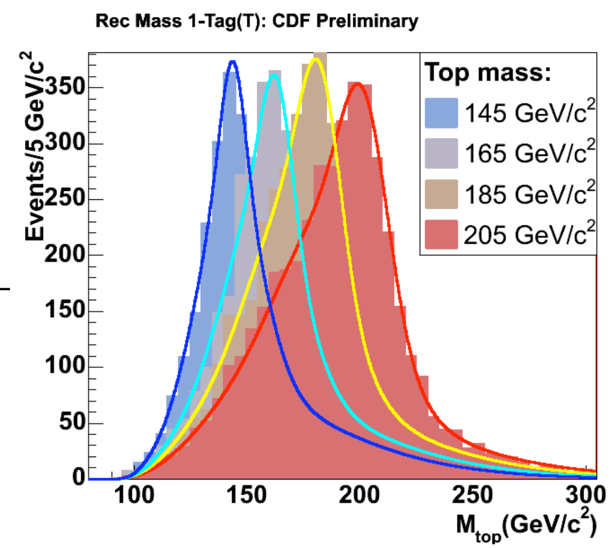


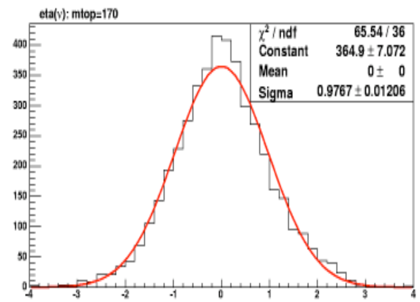


Given examples, deduce rule for *P*.



Apply rule to data, extract mass.

Neutrino Weighting



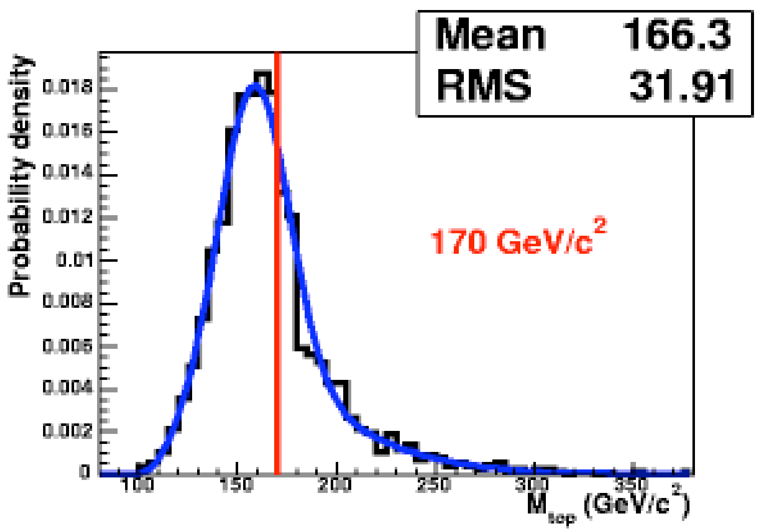
Inspired by Run1 neutrino weighting analyses

-Constrains Mw, Mt=Mt

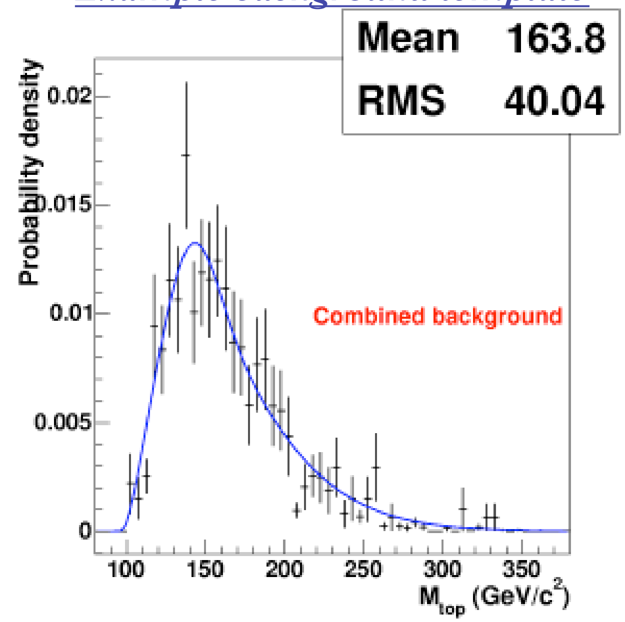
-Integrates over 2 neutrino eta distrubutions

-At specific Mt, weight factor derived from missing energy resolution

Example Signal template



Example background template



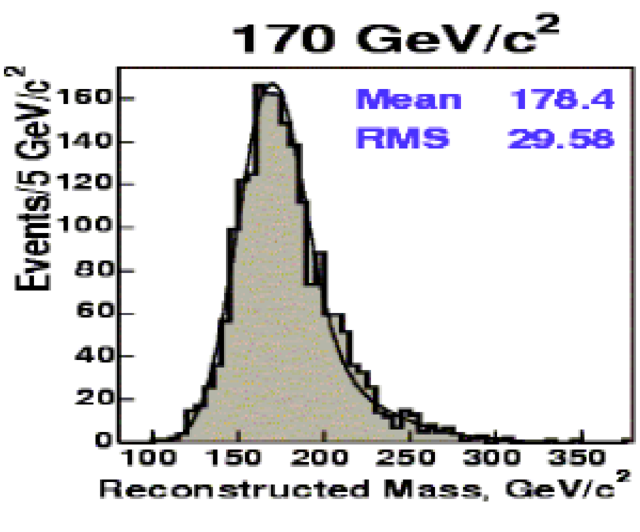
Neutrino phi

Neutrino angle method

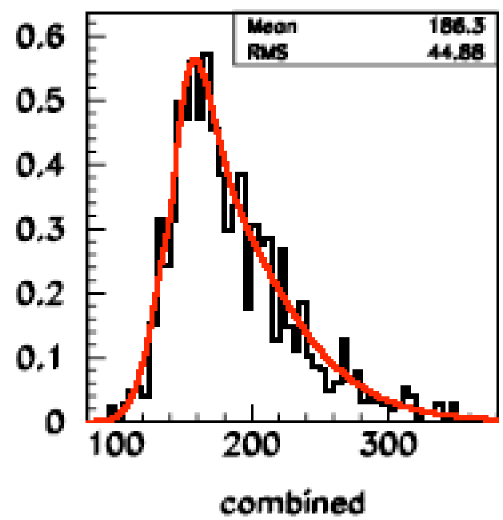
- Integration over two neutrino angles; flat distribution requires no MC prior
- Constrain Mt=Mt
- Find most probable top mass which minimizes:

$$\chi^2 = \sum_{l=1,2}\frac{(P_T^l - \tilde{P}_T^l)^2}{\sigma_l^2} + \sum_{j=1,2}\frac{(P_T^j - \tilde{P}_T^j)^2}{\sigma_j^2} + \sum_{i=x,y}\frac{(UE^i - \tilde{UE}^i)^2}{UE^{i2}} + \frac{(M_{l1\nu 1} - M_W)}{\Gamma_{M_W}^2} + \frac{(M_{l2\nu 2} - M_W)}{\Gamma_{M_W}^2} + \frac{(M_{j1l1\nu 1} - \tilde{m}_t)}{\Gamma_{M_t}^2} + \frac{(M_{j2l2\nu 2} - \tilde{m}_t)}{\Gamma_{M_t}^2}$$

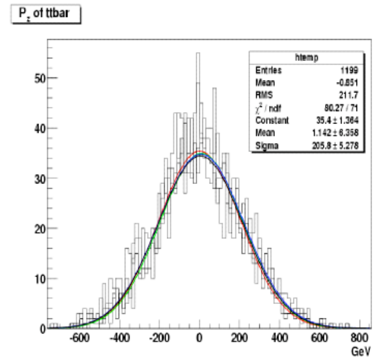
Example Signal template

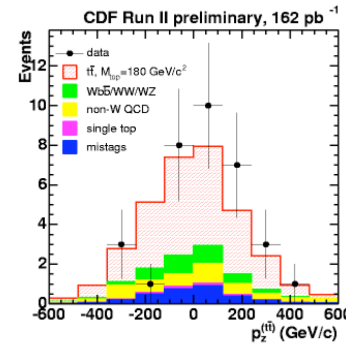


Example background template



ttbar pz





Method using ttbar Pz

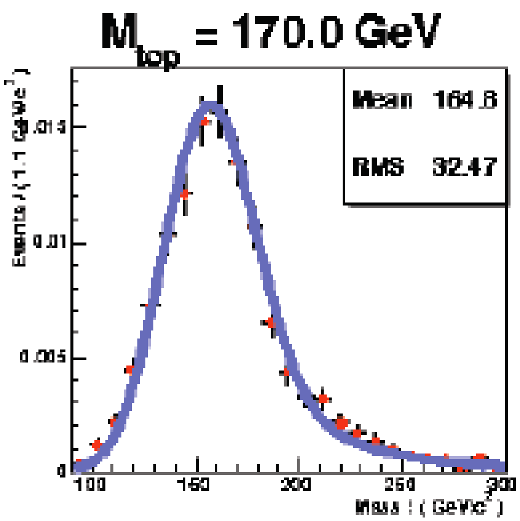
Assume Mw, Mt=Mt

Scan over Pz of ttbar system, prior from MC

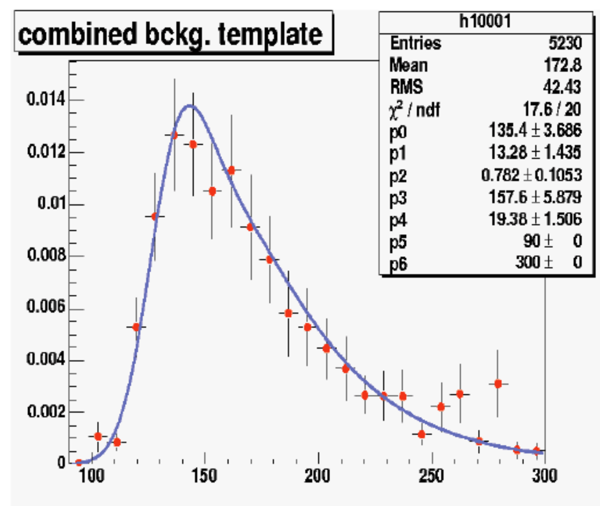
Smear jet Et, ttbar Pt, and missing Et

Choose most probable Mt

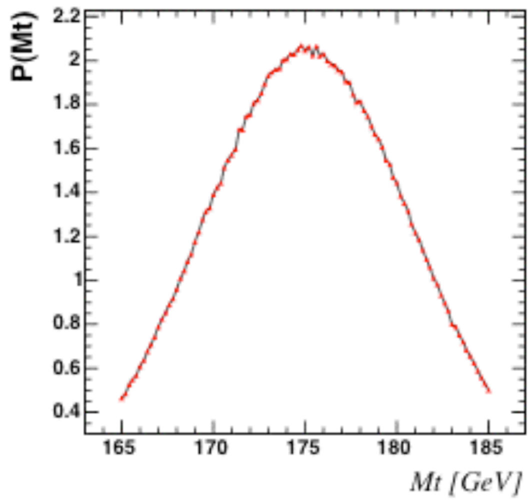
Example Signal template



Example background template

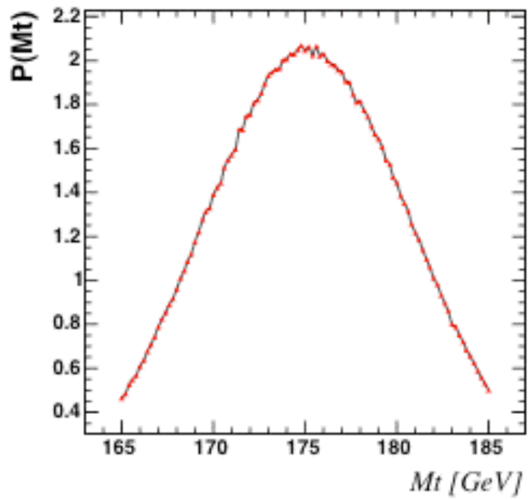


Matrix Element Method



Method

$$\mathbf{P}\,(\,\textbf{event x}\mid \mathbf{M_t}\,)\;:$$



$$= \frac{1}{\sigma(M_t)} \frac{d\sigma(M_t)}{d\,\mathrm{x}}$$

History

First ideas by K. Kondo, *et. al.* [J. Phys. Soc. Japan **57** 4136 (1988)]

First M_t measurement in Run1 by D0 in l+jets [Nature **429** 636 (2004)]

Applied in Run2 by CDF and D0 in l+jets [W&C 6/11/04 and 7/22/05]

Direct Calculation

Differential cross-section calculation:

$$\frac{d\sigma(M_t)}{d\,\mathbf{x}} = \int d\Phi_6 |\mathcal{M}_{t\bar{t}}(p; M_t)|^2 f_{PDF}(q_1) f_{PDF}(q_2)$$

Phase-space Integral

Matrix Element

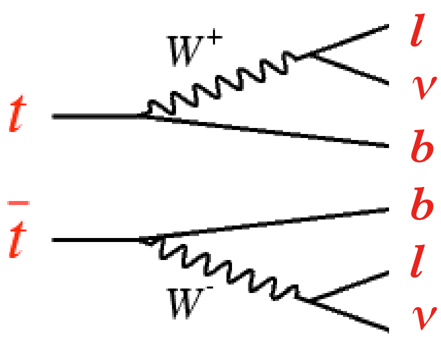
p **4-momentum of final partons**
q **4-momentum of initial partons**
x **measured event variables**

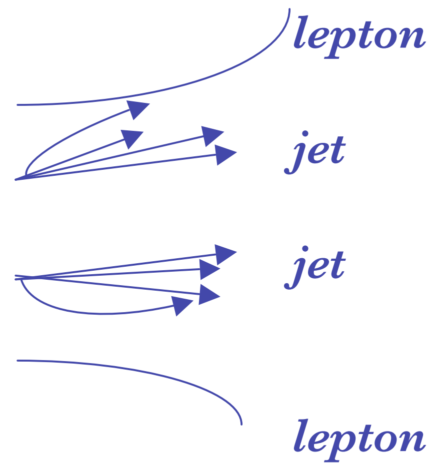
But there is no reference to our measured quantities *x*!

