Experimental Particle Physics Seminar University of Pennsylvania, October 17, 2007

# Search for Flavor Changing Neutral Currents in Top Quark Decays at CDF



Ulrich Husemann Yale University







### What are Flavor Changing Neutral Currents?

#### The CDF Experiment at the Tevatron

Top Quark Physics at CDF

Search for FCNC in Top Quark Decays

**Summary & Conclusions** 





### What are Flavor Changing Neutral Currents?

#### The CDF Experiment at the Tevatron

Fop Quark Physics at CDF

Search for FCNC in Top Quark Decays

**Summary & Conclusions** 



## **Standard Model of Particle Physics**





- Matter in the standard model: 12 fermions in three generations
  - Six quarks and their anti-particles
  - Six leptons and their anti-particles
- Forces in the standard model:
  - Strong force (carrier: gluon)
  - Electroweak force (carriers: photon, W<sup>±</sup> bosons, Z boson)
- Interactions can be described by "currents" coupling to gauge bosons, e.g. electromagnetic current





## **Flavor Changing Neutral Currents**



- Flavor changing neutral current (FCNC) interactions:
  - Transition from a quark of flavor A and charge Q to quark of flavor B with the same charge Q
  - Examples:  $b \rightarrow s\gamma$ ,  $t \rightarrow cH$ , ...
- 1960s: only three light quarks (u,d,s) known, mystery in neutral kaon system:





- Solution: "GIM Mechanism" (Glashow, Iliopoulos, Maiani, 1970)
  - Fourth quark needed for cancellation in box diagram: prediction of charm quark
  - Cancellation would be exact if all quarks had the same mass: estimate of charm quark mass



#### Standard model: no FCNC at Lagrangian level

Massless theory: weak neutral current is flavor-diagonal

$$J_{\mu}^{\rm NC} = J_{\mu}^3 - 2\sin^2\theta_{\rm W}j_{\mu}^{\rm em} = \bar{u}\left[\frac{1}{2}\gamma_{\mu}\left(1 - \gamma_5\right) - \frac{4}{3}\sin^2\theta_{\rm W}\gamma_{\mu}\right]u - \bar{d}\left[\frac{1}{2}\gamma_{\mu}\left(1 - \gamma_5\right) - \frac{2}{3}\sin^2\theta_{\rm W}\gamma_{\mu}\right]d$$

- Quark masses via Higgs mechanism:
  - Eigenstates of electroweak interactions are not mass eigenstates

$$\mathscr{L}_{Yuk} = -\frac{m_u^{\alpha\beta}}{\mu_L^{\prime\alpha}} \frac{\bar{u}_L^{\prime\alpha} u_R^{\prime\beta} - m_d^{\alpha\beta}}{\mu_L^{\prime\alpha}} \frac{\bar{d}_L^{\prime\alpha} d_R^{\prime\beta}}{\bar{d}_L^{\prime\alpha}} - \frac{1}{\sqrt{2}} f_u^{\alpha\beta} u_L^{\prime\alpha} h(x) u_R^{\prime\beta} - \frac{1}{\sqrt{2}} f_d^{\alpha\beta} d_L^{\prime\alpha} h(x) d_R^{\prime\beta} + \text{h.c.}$$
Mass Terms
Higgs Couplings

• Unitary transformation of Lagrangian to mass basis, i.e. for physical particles:

$\bar{u}_{\mathrm{L}} = \bar{u}_{\mathrm{L}}' \mathbf{U}_{\mathrm{L}}^{u}$	$u_{\rm R} = \mathbf{U}_{\rm R}^{u\dagger} u_{\rm R}'$	$\mathbf{m}_u = \mathbf{U}_{\mathrm{L}}^{u\dagger}  \mathbf{m}'_u  \mathbf{U}_{\mathrm{R}}^u$
$\bar{d}_{\mathrm{L}} = \bar{d}'_{\mathrm{L}} \mathbf{U}^{d}_{\mathrm{L}}$	$d_{\mathrm{R}} = \mathbf{U}_{\mathrm{R}}^{d\dagger} d_{\mathrm{R}}^{\prime}$	$\mathbf{m}_d = \mathbf{U}_{\mathrm{L}}^{d\dagger}  \mathbf{m}_d'  \mathbf{U}_{\mathrm{R}}^d$

- Kinetic terms: unchanged
- Higgs couplings proportional to mass terms: no flavor changing Higgs couplings
- Neutral currents have same structure as kinetic terms: unchanged  $\rightarrow$  no FCNC





Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix (obtained from transformation of charged current to mass basis):

$$J_{\mu}^{\text{CC}} = \bar{u}' \left( \frac{1}{2} \gamma_{\mu} \left( 1 - \gamma_{5} \right) \right) d' = \bar{u}'_{\text{L}} \gamma_{\mu} d'_{\text{L}} = \bar{u}_{\text{L}} \mathbf{U}_{\text{L}}^{u\dagger} \gamma_{\mu} \mathbf{U}_{\text{L}}^{d} d_{\text{L}} = \bar{u}_{\text{L}} \gamma_{\mu} \mathbf{V}_{\text{CKM}} d_{\text{L}},$$

• CKM matrix: unitary 3×3 matrix  $\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$  with  $\mathbf{V}_{CKM} \cdot \mathbf{V}_{CKM}^{\dagger} = \mathbf{V}_{CKM}^{\dagger} \cdot \mathbf{V}_{CKM} = \mathbf{1}$ 

yields unitarity relations, e.g. the unitary triangle of flavor physics (1st vs. 3rd column)  $V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} = 0$ or (used in top FCNC):  $V_{cd}^*V_{td} + V_{cs}^*V_{ts} + V_{cb}^*V_{tb} = 0$ 







FCNC are allowed via higher order mechanisms such as penguin diagrams, but heavily suppressed

- Suppression mechanism 1: GIM
  - Penguin matrix element depends on universal functions of single parameter  $x_i = m_i^2/m_W^2$

 $\mathscr{M} \propto F(x_{\rm d}) V_{\rm cd}^* V_{\rm td} + F(x_{\rm s}) V_{\rm cs}^* V_{\rm ts} + F(x_{\rm b}) V_{\rm cb}^* V_{\rm tb},$ 

• Compare to CKM unitarity relation:

 $V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$ 

Exact cancellation if masses of b, s, and d quarks were the same

- Quark masses more similar for down-type than for up-type: top FCNC more strongly suppressed than bottom FCNC, e.g. BR(t  $\rightarrow$  Zq)  $\approx 10^{-14}$  vs. BR(b $\rightarrow$  sy)  $\approx 10^{-4}$
- Suppression mechanism 2: smallness of relevant CKM matrix elements  $|V_{cd}^*V_{td}| \approx 0.002, |V_{cs}^*V_{ts}| \approx 0.04, |V_{cb}^*V_{tb}| \approx 0.04$









## FCNC are enhanced in many models of physics beyond the SM

- Enhancement mechanisms:
  - FCNC interactions at tree level
  - Weaker GIM cancellation by new particles in loop corrections
- Examples:
  - New quark singlets: Z couplings not flavor-diagonal → tree level FCNC
  - Two Higgs doublet models: modified Higgs mechanism
    - Flavor changing Higgs couplings allowed at tree level
    - Virtual Higgs in loop corrections
  - Supersymmetry: gluino/neutralino and squark in loop corrections









## FCNC are enhanced in many models of physics beyond the SM

- Enhancement mechanisms:
  - FCNC interactions at tree level
  - Weaker GIM cancellation by new particles in loop corrections
- Examples:
  - New quark singlets: Z couplings not flavor-diagonal → tree level FCNC
  - Two Higgs doublet models: modified Higgs mechanism
    - Flavor changing Higgs couplings allowed at tree level
    - Virtual Higgs in loop corrections
  - Supersymmetry: gluino/neutralino and squark in loop corrections









## FCNC are enhanced in many models of physics beyond the SM

- Enhancement mechanisms:
  - FCNC interactions at tree level
  - Weaker GIM cancellation by new particles in loop corrections
- Examples:
  - New quark singlets: Z couplings not flavor-diagonal → tree level FCNC
  - Two Higgs doublet models: modified Higgs mechanism
    - Flavor changing Higgs couplings allowed at tree level
    - Virtual Higgs in loop corrections
  - Supersymmetry: gluino/neutralino and squark in loop corrections









- Experimental tests of FCNC interactions: sensitive probes of new physics
  - Any signal above SM expectations would indicate new physics
  - Measurements constrain allowed phase space for new physics models
- Two types of searches for FCNC in the top sector:
  - Search for single top production (LEP, HERA, DØ)
  - Search for top quark decay via FCNC (CDF)
- Experiments usually report limits on
  - Branching fractions for specific processes, e.g.  $BR(t \rightarrow Zq)$
  - Coupling parameters of effective Lagrangian, e.g. for tZq coupling

$$\mathscr{L}_{\text{eff}} = -\frac{g}{2\cos\theta_W} \cdot \frac{\kappa}{\kappa} \cdot \left( x_L \cdot \bar{q}_L \gamma_\mu t_L + x_R \cdot \bar{q}_R \gamma_\mu t_R \right) Z^\mu + \dots$$