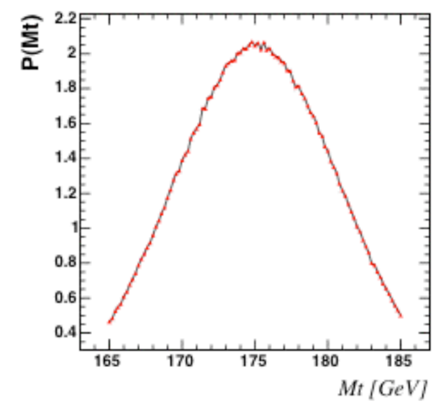
Method

Break into two pieces: parton-level process and showering/resolution effects

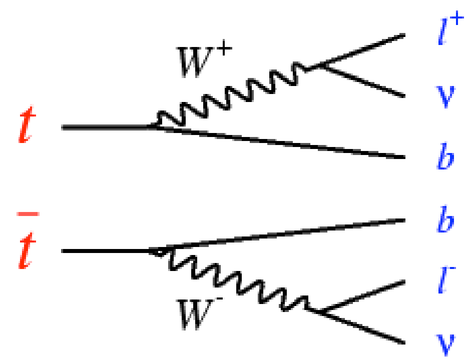
$$P(\text{partons} \mid M_t) \quad \times \quad P(\text{event } x \mid \text{partons}) \quad = \quad P(\text{event } x \mid M_t)$$







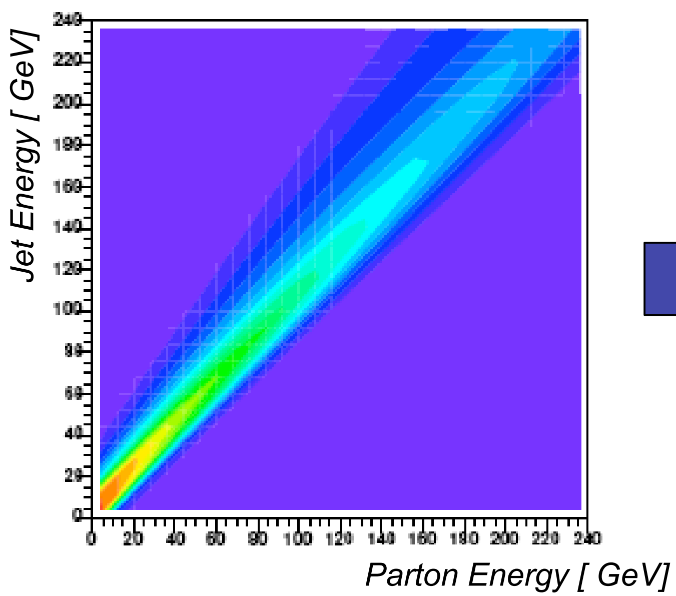
Direct Calculation

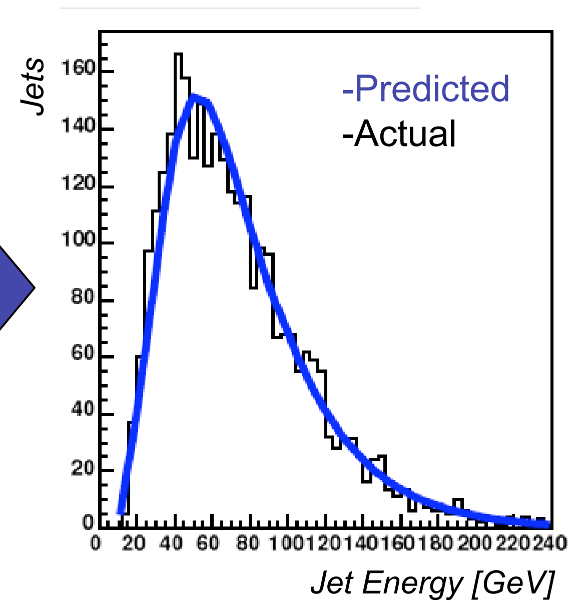


We know $P($ *partons* $|\ M_t)$

We can parameterize parts of $P($ *event x* $|$ *partons* $)$:

For jets:





Assumptions

- Initial state
  - No initial state radiation
  - Transverse energy of system is negligible

- Final state
  - Leptons
    - Energy well-measured
    - Direction well-measured

  - Jets
    - Jets arise from *b*-quarks
    - Direction well-measured
    - Energy can be parameterized from parton energy

- Assumptions make calculation tractable
  - balance sensitivity with computation time

Calculation

p 4-momentum of final partons
q 4-momentum of initial partons
x measured event variables

For each event, calculate differential cross-section:

$$P(\mathbf{x}|M_t) = \frac{1}{N}\int d\Phi_6 |\mathcal{M}_{t\bar{t}}(p;M_t)|^2 \prod_{jets} f(p_i,\mathbf{x})f_{PDF}(q_1)f_{PDF}(q_2)$$

Phase-space Integral

Matrix Element

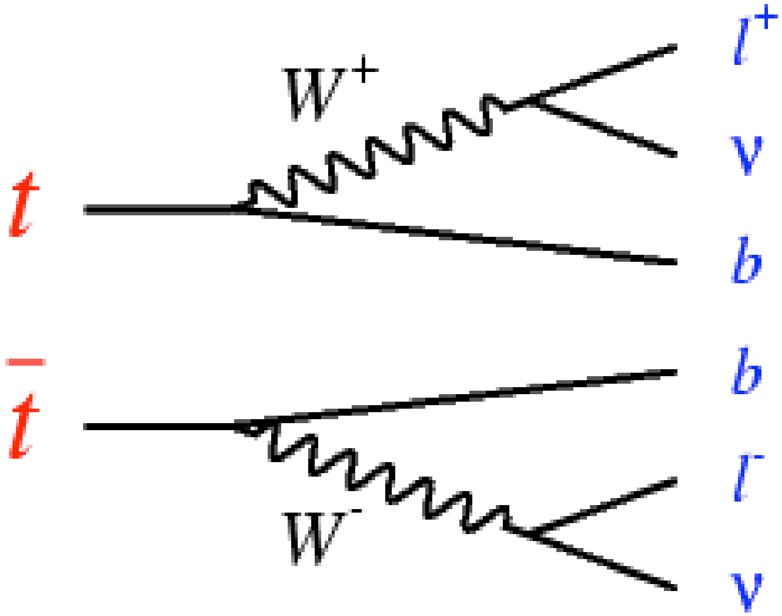
Transfer Functions

Only partial information available

Fix measured quantities

Integrate over unmeasured parton quantities consistent with tt production and measured event.

Unknowns



6 unknowns

3 (neutrino)

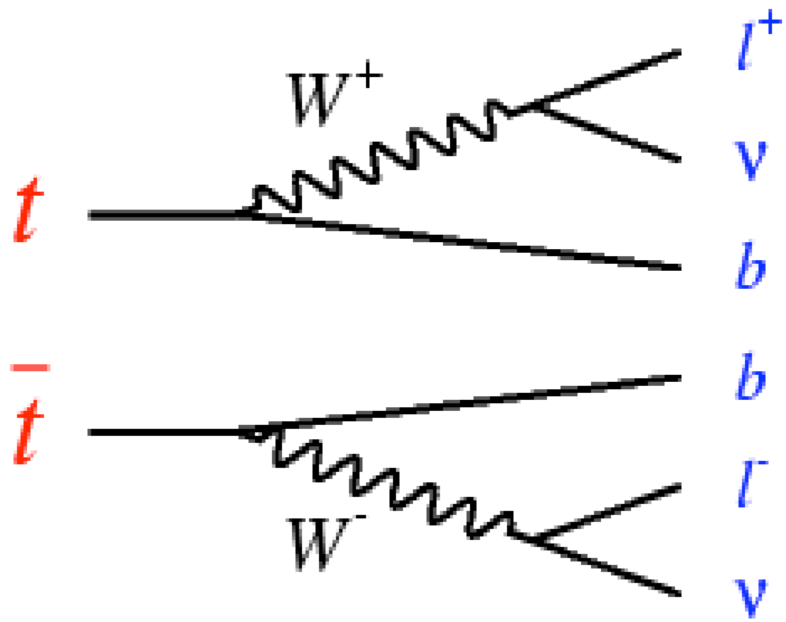
1 (quark energy)

1 (quark energy)

3 (neutrino)

-2 (P_T conservation)

Integration



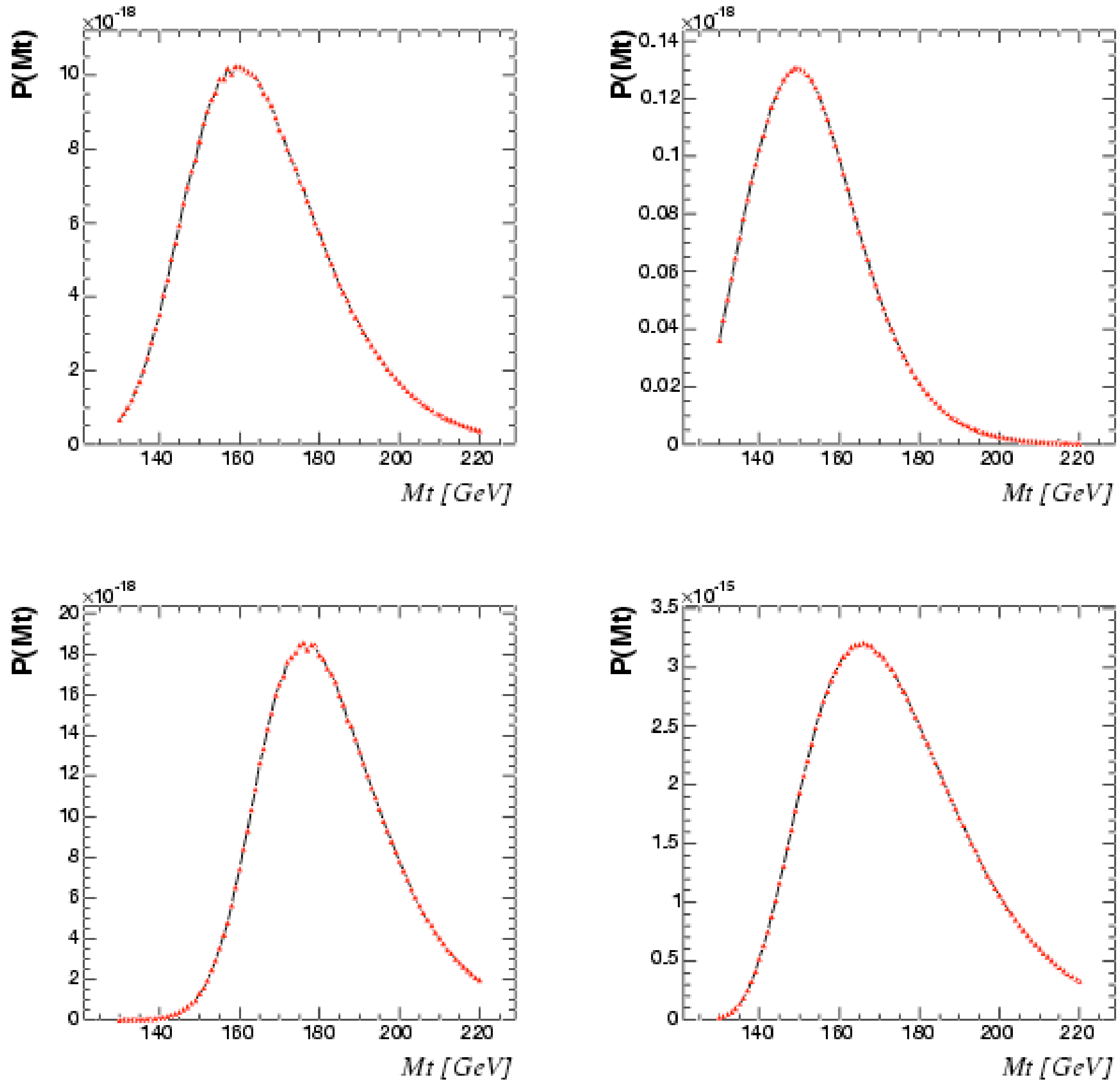
6 unknowns	6 integration variables
3 (neutrino)	*W,t* invariant masses
1 (quark energy)	quark energy
1 (quark energy)	quark energy
3 (neutrino) *-2 (P_T conservation)*	*W,t* invariant masses

Choice of variables

Transform phase space to exchange variables for those which are more efficient.
Requires numerical solution at each integration point.
Integration done with VEGAS.

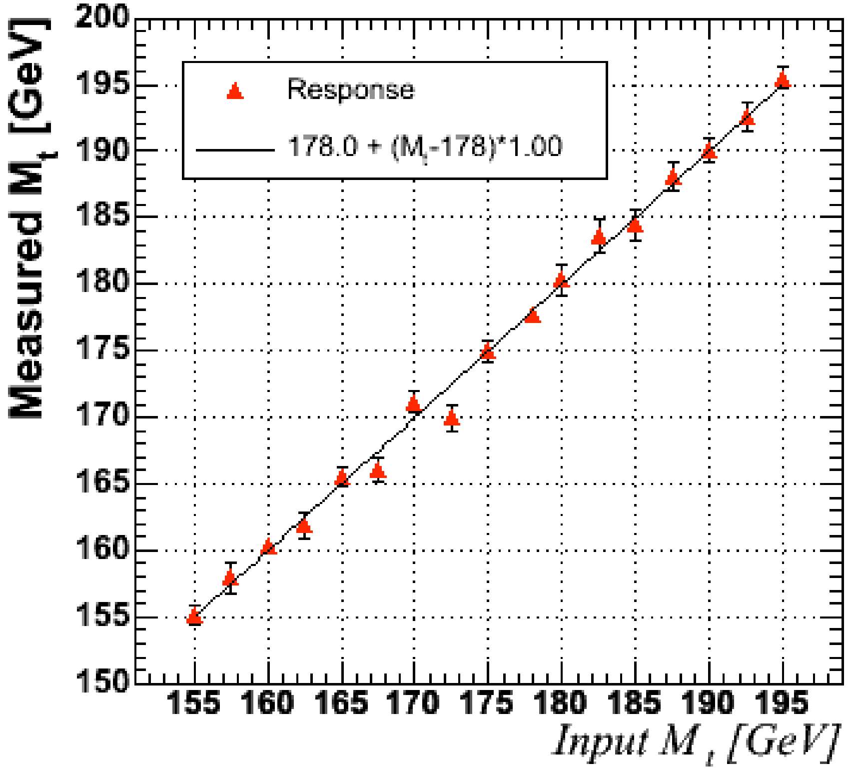
Examples

Four simulated events (Herwig, M_t=180 GeV)



To test performance, build joint probability in pseudo-experiments

Signal only results: full simulation



Response = $\langle M_{meas} \rangle$

Pull = $\dfrac{M_{meas} - M_{true}}{\sigma_{meas}}$

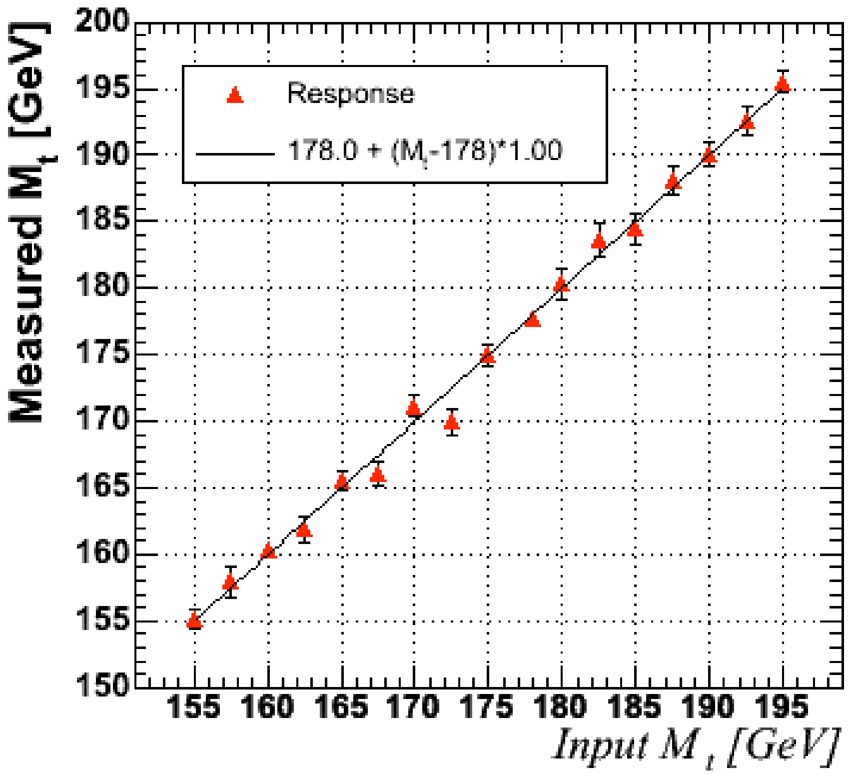
Response

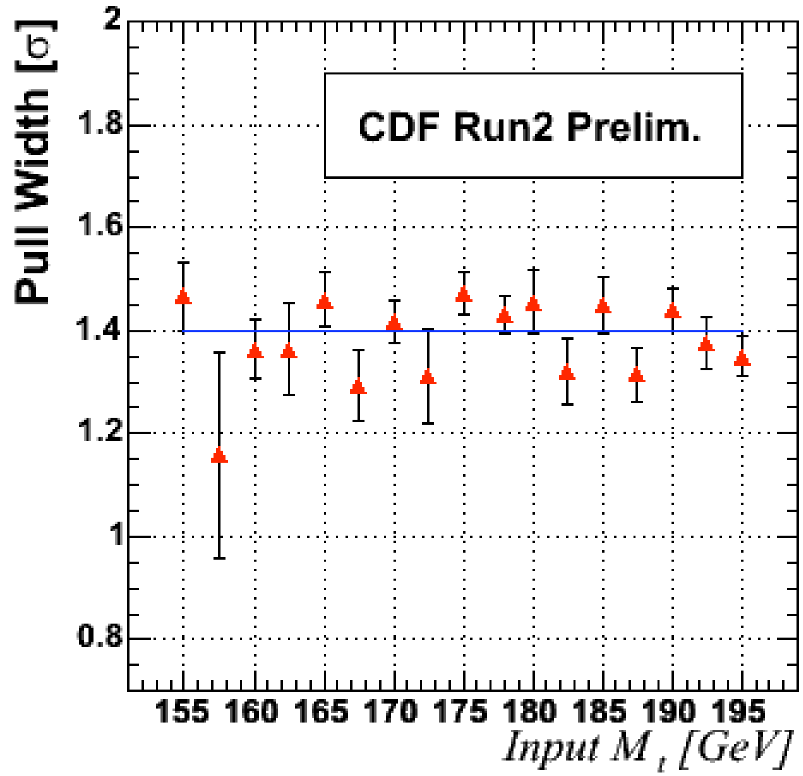
Linear.

Unbiased.

Signal only results: full simulation

Joint probability in signal-only pseudo-experiments





Response

Linear.

Unbiased.

Pull width

Flat.

Width is ~1.4.

In parton-level tests, width is ~1.0.

(template methods have ~1.05-1.1)

Pull width

> **PW inflated because probability contains assumptions broken by a subset of events.**

Assumptions held

Parton events: *width = 1.0*

Assemptions broken

Full simulation : *width = 1.4*

+ Two matched jets: *width ➡ 1.2*

+ Well measured leptons: *width ➡ 1.1*

+ Small parton-jet angle: *width ➡ 1.0*

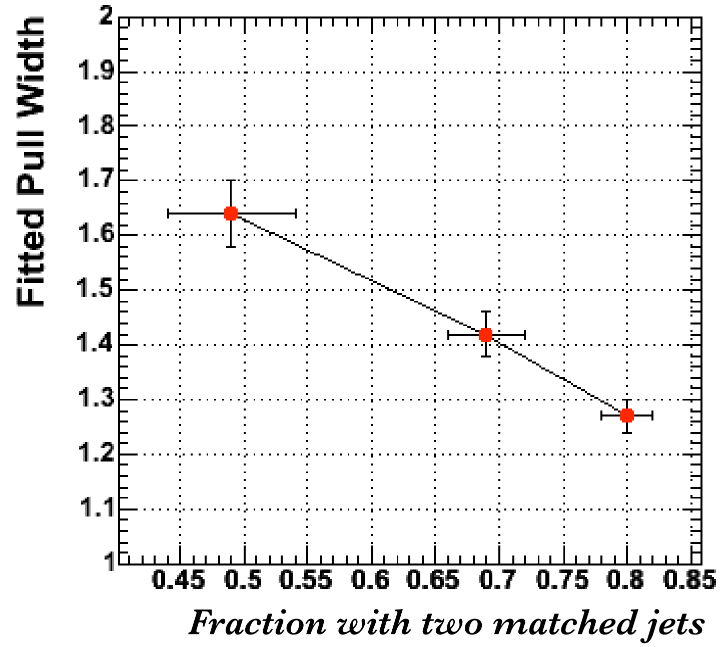
Scale factor for error

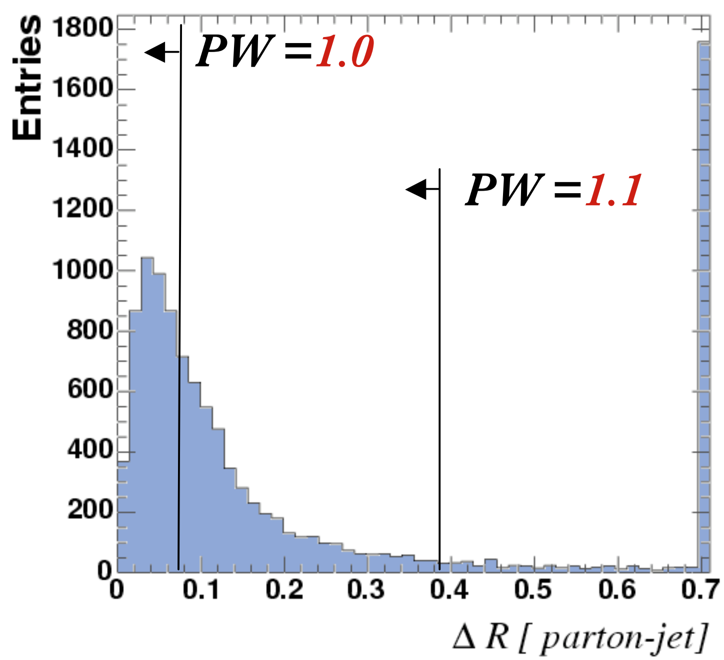
Flat in top mass

Flat in measured statistical error

Insensitive to systematic variations

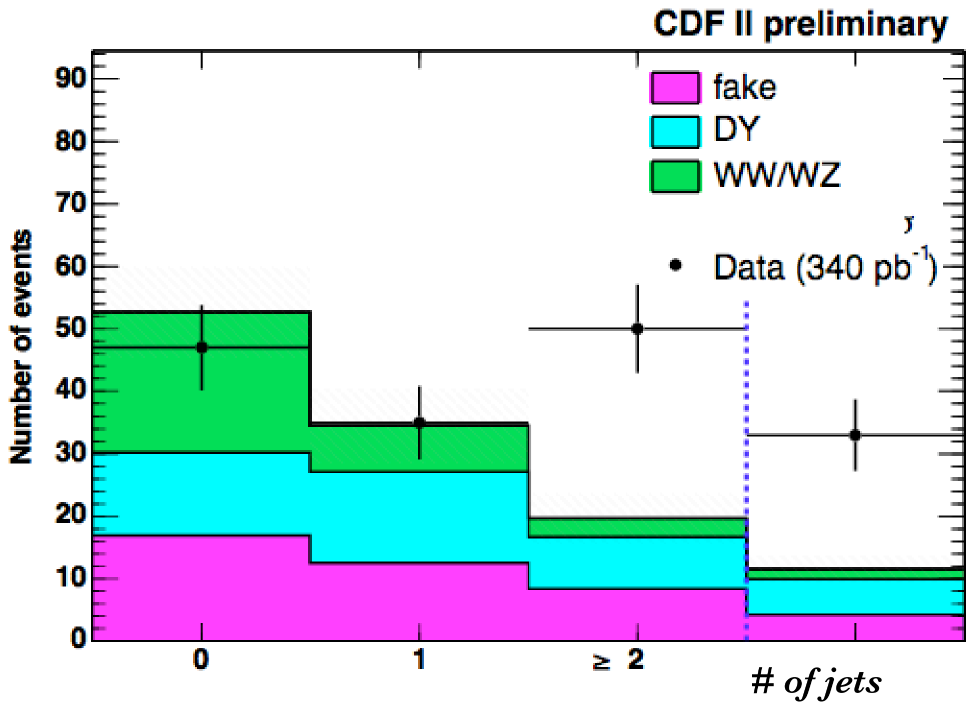
Error on scale factor is ~ 0.03





DIL Sample composition

DIL Sample is expected to be *3:2* signal to background



Final Cuts

Sample	N_{events}
tt (M_t=175GeV, σ = 6.7 pb)	17.2
$Z \rightarrow ee/\mu\mu$	4.9
MisID lepton	4.2
WW, WZ	1.6
$Z \rightarrow \tau\tau$	0.8
Total	28.7 ± 2.1
Observed	33

Probability expression needs to reflect presence of background

Background Likelihood

We generalize the probability to be a weighted sum of signal & bg probabilities

$$P(\mathbf{x}|M_t) = P_s(\mathbf{x}|M_t)p_s + P_{bg1}(\mathbf{x})p_{bg1} + P_{bg2}(\mathbf{x})p_{bg2}\ldots$$

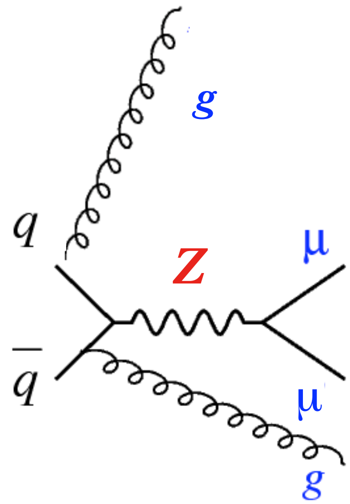
Where the weights are the expected sample fraction:

$$p_s(M_t) = \frac{\lambda_s(M_t)}{\lambda_s(M_t) + \lambda_b}$$

$$p_b = \frac{\lambda_b}{\lambda_s(M_t) + \lambda_b}$$

λ = expected number of events

Z+jets

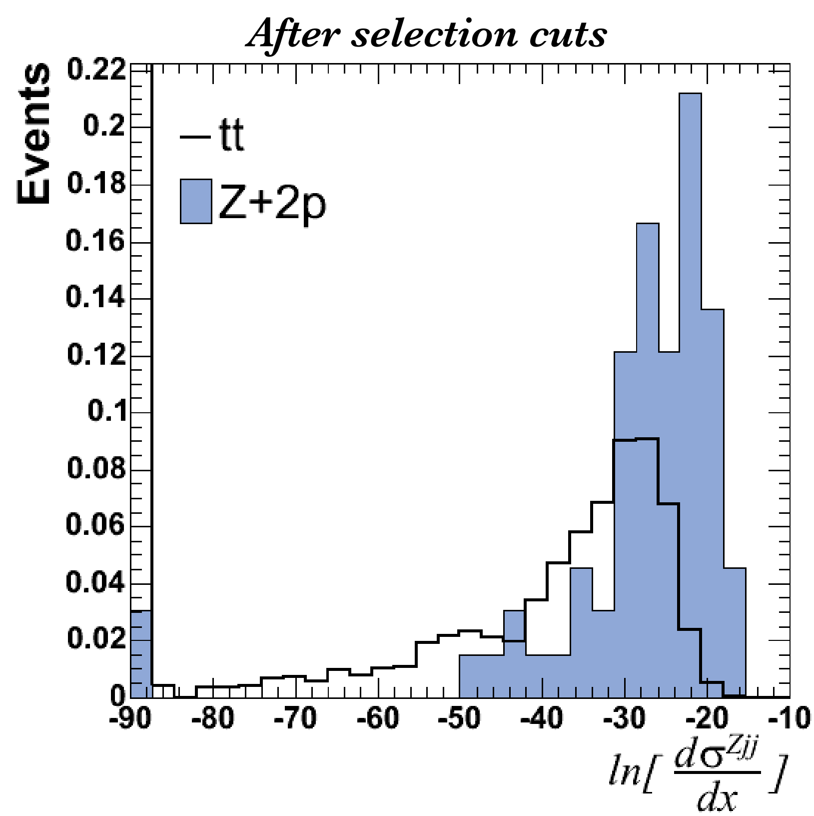


0 unknowns

1 parton energy

1 parton energy

-2 (*P_T conservation*)



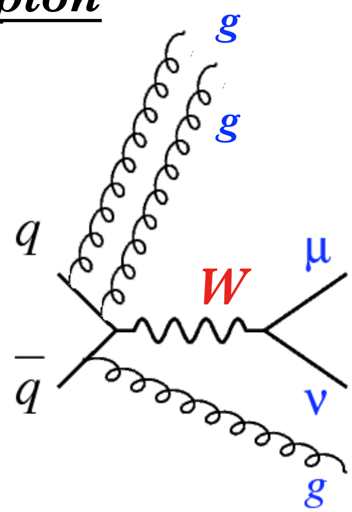
Matrix Element & Integrals

Add 2 integrals for P_T of Zjj system

Alpgen subroutine for Z+2p

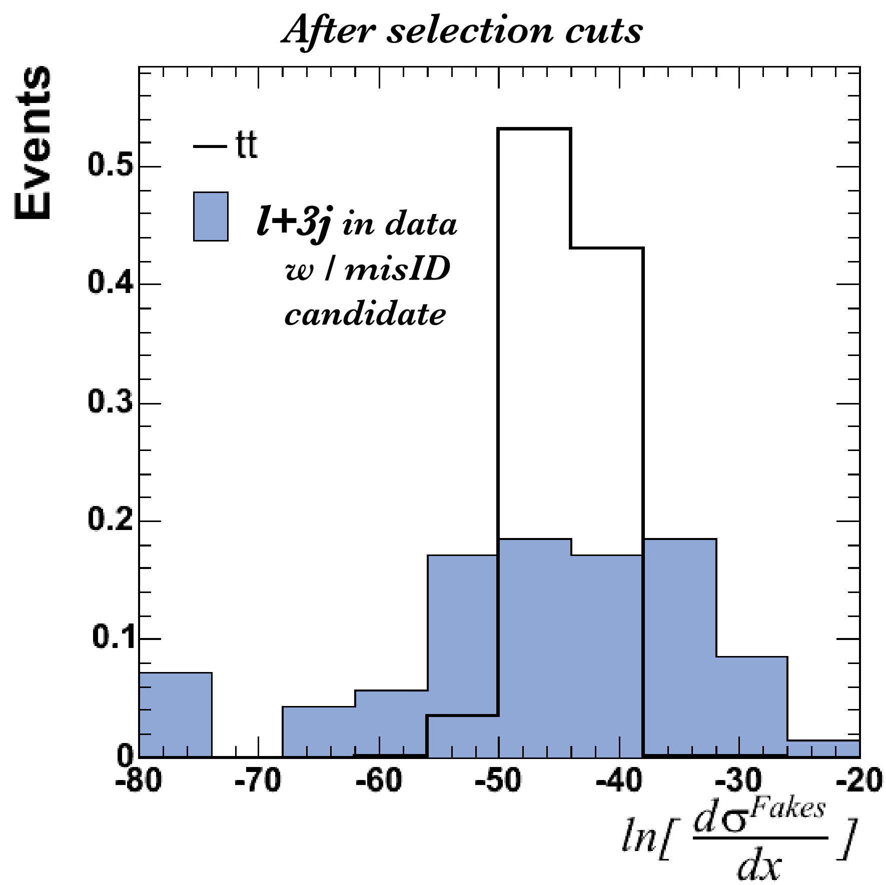
Integration with Vegas

Mis-ID Lepton



3 unknowns

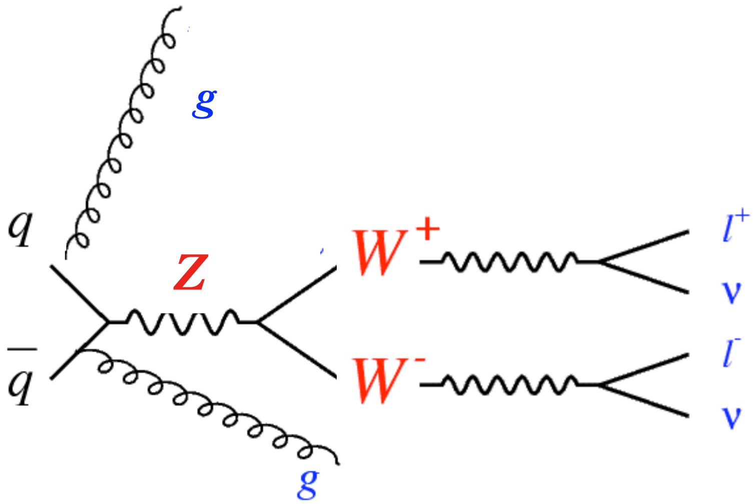
1 parton energy
1 parton energy
3 neutrino *(P components)*
-2 *(P_T conservation)*