- CDF Run I search:
   F. Abe *et al.*, PRL 80 (1998) 2525.
  - Signature:  $Z \rightarrow I^+I^- + 4$  jets (1 b-jet)
  - Limit on BR(t $\rightarrow$ Zq): 33%

#### • LEP searches:

- P. Achard *et al.* (L3), Phys. Lett. **B549** (2002) 290.
- G. Abbiendi *et al.* (Opal), Phys. Lett. **B521** (2001) 181.
- J. Abdallah *et al.* (Delphi), Phys. Lett. **B590** (2004) 21.
- A. Heister et al. (Aleph), Phys. Lett. B453 (2002) 173.
- Hadronic top decay (4 jets) or semileptonic top decay (2 jets & lepton)
- Very similar results among all LEP experiments, best limit on BR(t→Zq):13.7% (L3)

#### • HERA searches:

- A. Aktas et al. (H1), Eur. Phys. J. C33 (2004) 9.
- S. Chekanov et al. (ZEUS), Phys. Lett. B559 (2003) 153.
- Hadronic top decay (3 jets) or semileptonic top decay (lepton & jet)
- Most sensitive to tγq vertex, preference for u over c quarks (proton sea)





## **Best Limits 2006**





The H1 result caused some excitement:

**Abstract.** [...] In the leptonic channel, 5 events are found while  $1.31 \pm 0.22$  events are expected from the Standard Model background. In the hadronic channel, no excess above the expectation for Standard Model processes is found. [...]



## DØ 2007: Single Top via FCNC



• Study single Top production via FCNC:



- Artificial neural network to discriminate signal from background
- World's best limit on t-c-g and t-u-g couplings  $(\kappa/\Lambda)^2 \rightarrow$  previous limits improved by order of magnitude



[V. M. Abazov et al., hep-ex/0702005, submitted to PRL]







14

### What are Flavor Changing Neutral Currents?

### The CDF Experiment at the Tevatron

### **Fop Quark Physics at CDF**

## Search for FCNC in Top Quark Decays

**Summary & Conclusions** 









[Fermilab Visual Media Service]

- Proton-antiproton collider:  $\sqrt{s} = 1.96$  TeV
- 36×36 bunches, collisions every 396 ns
- Record instantaneous peak luminosity: 292 µb<sup>-1</sup> s<sup>-1</sup> (1 µb<sup>-1</sup> s<sup>-1</sup> = 10<sup>30</sup> cm<sup>-2</sup> s<sup>-1</sup>)
- Luminosity goal:
   5.5–6.5 fb<sup>-1</sup> of integrated luminosity by 2009, running in 2010 currently under discussion
- Two multi-purpose detectors: CDF and DØ





- Tevatron continues to perform very well:
  - More than 3 fb<sup>-1</sup> delivered up to Summer 2007 shutdown
  - More than 2.5 fb<sup>-1</sup> recorded by CDF





































### What are Flavor Changing Neutral Currents?

#### The CDF Experiment at the Tevatron

Top Quark Physics at CDF

## Search for FCNC in Top Quark Decays

**Summary & Conclusions** 



Brief history of top quark discovery:

- 1977: Y discovery bottom quark
- 1980s: Searches for "light" top (mass smaller than W boson mass) as isospin partner of bottom at PETRA, SppS, LEP, CDF Run 0
- 1992/3: Tevatron Run I starts, first indications for top quark production
- March 2, 1995: CDF and DØ announce top quark discovery

## The Discovery of the Top Quark

Finding the sixth quark involved the world's most energetic collisions and a cast of thousands

by Tony M. Liss and Paul L. Tipton

[Scientific American, September 1997]

VIOLENT COLLISION between a proton and an antiproton (*center*) creates a top quark (*red*) and an antitop (*blue*). These decay to other particles, typically producing a number of jets and possibly an electron or positron.