Matrix Element & Integrals

Alpgen subroutine for W+3p
Integration with Vegas

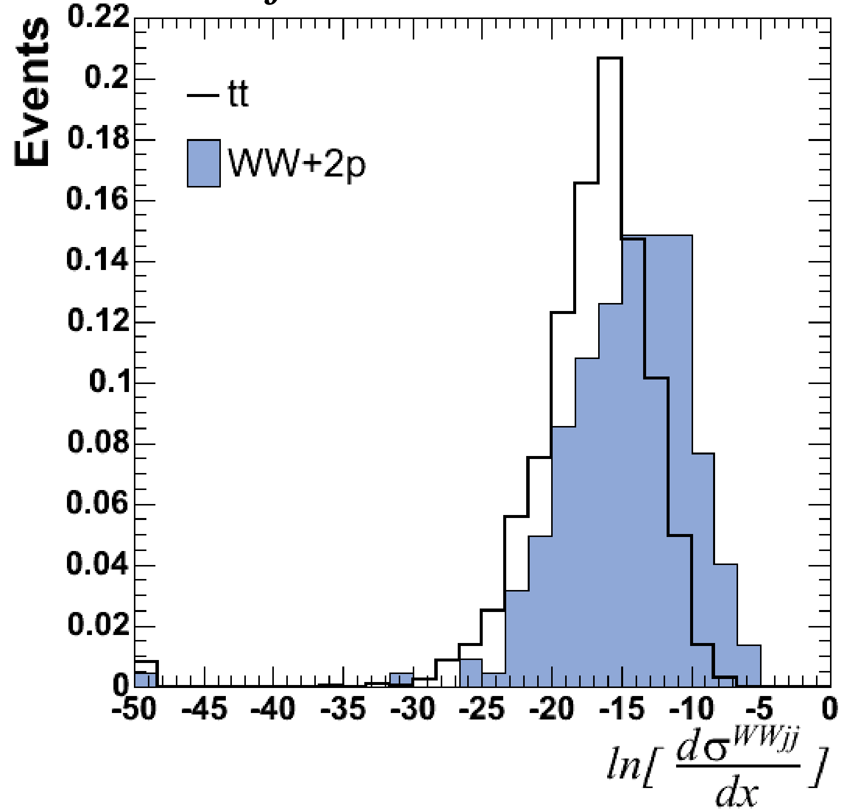
WW+jets



6 unknowns

Parton energy
Parton energy
3 neutrino *(P components)*
3 neutrino *(P components)*
-2 *(P_T conservation)*

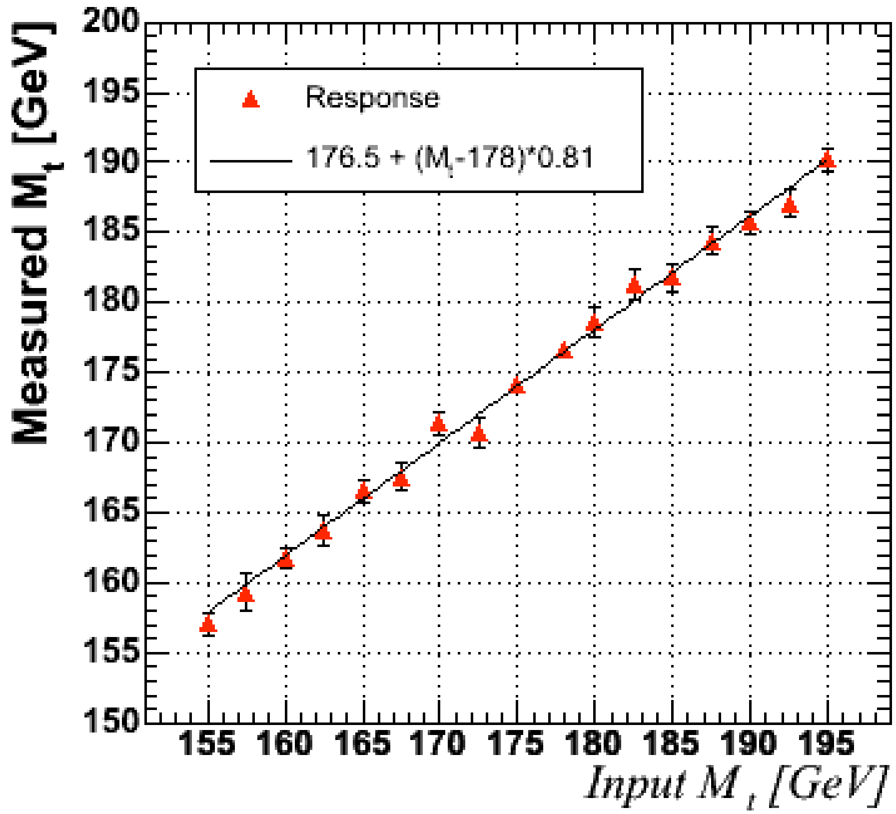
After selection cuts

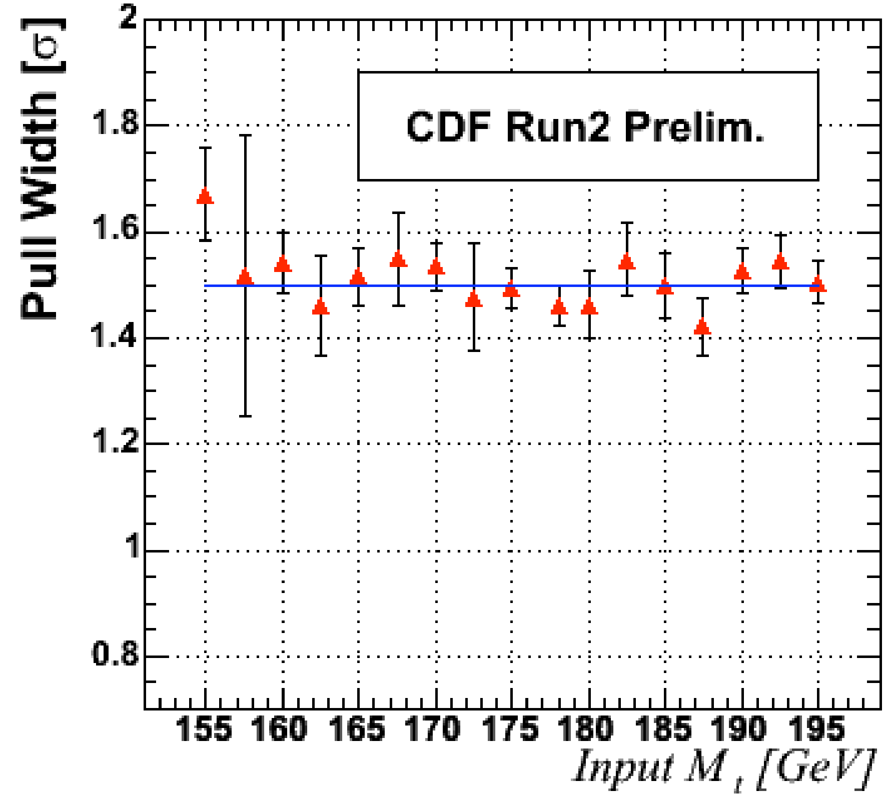


Matrix Element & Integrals

Alpgen subroutine for WW+2p
Integration with Vegas

Response & Pulls





Response

Linear.

Slope < 1 due to backgrounds.

Error is reduced 15% by P_{bg}

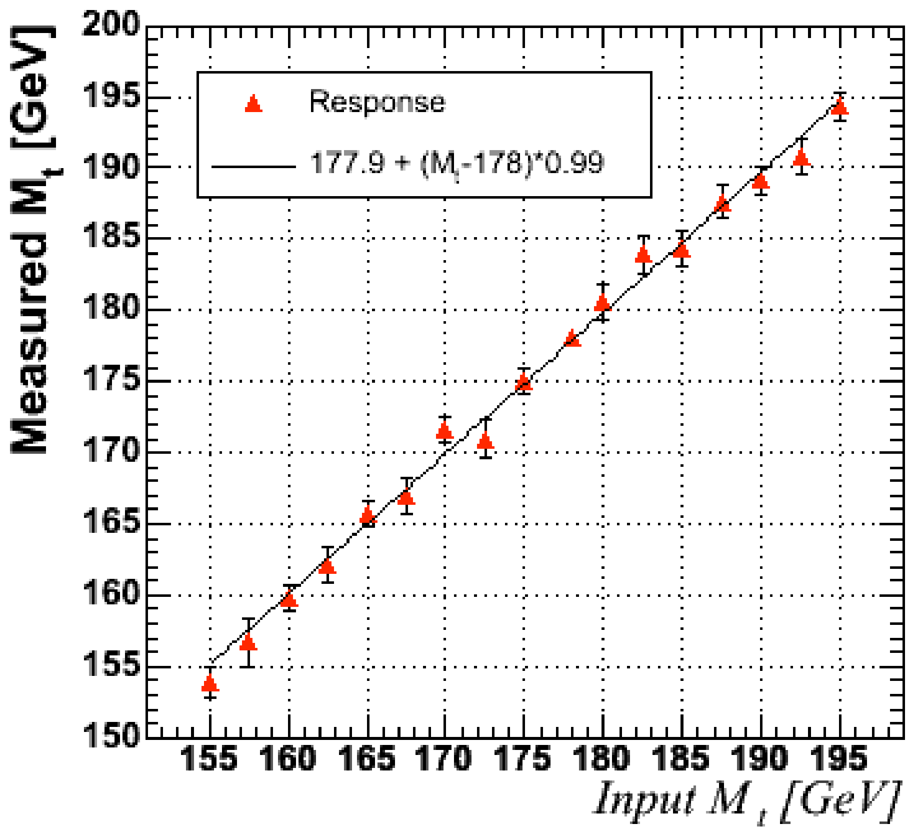
Slope is improved by 20% by P_{bg}

Pull width

Flat.

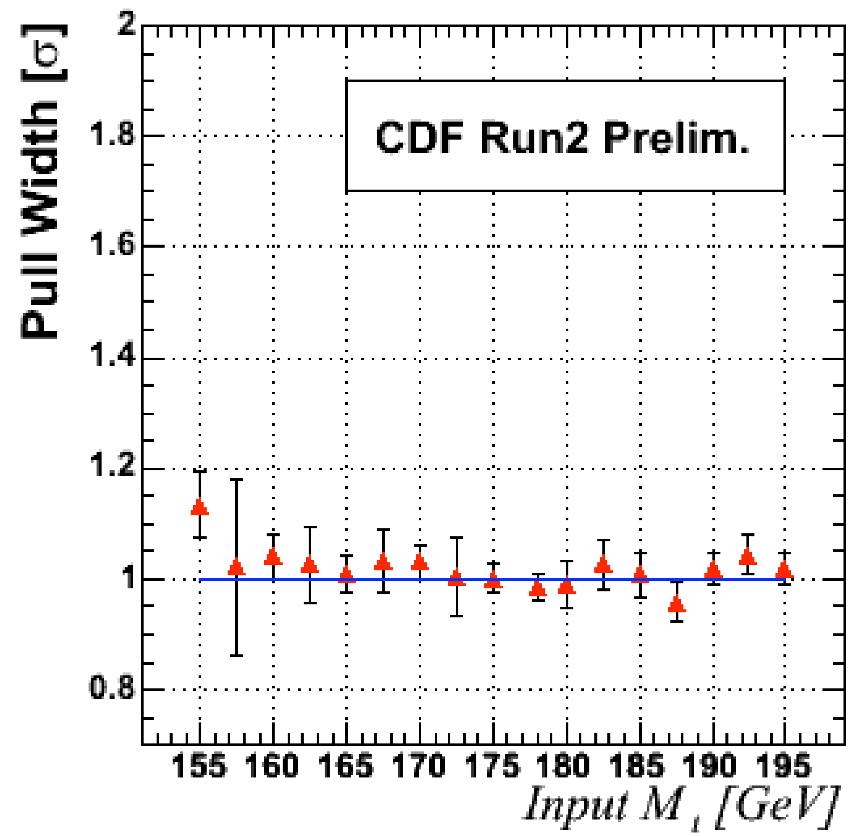
Width increased due to unmodelled bg (*Z->ττ, WZ,ZZ*)

Response & Pulls



Response

After slope correction.

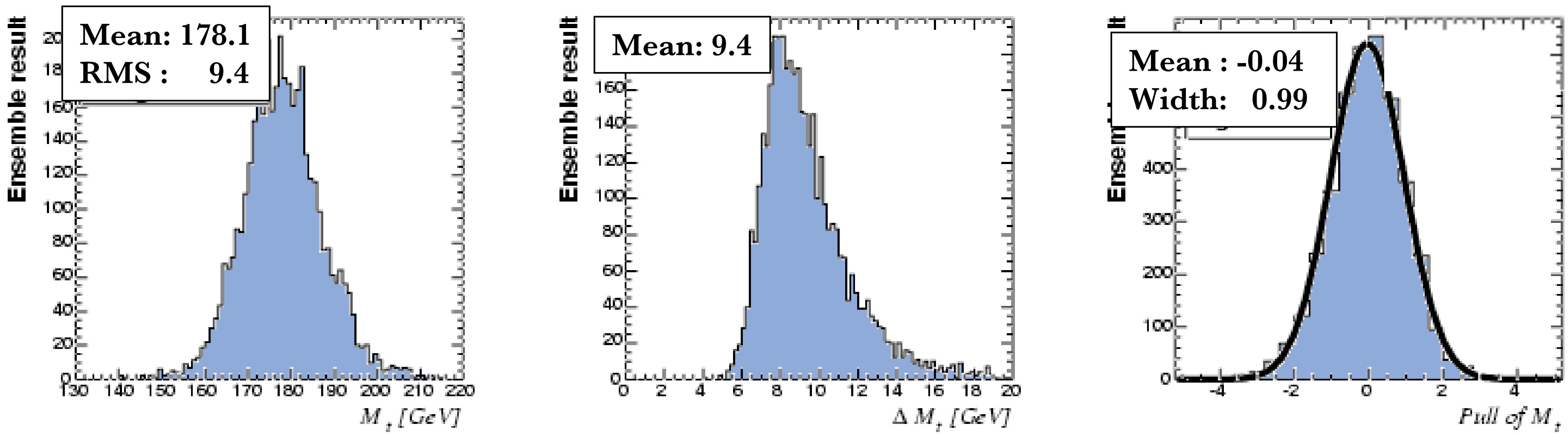


Pull width

After error correction.

Response at M_t=178 GeV

Residual, error, and pull distribution



Method	*Mean Error (M_t=178 GeV)*
Matrix Element	9.4 GeV
Neutrino Weighting	12.8 GeV
Kinematics	14.6 GeV
Neutrino Phi	14.9 GeV

Systematic Errors

Source	Size
Jet Energy Scale	2.6 GeV
Generator	1.0 GeV
Method	0.6 GeV
Sample composition uncertainty	0.7 GeV
Background statistics	1.5 GeV
Background modelling	0.8 GeV
FSR modelling	0.5 GeV
ISR modelling	0.5 GeV
PDFs	1.1 GeV
Total	3.6 GeV

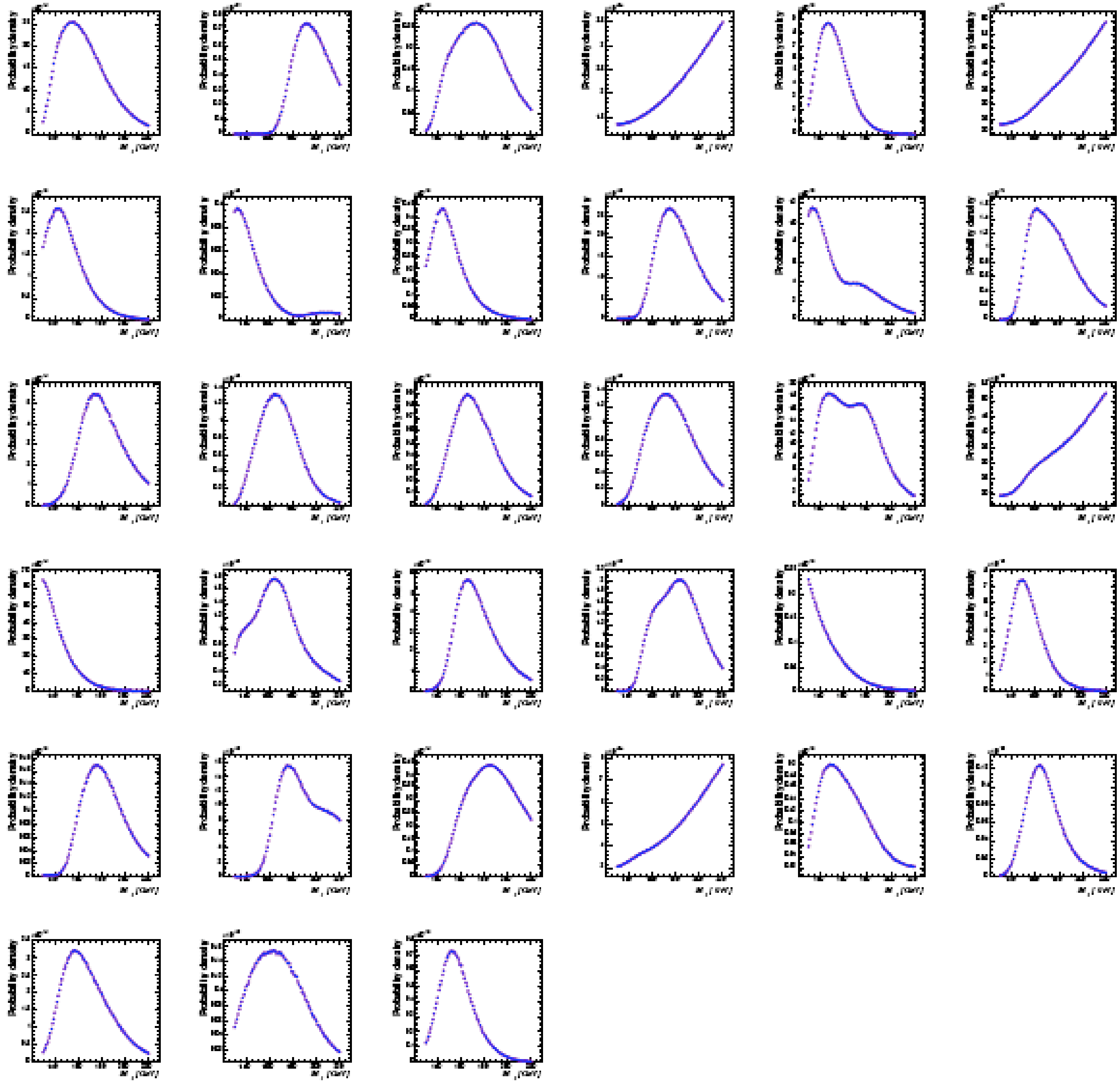
Data

33 candidates

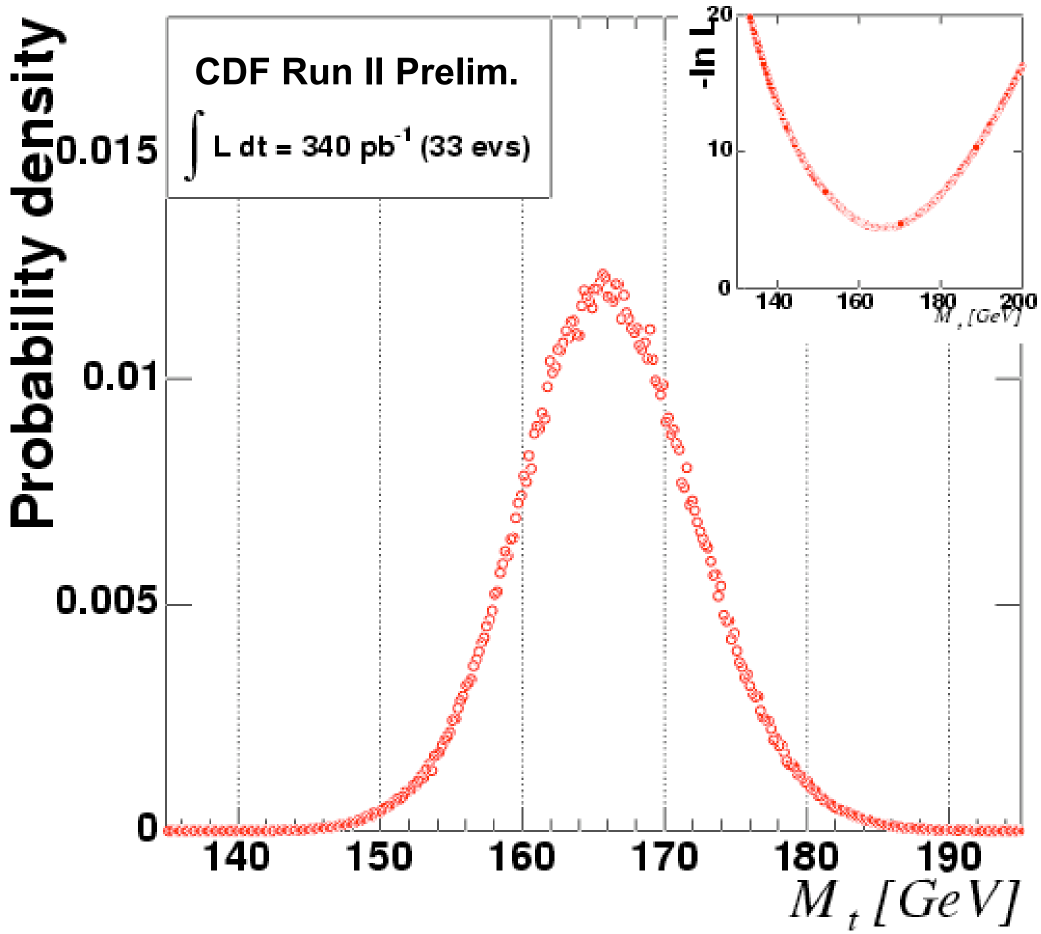
signal and bg probabilities

Slope and error corrections not applied

Range is 130-220 GeV/c^2



Measurement!



$$M_t = 165.3 \pm 6.3\ _{stat} \pm 3.6_{syst}\ \mathrm{GeV/c^2}$$

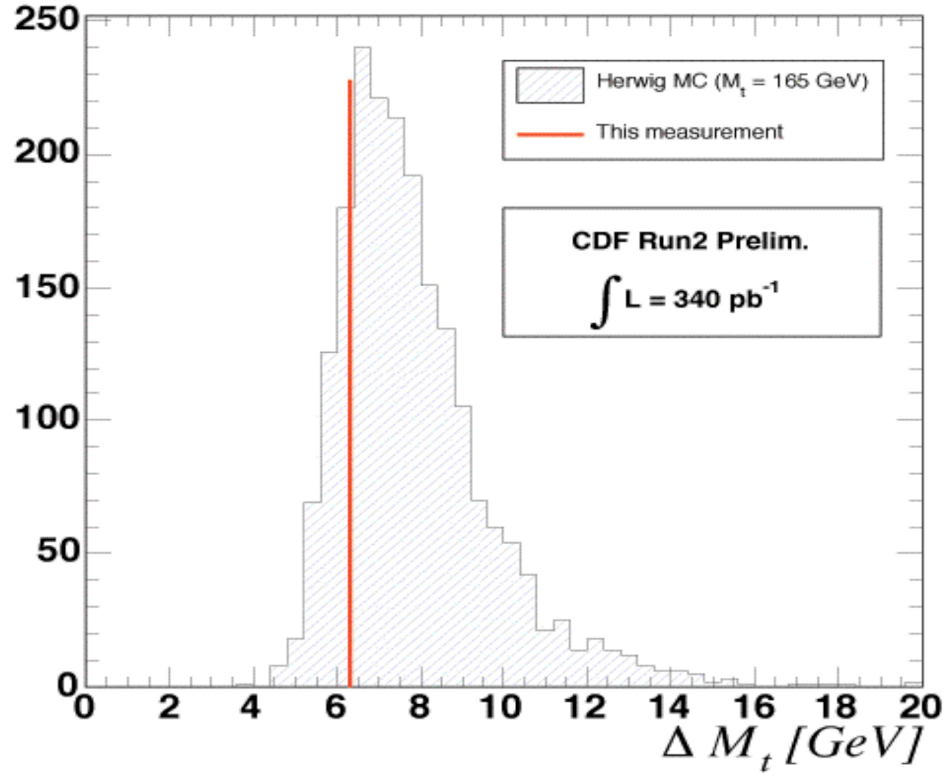
Most precise single dilepton measurement to date.

Expected Sensitivity

Sensitivity

If M_t = 165 GeV/c^2,
mean stat error is 7.8 GeV/c^2

If M_t = 178 GeV/c^2
mean stat error is 9.4 GeV/c^2



17% of pseudo-experiments
give smaller errors

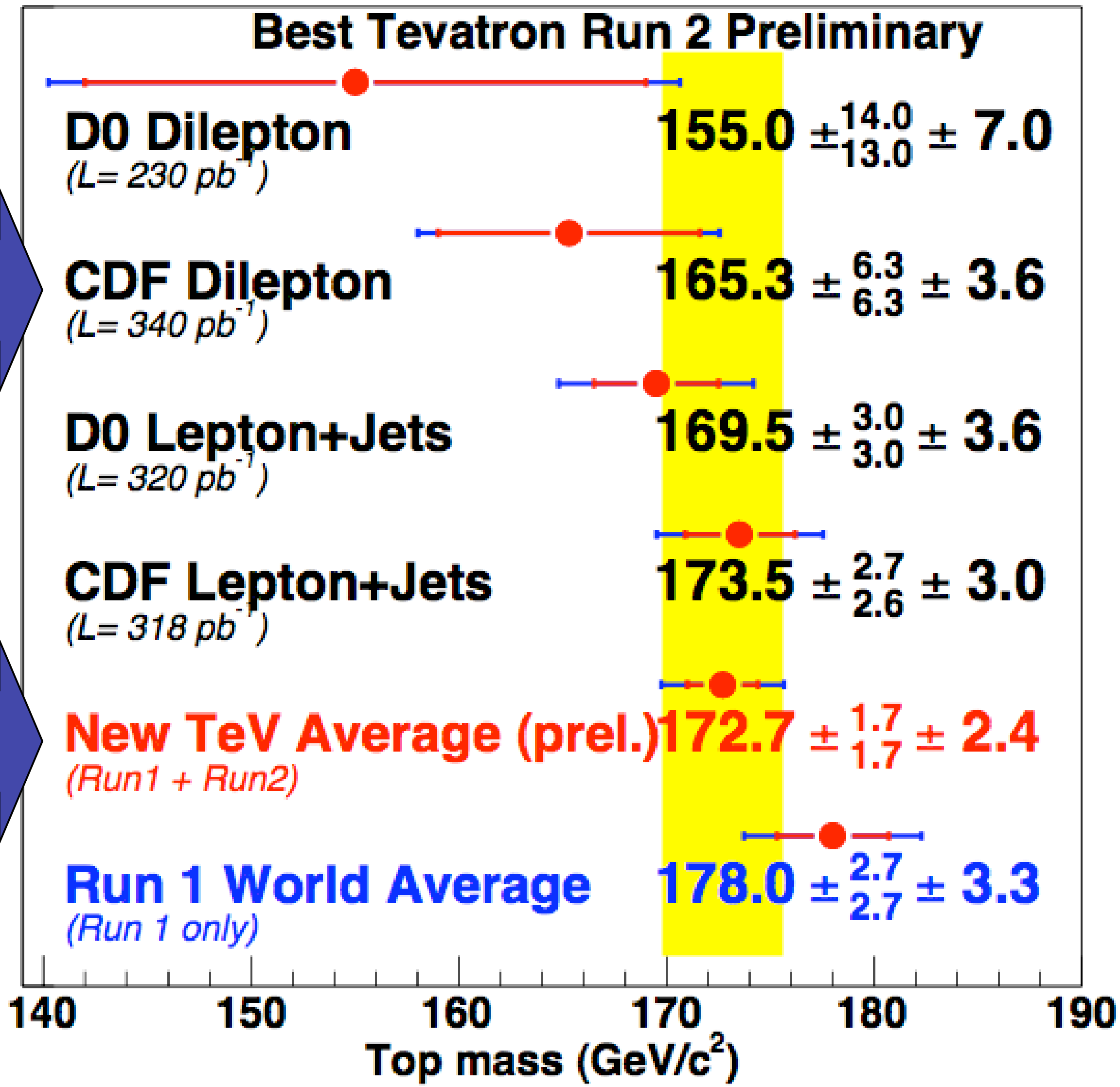
Impact

In World Average

Weight = 8%

Pull = - 1.1

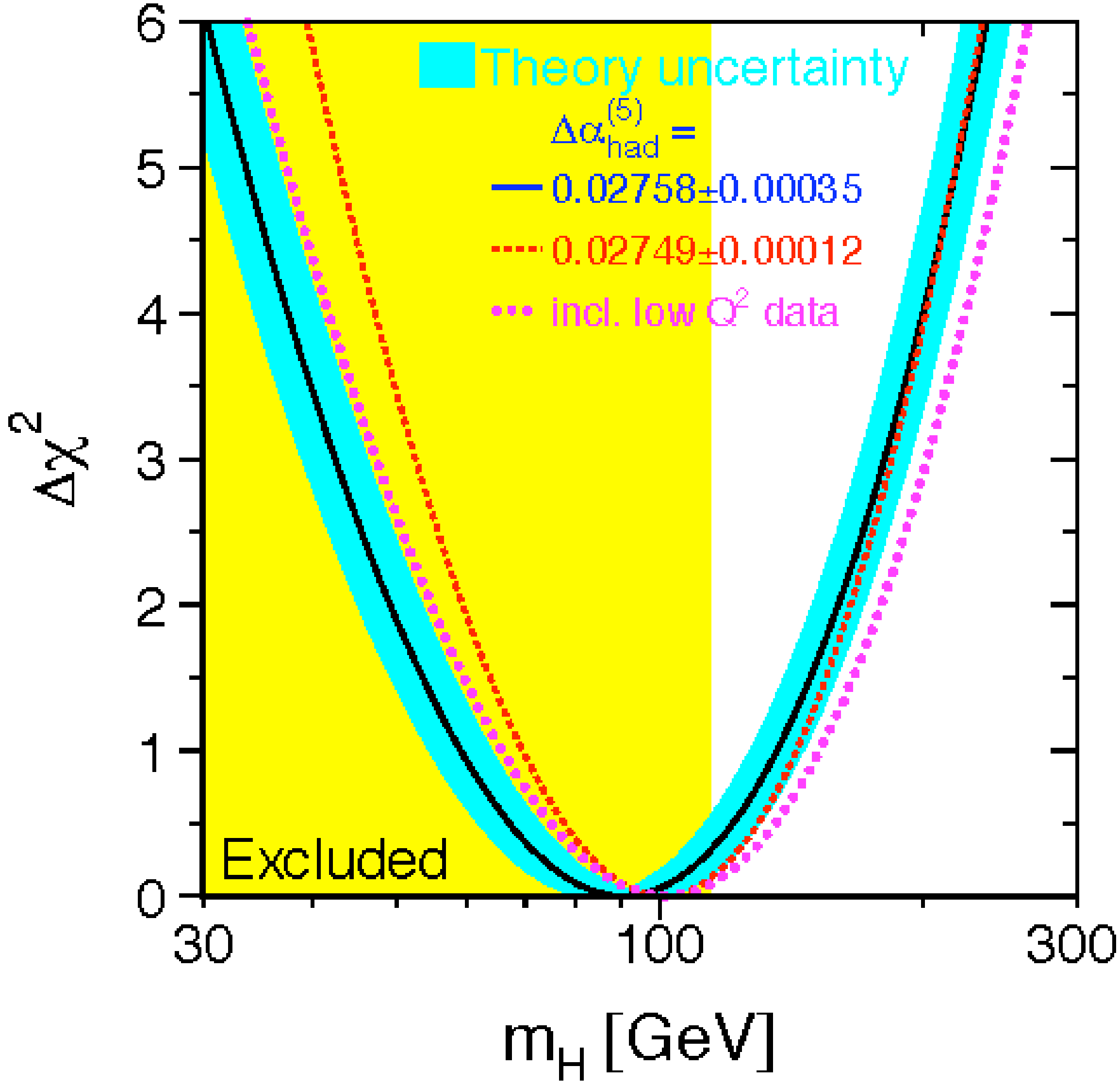
Total result has error of 2.9 GeV, systematics limited.