- The top is heavy:  $m_t \approx 170 \text{ GeV}/c^2$ (40× $m_b$ , approx. mass of gold atom)
- Mass close to scale of electroweak symmetry breaking (EWSB), top Yukawa coupling *f* ≈1:

$$\mathscr{L}_{\mathrm{Yuk},t} = f \frac{v}{\sqrt{2}} \, \overline{t}_L t_R \equiv m_t \, \overline{t}_L t_R$$

(vacuum expectation value of Higgs field:  $v/\sqrt{2} \approx 178 \text{ GeV}$ ) → Important role in EWSB models

Top is the only "free" quark: lifetime shorter than hadronization time

$$\tau = \frac{1}{\Gamma} \approx \frac{1}{1.5 \,\text{GeV}} < \frac{1}{\Lambda_{\text{QCD}}} \approx \frac{1}{0.2 \,\text{GeV}}$$

- $\rightarrow$  No spectroscopy of bound states
- $\rightarrow$  Spin transferred to decay products



## **Top Pair Production at the Tevatron**









	$W \rightarrow$	hadrons	τ	μe
hadrons	, (	All Hadronic S/B ≈ 0.04)	Lepton+ τ	Lepton + Jets (S/B ≈ 1)
Ч		Lepton+ τ		
Μ⁺ μ Β	Le	epton + Jets (S/B ≈ 1)		Dilepton $(S/B \approx 3)$

- Top decay in the Standard Model: t → Wb (BR ≈ 100%)
- tt decay signatures characterized by W decays:
  - All-Hadronic (45% of all decays)
  - Lepton+Jets (30% of all decays)
  - Dilepton (5% of all decays)
- Main background process: production of W bosons in association with Jets
- tt events contain two b quarks:
   b quark identification ("b-tagging") crucial











Single	σ <sub>tt</sub> = 8.2 ± 0.5 (stat) ±
B-Tag	0.8 (syst) ± 0.5 (lum) pb
Double	σ <sub>tt</sub> = 8.8 ± 0.8 (stat) ±
B-Tag	1.2 (syst) ± 0.5 (lum) pb



24

## **CDF's Top Properties Program**

- From top discovery in 1995 to precision physics in 2007:
  - Dataset: 1000s of top events
  - Mass & cross section very precisely measured
  - Evidence for single top production
- Broad program to study properties of the top quark: production, decay, quantum numbers, ...
- Measurements of top properties try to answer:

Is the top really the Standard Model top?



25





### What are Flavor Changing Neutral Currents?

#### The CDF Experiment at the Tevatron

Fop Quark Physics at CDF

Search for FCNC in Top Quark Decays

**Summary & Conclusions** 





- Basic question: how often do top quarks decay into Zq?
    $\rightarrow$  set limit on branching fraction BR(t  $\rightarrow$  Zq)
- Selection of decay channels for  $t\bar{t} \rightarrow Zq$  Wb:
  - Z → charged leptons: very clean signature, lepton trigger
  - W → hadrons: large branching fractions, no neutrinos
     → event can by fully reconstructed
  - Final signature: Z + ≥4 jets

#### Analysis Outline:

- I. Baseline Event Selection
- II. Initial Background Estimate
- **III.** Optimization of Event Selection
- IV. Systematic Uncertainties
- V. Final Limit Calculation







## **Blind Analysis**





- Event signature:  $Z \rightarrow I^+I^- + 4$  jets
- Motivation for blind analysis: avoid biases by looking into the data too early
- Blinding & unblinding strategy:
  - Initial blinded region:  $Z + \ge 4$  jets
  - Later: add control region in Z + ≥ 4 jets from kinematic constraints
  - Optimization of event selection, prediction of backgrounds, and systematic uncertainties on data control regions and Monte Carlo (MC) simulation only
  - Very last step: "opening the box", i.e. look into signal region in data





#### • Monte Carlo (MC) simulation of FCNC decay t $\rightarrow$ Zq with PYTHIA

- $t \rightarrow Zq$  vertex unknown to PYTHIA
- Decay generated flat in cos θ\* (angle between top boost direction and lepton of same charge sign from Z decay, in Z rest frame)



• Solution: reweight according to expectation from standard model Higgs mechanism:  $\frac{d\sigma}{d\cos(\theta^*)} = f^0 \cdot \frac{3}{4} \left(1 - \cos(\theta^*)^2\right) + f^- \cdot \frac{3}{8} \left(1 - \cos(\theta^*)\right)^2 + f^+ \cdot \frac{3}{8} \left(1 + \cos(\theta^*)\right)^2$ with  $f^0 = 0.65$  ("longitudinal),  $f^- = 0.35$ ("left-handed"),  $f^+ = 0$  ("right-handed")

• Main FCNC signal sample: one top decays  $t \rightarrow Zc$ , other decays  $t \rightarrow Wb$ 

- Additional sample required for decay  $t \rightarrow Zu$
- Additional sample for "double FCNC" events, i.e. both tops decay via FCNC t  $\rightarrow$  Zq







Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC

30














Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC



## **Search for FCNC: Ingredients**









- Simple trigger: single electron or muon, transverse momentum >18 GeV/c
- Sharp Z resonance, good lepton momentum resolution  $\rightarrow$  cut on lepton pair invariant mass: 76 GeV/ $c^2 < M_{\parallel} < 106$  GeV/ $c^2$
- Enhancing the Z acceptance:
  - Tracking systems have better coverage than calorimeter and muon detectors: allow second lepton to be isolated track
     → doubles acceptance w.r.t. standard lepton selection
  - Electron tracks lose momentum via bremsstrahlung: correct track momentum with calorimeter energy → 3% more dielectron pairs





## **Adding Jets**

- FCNC: four jet assignments
  - 1 b-jet from  $t \rightarrow Wb$  decay
  - 2 jets from subsequent W decay
  - 1 jet from  $t \rightarrow Zq$  decay
- For all 12 possible combinations of first four jets in the event:
  - 1. Combine jets #1 and #2 to W, calculate invariant mass *m*<sub>W,rec</sub>
  - 2. Vary momenta of jets #1 and #2 within their resolutions to match PDG W mass ("fix W mass")
  - 3. Add jet #3 to fixed W, calculate invariant mass  $m_{t \rightarrow Wb, rec}$
  - Vary momenta of leptons within their resolutions to match PDG Z mass ("fix Z mass")
  - 5. Add jet #4 to fixed Z, calculate invariant mass  $m_{t \rightarrow Zq, rec}$



• Widths reflect mass resolutions as measured in MC simulation:  $\sigma_{W,rec} = 15 \text{ GeV}/c^2$ ,  $\sigma_{t \rightarrow Wb,rec} = 24 \text{ GeV}/c^2$  $\sigma_{t \rightarrow Zq,rec} = 21 \text{ GeV}/c^2$ 





- How do you search for a signal that is likely not there? Understand the background!
- Standard model processes that can mimic Z + ≥4 jets signature:
  - Z+Jets: Z boson production in association with jets
     → dominant background for top FCNC search, most difficult to estimate
  - Standard model tt
     t
     production
     → small background
  - Dibosons: WZ and ZZ diboson production  $\rightarrow$  small background
  - W+Jets, WW: negligible
- Top FCNC background estimate: mixture of data driven techniques and MC predictions







- How do you search for a signal that is likely not there? Understand the background!
- Standard model processes that can mimic Z + ≥4 jets signature:
  - Z+Jets: Z boson production in association with jets
     → dominant background for top FCNC search, most difficult to estimate
  - Standard model tt
     t
     f
     production
     → small background
  - Dibosons: WZ and ZZ diboson production  $\rightarrow$  small background
  - W+Jets, WW: negligible
- Top FCNC background estimate: mixture of data driven techniques and MC predictions

## Standard Model tt Production

- Small background: no real Z, need extra jets from gluon radiation and/or "fake lepton"
  - Dilepton channel (tt̄ → Wb Wb → Ivb Ivb): dilepton invariant mass can fall into Z mass window
  - Lepton+Jets channel (tt̄ → Wb Wb → lvb qq'b): misreconstruct one jet as a lepton ("fake"), invariant mass of lepton and fake lepton can fall into Z mass window
- Large fraction of heavy flavor jets: more important in b-tagged samples
- Estimated from MC simulation





- How do you search for a signal that is likely not there? Understand the background!
- Standard model processes that can mimic Z + ≥4 jets signature:
  - Z+Jets: Z boson production in association with jets
     → dominant background for top FCNC search, most difficult to estimate
  - Standard model tt
     t
     production
     → small background
  - Dibosons: WZ and ZZ diboson production → small background
  - W+Jets, WW: negligible
- Top FCNC background estimate: mixture of data driven techniques and MC predictions

## **Diboson Production: WZ, ZZ**

- Small background (similar in size to standard model tt production)
  - Small cross section but real Z
  - Need extra jets from gluon radiation
- ZZ: Heavy flavor contribution from Z→bb̄ decay
- Estimated from MC simulation



## **Z+Jets Production**



#### MC tool for Z+Jets: ALPGEN

- Modern MC generator for multiparticle final states
- "MLM matching" prescription to remove overlap between jets from matrix element and partons showers
- Comparing ALPGEN with data:
  - Leading order generator: no absolute prediction for cross section
  - Underestimate of number of events with large jet multiplicities, large uncertainties
- Our strategy: only shapes of kinematic distributions from MC, normalization from control samples in data







35

• Mass  $\chi^2$ : combination of mass constraints – best discriminator

$$\chi^{2} = \left(\frac{m_{W,\text{rec}} - m_{W,\text{PDG}}}{\sigma_{W,\text{rec}}}\right)^{2} + \left(\frac{m_{t \to Wb,\text{rec}} - m_{t,\text{PDG}}}{\sigma_{t \to Wb}}\right)^{2} + \left(\frac{m_{t \to Zq,\text{rec}} - m_{t,\text{PDG}}}{\sigma_{t \to Zq}}\right)^{2}$$

Transverse mass: FCNC top decays are more central than Z+jets

$$M_T = \sqrt{\left(\sum E_T\right)^2 - \left(\sum \vec{p}_T\right)^2}$$

 Jet transverse energies: FCNC signal has four "hard" jets, background processes: jets have to come from gluon radiation





## To B-Tag or not to B-Tag?



- Advantage of requiring b-tag: Better discrimination against main Z+jets background (heavy flavor backgrounds rather small: SM tt̄, Zbb̄ + jets)
- Disadvantage: Reduction of data sample size
- Solution: use both!
  - Split sample in tagged and anti-tagged
  - Optimize cuts individually for tagged and anti-tagged samples
  - Combine samples in limit calculation
- Main difficulty of this approach: event migration between samples
  - Systematics may be correlated or anticorrelated between samples
  - Taken into account in limit calculation







- Question: best choice for cut values?
- Goal: derive limit on branching fraction of FCNC process  $t \rightarrow Zq$
- No prediction for amount of signal: "signal over background" et al. do not work
- Solution: optimize cuts for best expected limit (assuming no signal)

 $\sum_{n_{\rm obs}} P(n_{\rm obs}|n_{\rm back}) \cdot \operatorname{Lim}(n_{\rm obs}|A, n_{\rm back})$ 

- P: Poisson probability
- L: any limit calculation method
- Our analysis: faster objective Bayesian limits for optimization, "better" Feldman-Cousins limits for final result (both including systematic uncertainties)
- Correlations among variables: multi-dimensional optimization

#### **Final Event Selection**

Kinematic Variable	Optimized Cut
Z Mass	$\in$ [76,106] GeV/ $c^2$
Leading Jet $E_T$	$>40\mathrm{GeV}$
Second Jet $E_T$	$> 30 \mathrm{GeV}$
Third Jet $E_T$	> 20  GeV
Fourth Jet $E_T$	> 15  GeV
Transverse Mass	$> 200 \mathrm{GeV}$
$\sqrt{\chi^2}$	< 1.6 ( <i>b</i> -tagged)
	< 1.35 (anti-tagged)





# **Background: Putting it all Together**



- Total background prediction from control region in data: 130 ± 28 events
  - Tail of mass  $\chi^2$  distribution
  - Average of cuts at  $\sqrt{\chi^2}$  = 3.0, 3.2
- Tagging rate: 15% ± 4%
  - Tail of mass  $\chi^2$ : 16% ± 7% (small sample  $\rightarrow$  large uncertainties)
  - MC prediction of tagging rate: 11% (but: 30% too low for Z+≤ 3 Jets)
  - Template fit of MC tagging probabilities vs. number of jets: 14%



Source	Without <i>b</i> -tag	Loose SECVTX b-tag
Z+Jets	123.3±28	17.6±6
Standard Model $t\bar{t}$	$2.4{\pm}0.3$	$1.7{\pm}0.2$
Diboson ( $WZ$ , $ZZ$ )	$4.3 \pm 0.2$	$0.7{\pm}0.1$
WW, W+Jets	< 0.1	negligible
Total Backgrounds:	130±28	20±6





- Question: how to get from event counts to limit on BR(t→Zq)?
  - Circular dependency #1: Limit calculation requires knowledge of signal acceptance, but signal acceptance depends on limit
  - Circular dependency #2: Measure limit on fraction of tt production cross section, but cross section changes with changing FCNC contribution
- Solution: "running acceptance" functional form of above dependencies implemented in limit machinery
  - Signal acceptance dynamically adjusted as a function of BR(t→Zq)
  - Signal normalized to measured tt
     production cross section measurement
  - $t\bar{t}$  cross section re-interpreted as a function of BR(t $\rightarrow$ Zq) to allow for FCNC contribution