Higgs Impact

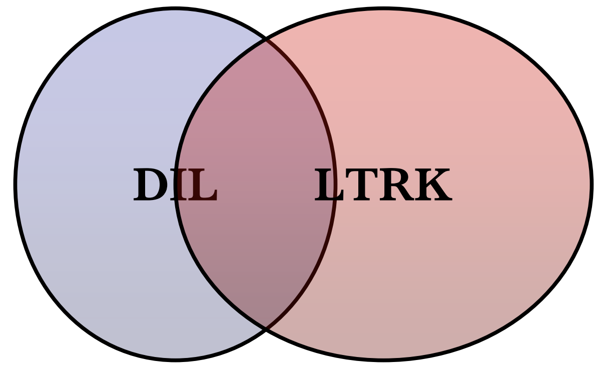
$M_{higgs} = 91^{+45}_{-32}$ GeV/c²

$M_{higgs} < 186$ GeV/c² (95% conf)



Combining dilepton results

Two overlapping datasets



Statistical Correlation

Data samples overlapping

Measure correlation in pseudo-experiments which model common data

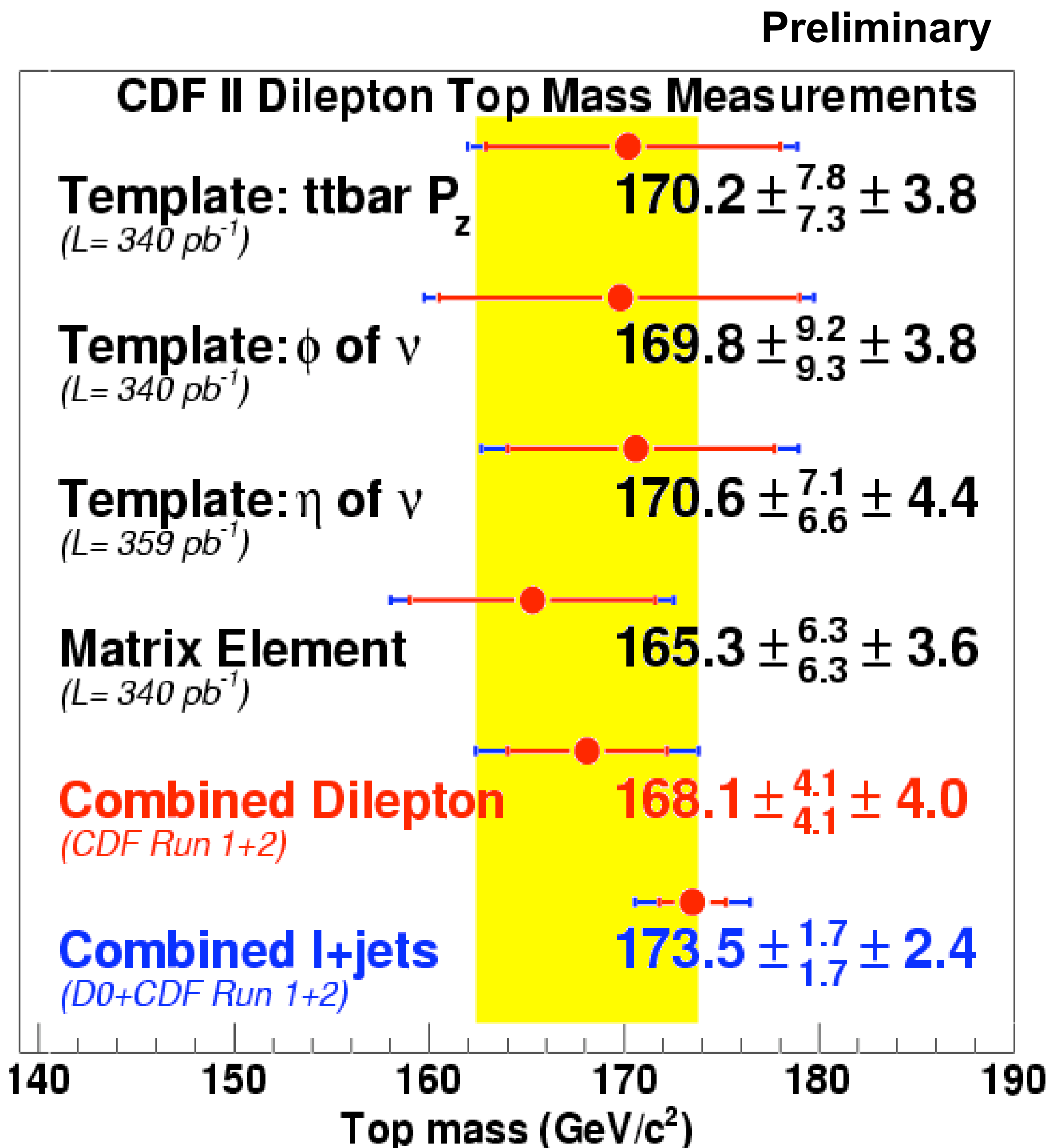
***Statistical Correlation Matrix* [CDF Run2 Preliminary]**

Sample		*ME*	*KIN*	*PHI*	*NWA*	*Run1*
DIL sample	Matrix Element	1.00				
DIL sample	Kinematic	0.33	1.00			
DIL sample	Neutrino Phi	0.15	0.18	1.00		
LTRK sample	Neutrino Weight	0.06	0.17	0.26	1.00	
	Run1	0.00	0.00	0.00	0.00	1.00

$\Delta\rho \sim 0.06$

Combined Dilepton Result

	Weight	Pull
Matrix El.	36%	-0.60
ν Weight	27%	0.42
Kinematic	13%	0.33
ν Phi	10%	0.20
Run1	14%	-0.07



Summary & Outlook

Precision Measurement

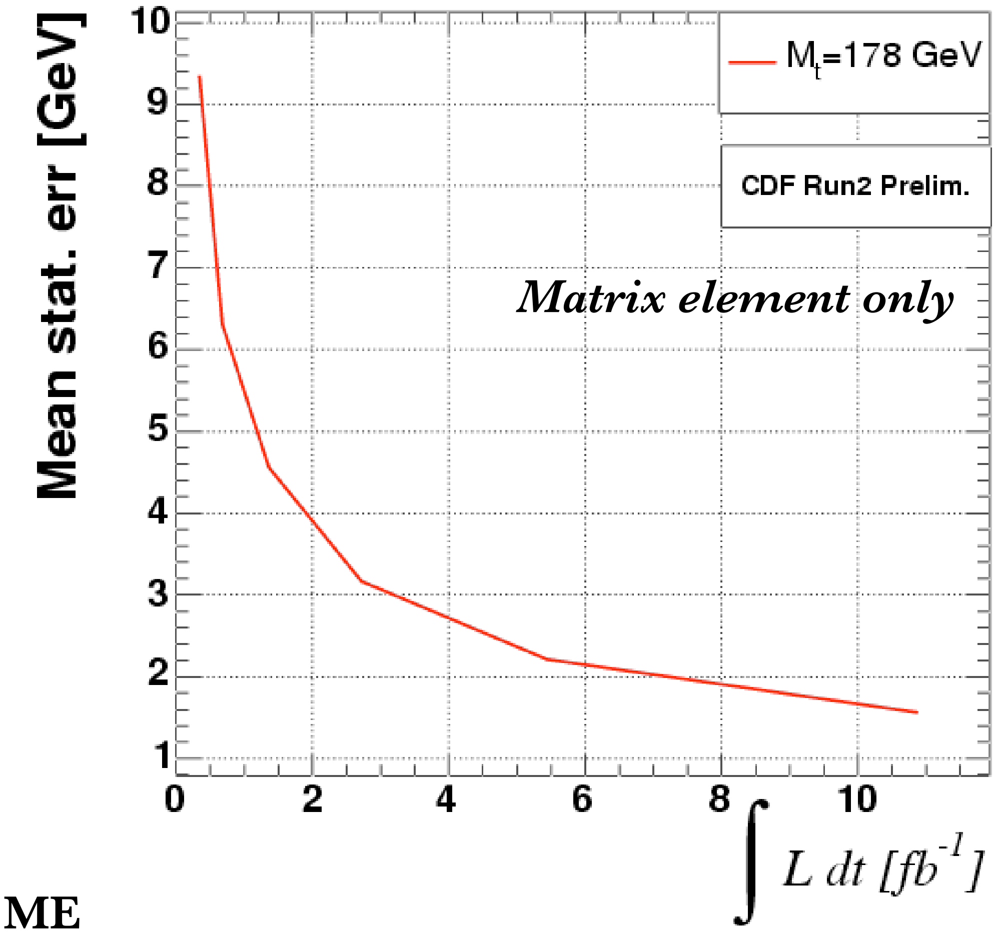
We have measured the top mass in dileptons with a stat. error of 4.1 GeV

Results consistent with l+jets

Room for Improvement

Identified assumptions which are critical to sensitivity

High Precision Future

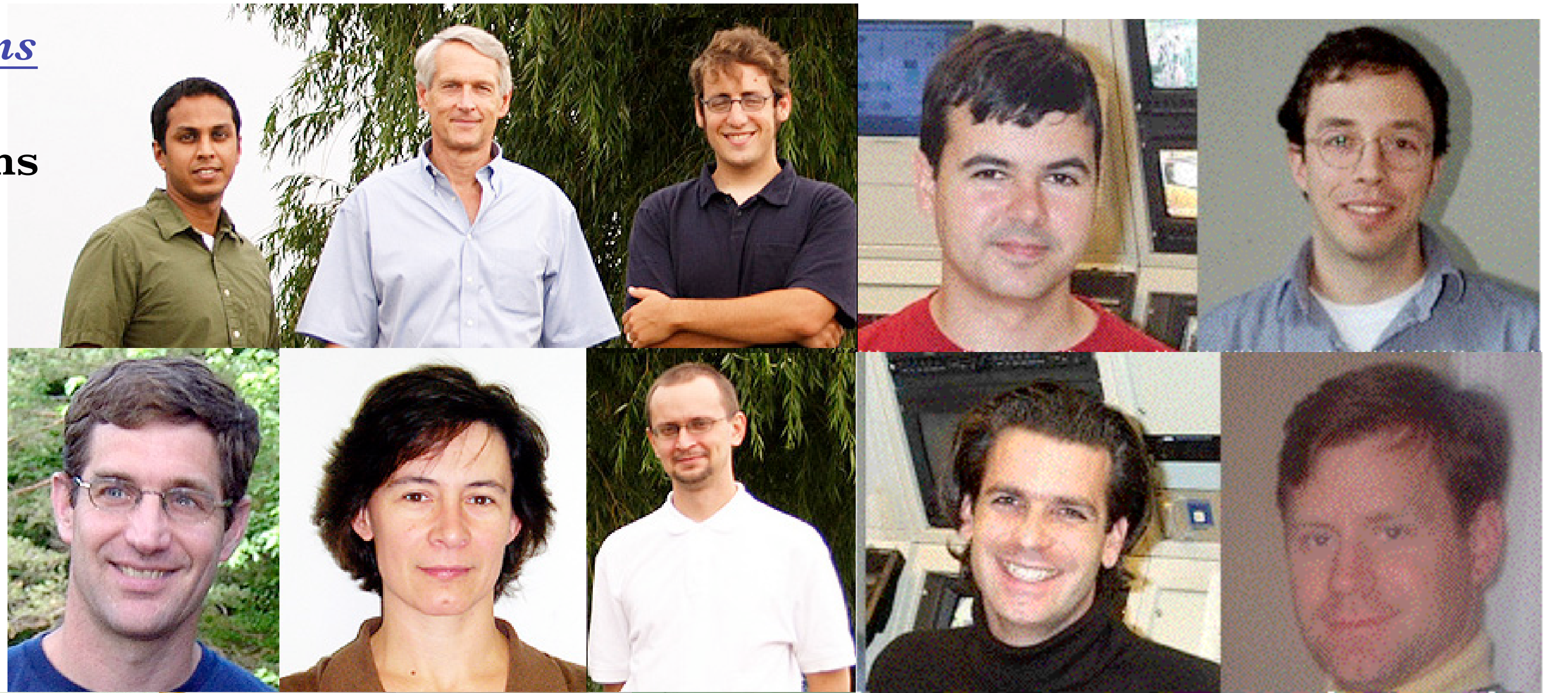


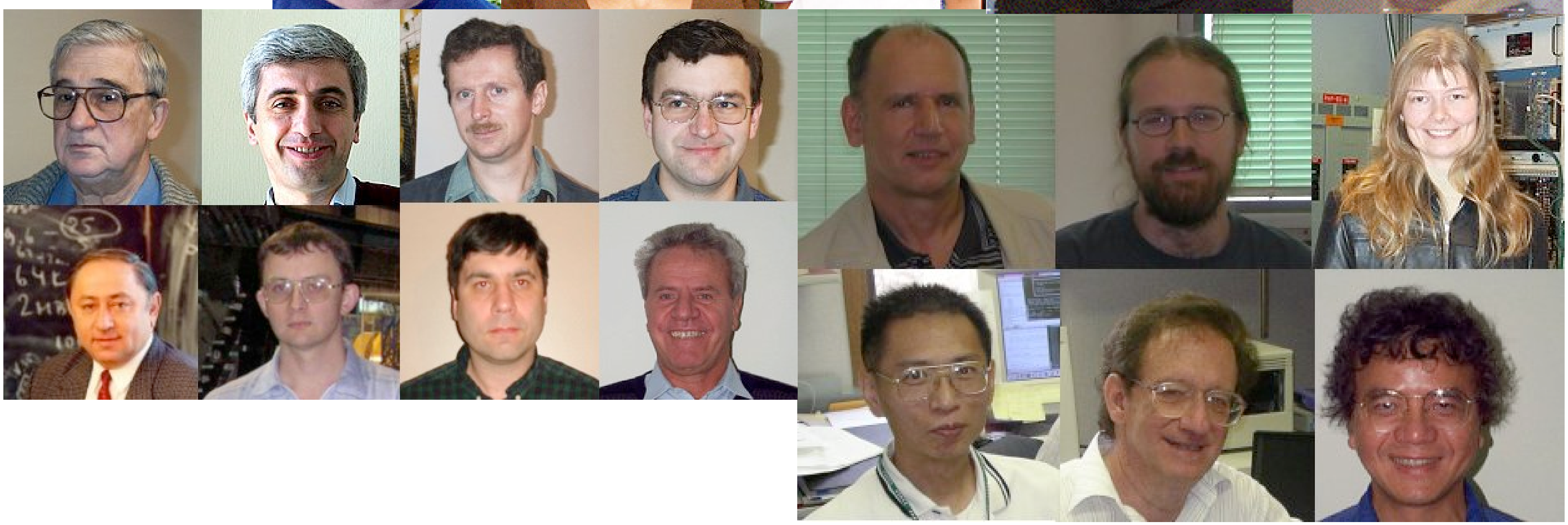
With no improvements, in $4 fb^{-1}$,

- expect 2.5 GeV *statistical* error for ME
- Dileptons become systematics limited.
- l+jets and dileptons approach equal weight
- Comparison of channels to test top hypothesis, look for new physics
- Systematic challenge: reduce jet energy scale (no W->jj decay for calibration)