- Signal count: probability for one or both tops to decay via FCNC  $\mathscr{P}(t\bar{t} \rightarrow ZcWb, ZcZc,...)$
- Normalization to double-tagged tt cross section measurement:
  - Double-tagged: smallest overlap between acceptances
  - Luminosity uncertainties cancel, other uncertainties reduced

$$\mathscr{B}_Z \equiv \mathscr{B}(t \to Zc) = 1 - \mathscr{B}(t \to Wb)$$

- $\mathscr{A}_{WZ} \equiv \text{FCNC Acceptance}$
- $\mathscr{A}_{ZZ} \equiv$  Double FCNC Acceptance
- $\mathscr{A}_{LJ_{WW}} \equiv L+J$  Acceptance for SM  $t\bar{t}$
- $\mathscr{A}_{LJ_{WZ}} \equiv L+J$  Acceptance for FCNC

$$\mathscr{A}_{LJ_{ZZ}} \equiv L+J$$
 Acceptance for Double FCNC

$$K_{ZZ/WZ} \equiv \mathscr{A}_{ZZ}/\mathscr{A}_{WZ}$$
$$\mathscr{R}_{WZ/WW} \equiv \mathscr{A}_{LJ_{WZ}}/\mathscr{A}_{LJ_{WW}}$$

$$\mathscr{R}_{ZZ/WW} \equiv \mathscr{A}_{LJ_{ZZ}}/\mathscr{A}_{LJ_{WW}}$$

#### **Acceptance Master Formula:**

$$N_{\text{signal}} = [(\mathscr{P}(t\bar{t} \to WbZc) \cdot \mathscr{A}_{WZ}) + (\mathscr{P}(t\bar{t} \to ZcZc) \cdot \mathscr{A}_{ZZ})] \cdot \sigma_{t\bar{t}} \cdot \int \mathscr{L} dt$$

$$\dots 1/2 \text{ page of algebra.} \dots$$

$$= \mathscr{B}_{Z} \cdot (N_{LJ} - B_{LJ}) \cdot \frac{\mathscr{A}_{WZ}}{\mathscr{A}_{LJww}} \cdot \frac{(2 \cdot (1 - \mathscr{B}_{Z}) + K_{ZZ/WZ} \cdot \mathscr{B}_{Z})}{(1 - \mathscr{B}_{Z})^{2} + 2\mathscr{B}_{Z} \cdot (1 - \mathscr{B}_{Z}) \cdot \mathscr{R}_{wz/ww} + \mathscr{B}_{Z}^{2} \cdot \mathscr{R}_{ZZ/ww}}$$

$$\stackrel{\text{``Running'' Acceptance Correction}$$





- Signal count: probability for one or both tops to decay via FCNC  $\mathscr{P}(t\bar{t} \rightarrow ZcWb, ZcZc,...)$
- Normalization to double-tagged tt cross section measurement:
  - Double-tagged: smallest overlap between acceptances
  - Luminosity uncertainties cancel, other uncertainties reduced

$$\mathscr{B}_Z \equiv \mathscr{B}(t \to Zc) = 1 - \mathscr{B}(t \to Wb)$$

- $\mathscr{A}_{WZ} \equiv \text{FCNC Acceptance}$
- $\mathscr{A}_{ZZ} \equiv$  Double FCNC Acceptance
- $\mathscr{A}_{LJ_{WW}} \equiv L+J$  Acceptance for SM  $t\bar{t}$
- $\mathscr{A}_{LJ_{WZ}} \equiv L+J$  Acceptance for FCNC

$$\mathscr{A}_{LJ_{ZZ}} \equiv L+J$$
 Acceptance for Double FCNC

40

$$K_{ZZ/WZ} \equiv \mathscr{A}_{ZZ}/\mathscr{A}_{WZ}$$
$$\mathscr{R}_{WZ/WW} \equiv \mathscr{A}_{LJ_{WZ}}/\mathscr{A}_{LJ_{WW}}$$

$$\mathscr{R}_{ZZ/WW} \equiv \mathscr{A}_{LJ_{ZZ}}/\mathscr{A}_{LJ_{WW}}$$

#### **Acceptance Master Formula:**

$$N_{\text{signal}} = [(\mathscr{P}(t\bar{t} \to WbZc) \cdot \mathscr{A}_{WZ}) + (\mathscr{P}(t\bar{t} \to ZcZc) \cdot \mathscr{A}_{ZZ})] \cdot \sigma_{t\bar{t}} \cdot \int \mathscr{L} dt$$