CDF Dileptons

24 people
9 institutions
7 countries



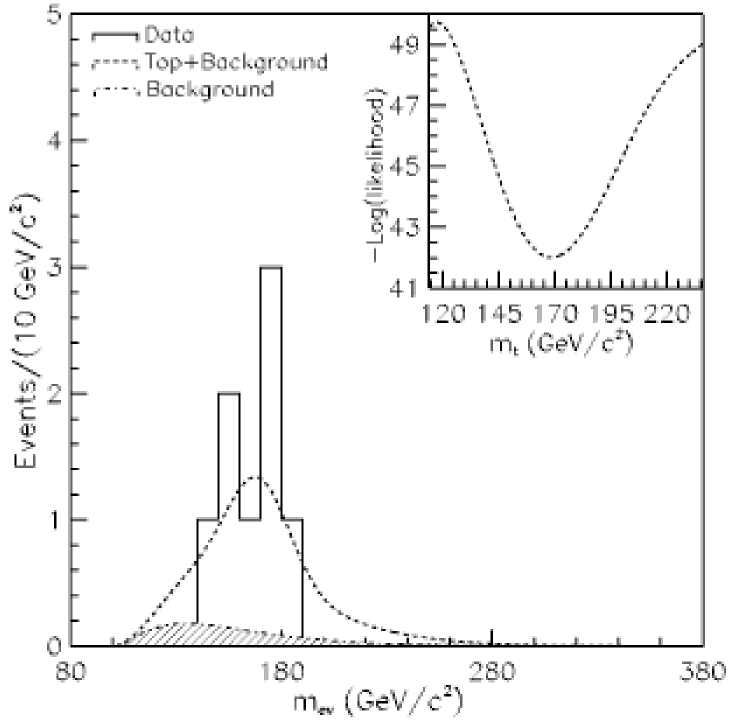


Backup material

Dilepton History: Run I

CDF: 8 events

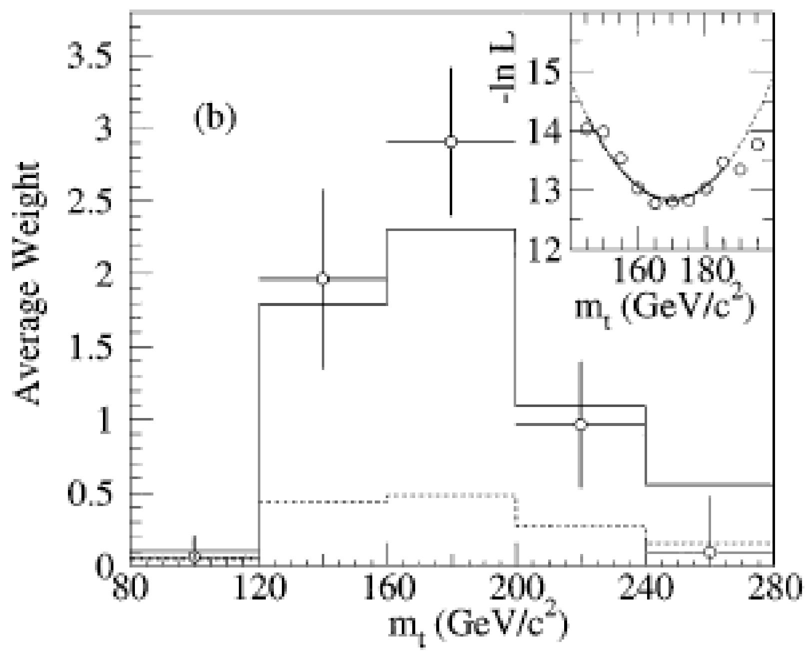
Mt = 167.4 +- 10.3 +- 4.8 GeV



Phys. Rev. Lett. ***82****, 271 (1999)*

D0: 6 events

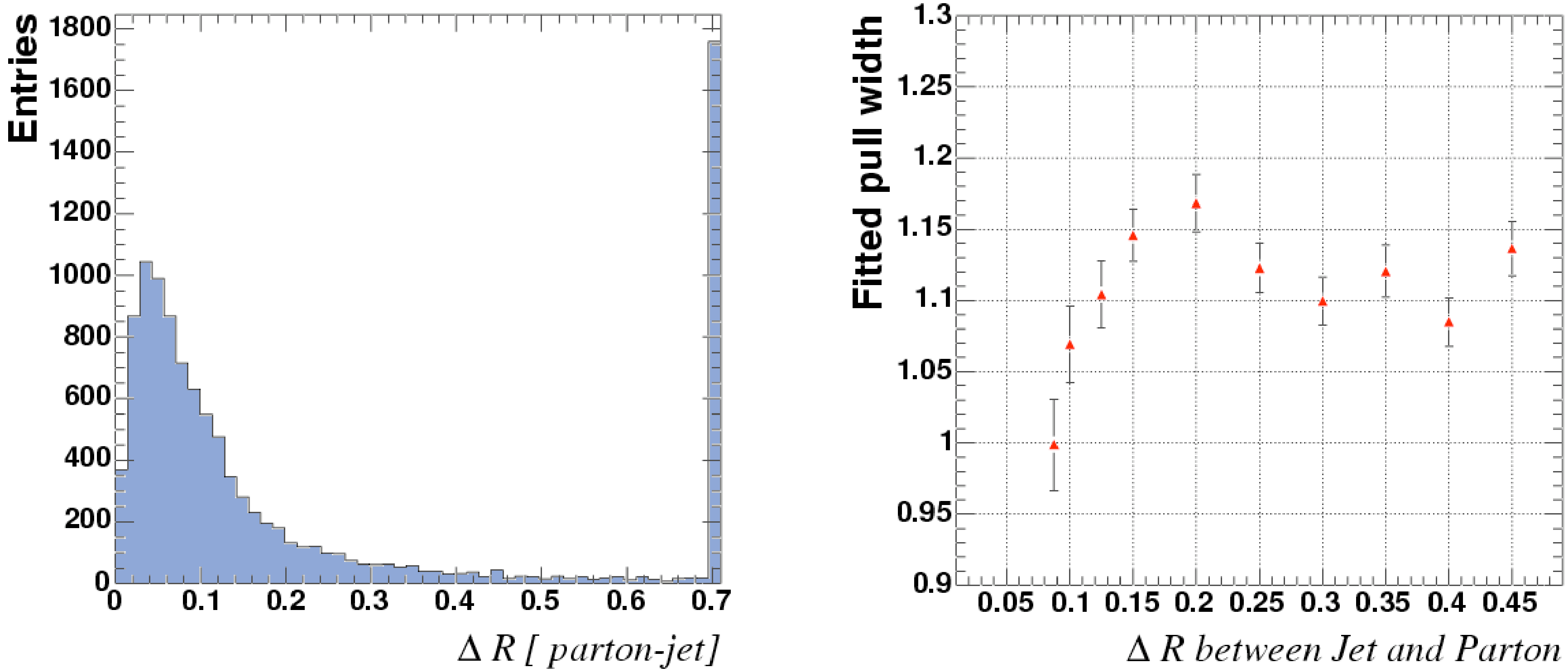
Mt= 168.4 +- 12.4 +- 3.6 GeV



Phys Rev Lett 82, 2063 (1998)

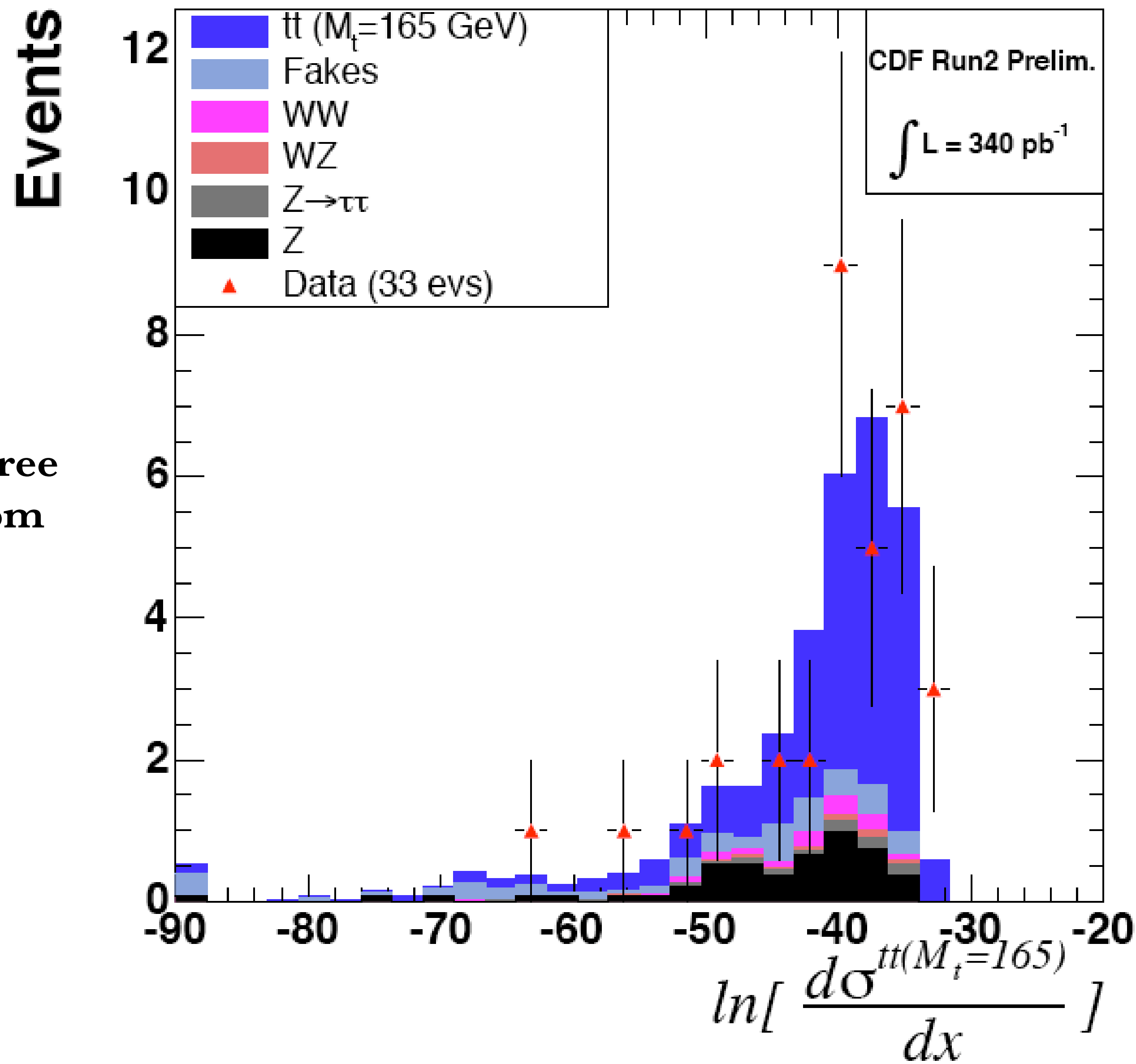
Pull Width: jet angles

Requring two matched b-jets
Requiring well-measured leptons



Pull width decreases to ~1 as angle improves

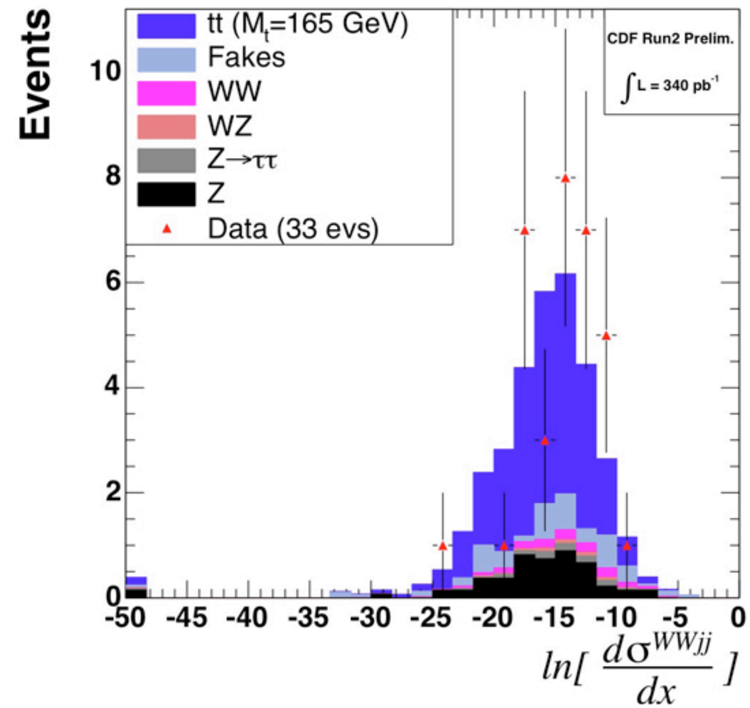
Signal probability in data

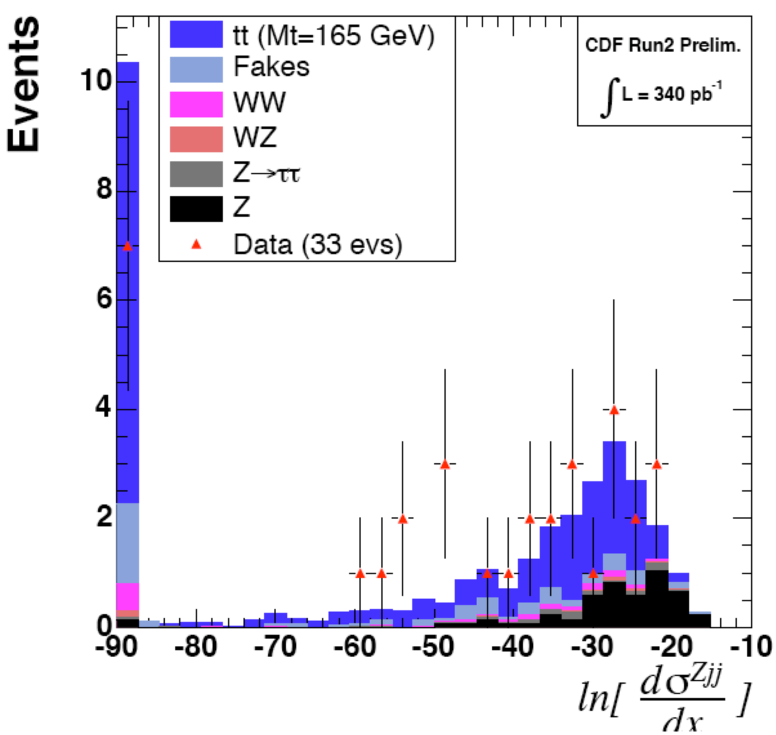


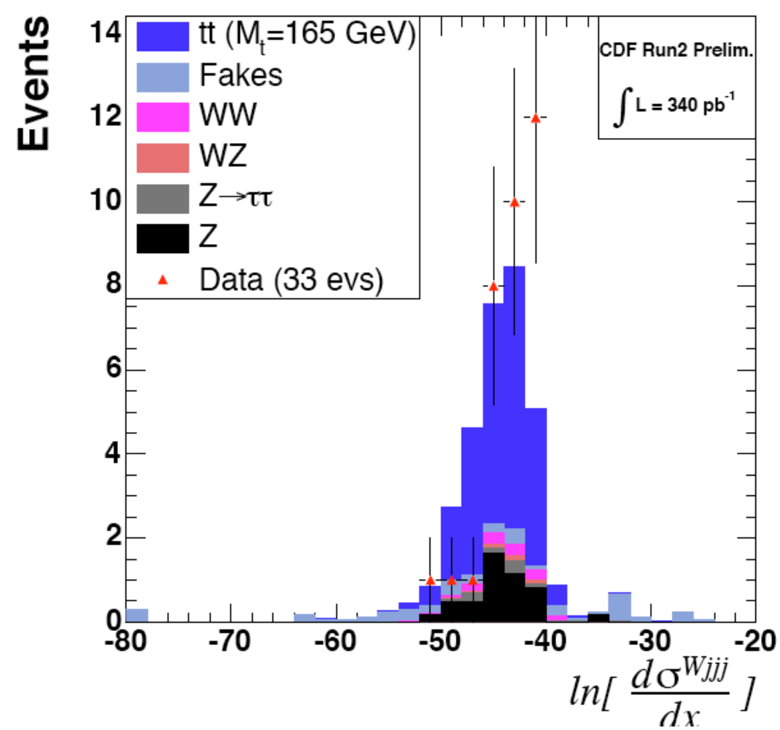
Distribution of un-normalized signal probability in data agree with that expected from MC

Background probabilities in data

Distribution of un-normalized background probabilities in data agree with that expected from MC

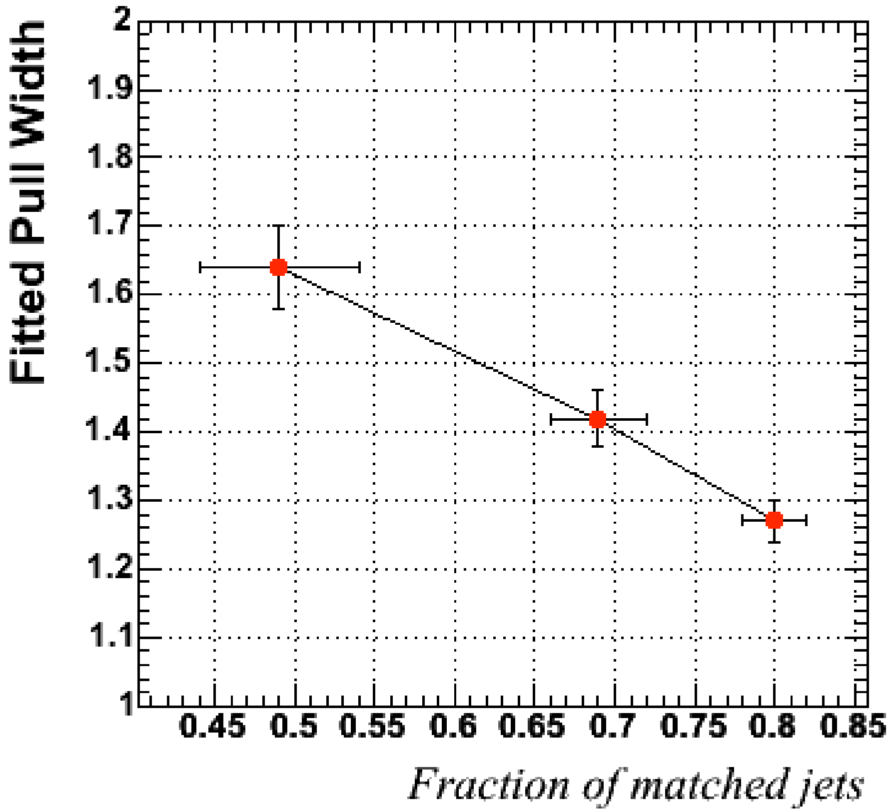


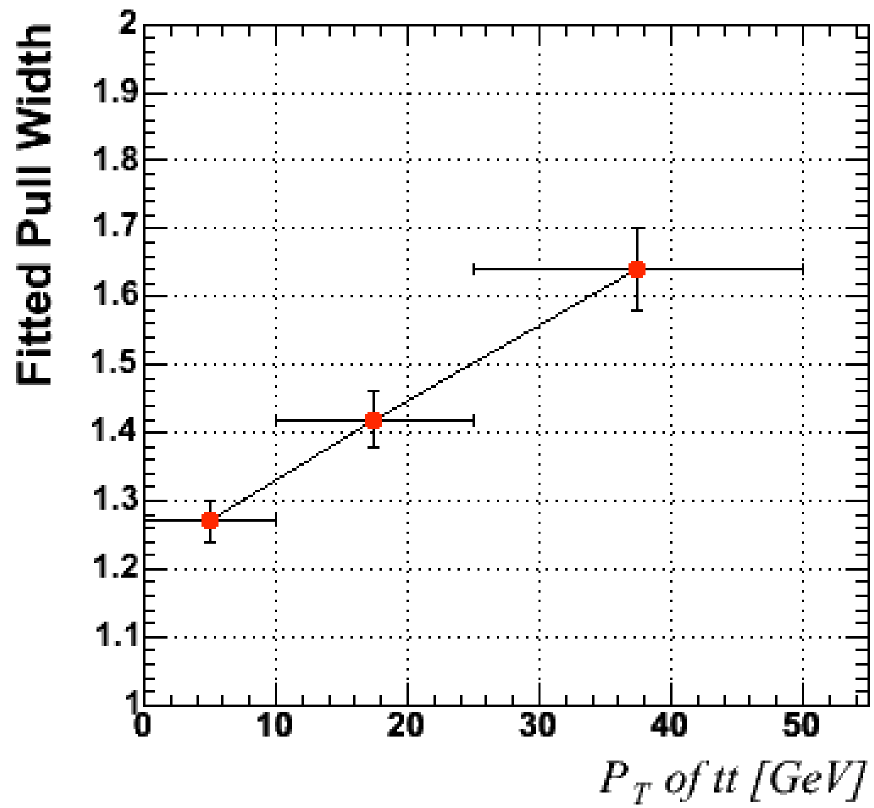




Pull width: mismatched jets

Pull width is affected by wrong jets

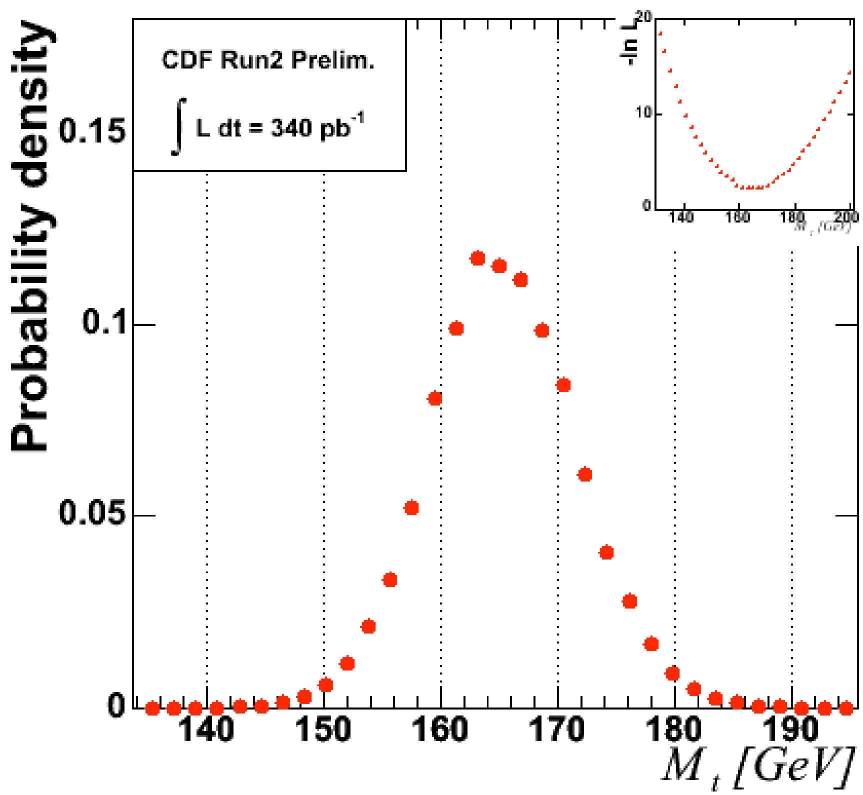


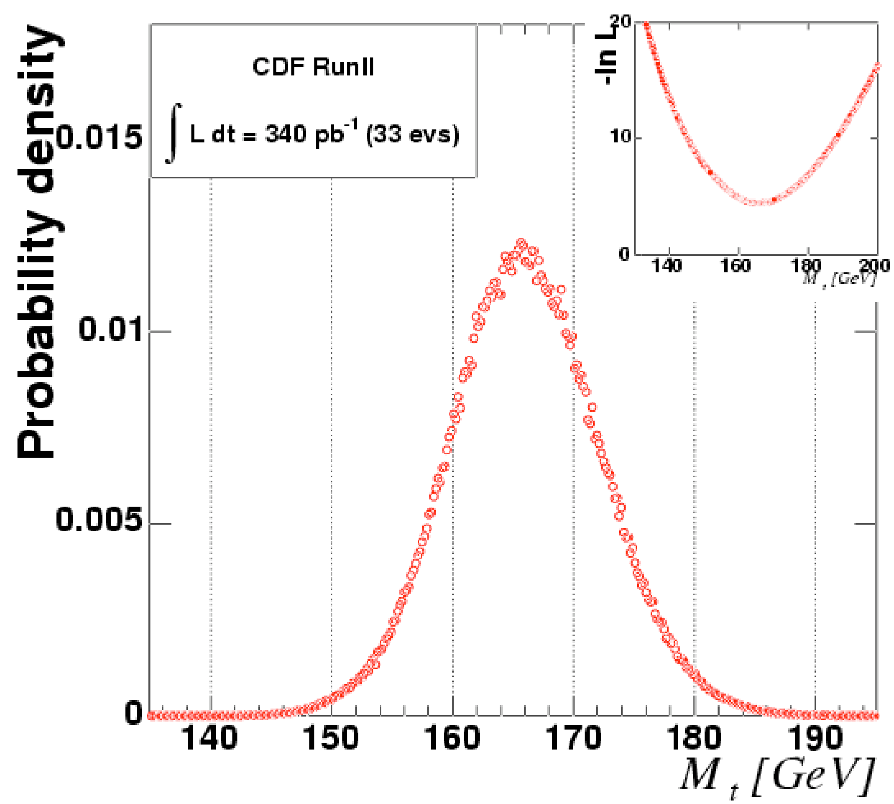


Most wrong jets come from initial state radiation, which can be probed by examining P_T of ttbar

Mass steps refined

We scanned the space in Mt with finer steps to probe the shape:





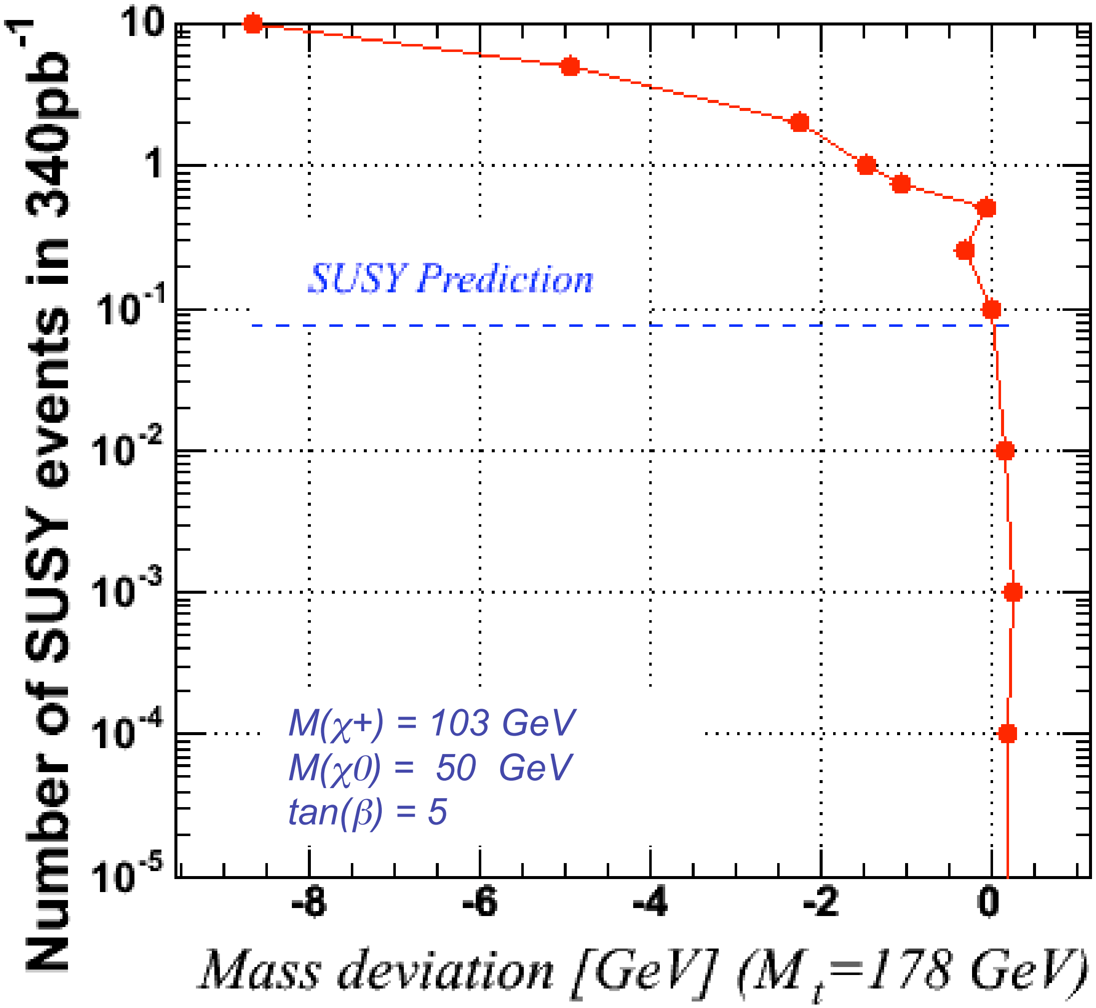
Effect of SUSY events on dilepton mass measurement

Chargino/Neutralino

Topology is lll+2j or llqq

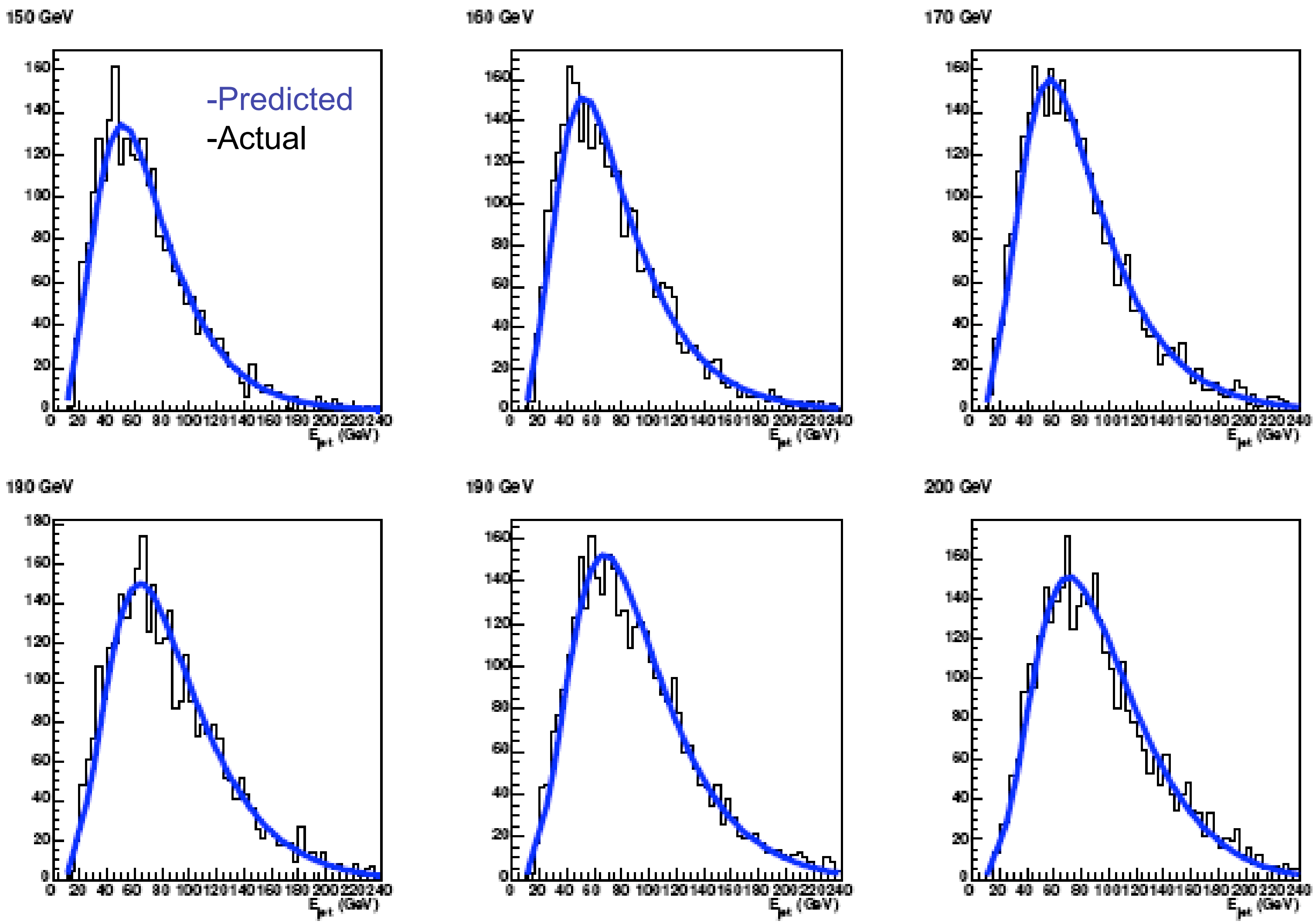
M(χ+) = 103 GeV
M(χ0) = 50 GeV
tan(β) = 5

*Sigma*br = 150 fb*
Acc = 0.15%

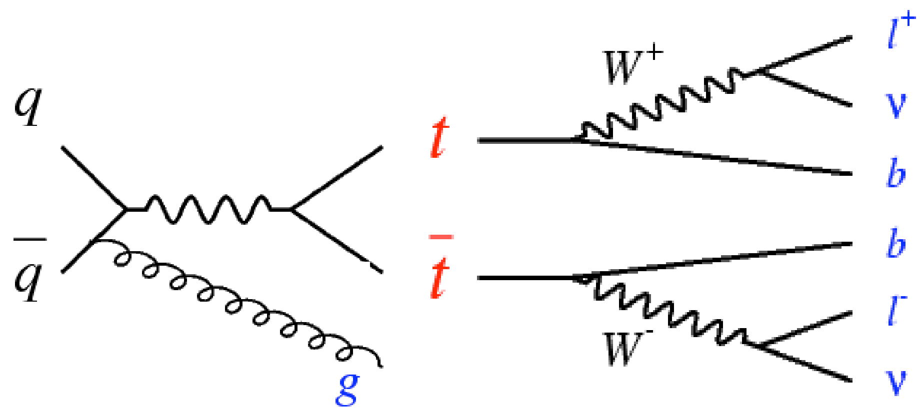


TFS

Transfer functions predict jet energy spectrum at varying top masses.



Future work



Statistical error

-Improve handling of extra jets
(approximates NLO effect)

Systematic error

-Apply jet energy calibration from Z->bb
-Improve sophistication of background modelling