$$\dots 1/2 \text{ page of algebra.} \dots$$

$$= \mathscr{B}_{Z} \cdot (N_{LJ} - B_{LJ}) \cdot \mathscr{A}_{WZ} \cdot (2 \cdot (1 - \mathscr{B}_{Z}) + K_{ZZ/WZ} \cdot \mathscr{B}_{Z})$$

$$(1 - \mathscr{B}_{Z})^{2} + 2\mathscr{B}_{Z} \cdot (1 - \mathscr{B}_{Z}) \cdot \mathscr{R}_{WZ/WW} + \mathscr{B}_{Z}^{2} \cdot \mathscr{R}_{ZZ/WW}$$

$$\overset{\text{(Running" Acceptance Correction}}{(1 - \mathscr{B}_{Z})^{2} + 2\mathscr{B}_{Z} \cdot (1 - \mathscr{B}_{Z})} \cdot \mathscr{R}_{WZ/WW} + \mathscr{B}_{Z}^{2} \cdot \mathscr{R}_{ZZ/WW}$$





- Signal systematic evaluated for acceptance ratio A<sub>WZ</sub>/A<sub>LJ</sub>
- Distinguish uncertainties: correlated or anti-correlated between selections
  - Correlated: shift anti-tagged & tagged selection into same direction (e.g. lepton SF)
  - Anti-correlated: shift anti-tagged & tagged into opposite directions (e.g. b-tagging)

Systematic Uncertainty	<b>Base Selection</b> (%)	Anti-Tagged (%)	Loose Tag (%)
Lepton Scale Factor	0.5	0.5	0.5
Trigger Efficiency	0.2	0.2	0.2
Jet Energy Scale	3.1	2.6	1.9
ISR/FSR	1.3	2.6	6.5
Helicity Re-Weighting	3.5	3.4	3.2
Parton Distribution Functions	0.9	0.9	0.9
Total Correlated	5.0	5.1	7.5
<b>B-Tagging Scale Factor</b>	10.2	16.3	5.5
Mistag $\alpha\beta$ Correction	0.6	1.0	0.4
$\mathscr{B}(t \to Zc)$ versus $\mathscr{B}(t \to Zu)$	0.0	4.0	4.0
Total Anti-Correlated	10.2	16.8	6.8





- Signal systematic evaluated for acceptance ratio A<sub>WZ</sub>/A<sub>LJ</sub>
- Distinguish uncertainties: correlated or anti-correlated between selections
  - Correlated: shift anti-tagged & tagged selection into same direction (e.g. lepton SF)
  - Anti-correlated: shift anti-tagged & tagged into opposite directions (e.g. b-tagging)

Systematic Uncertainty	<b>Base Selection</b> (%)	Anti-Tagged (%)	Loose Tag (%)
Lepton Scale Factor	0.5	0.5	0.5
Trigger Efficiency	0.2	0.2	0.2
Jet Energy Scale	3.1	2.6	1.9
ISR/FSR	1.3	2.6	6.5
Helicity Re-Weighting	3.5	3.4	3.2
Parton Distribution Functions	0.9	0.9	0.9
Total Correlated	5.0	5.1	7.5
B-Tagging Scale Factor	10.2	16.3	5.5
Mistag $\alpha\beta$ Correction	0.6	1.0	0.4
$\mathscr{B}(t \to Zc)$ versus $\mathscr{B}(t \to Zu)$	0.0	4.0	4.0
Total Anti-Correlated	10.2	16.8	6.8



## **Background Systematics**



- Background systematics dominated by yield uncertainties
  - Total background yield: 130 ± 28 (21.5% relative uncertainty)
  - Tagging rate: 15% ± 4% (relative uncertainty: 26.7% tagged, 4.7% anti-tagged)
- Remaining uncertainties: efficiency of χ<sup>2</sup> cut
  - Ratio of events with  $\sqrt{\chi^2} < 1.6$  (signal region) vs.  $\sqrt{\chi^2} > 3.0$  (control region)
  - Dominated by choice of MC generator and jet energy scale





## **Background Systematics**



- Background systematics dominated by yield uncertainties
  - Total background yield: 130 ± 28 (21.5% relative uncertainty)
  - Tagging rate: 15% ± 4% (relative uncertainty: 26.7% tagged, 4.7% anti-tagged)
- Remaining uncertainties: efficiency of χ<sup>2</sup> cut
  - Ratio of events with  $\sqrt{\chi^2} < 1.6$  (signal region) vs.  $\sqrt{\chi^2} > 3.0$  (control region)
  - Dominated by choice of MC generator and jet energy scale







#### Opening the box with 1.12 fb<sup>-1</sup>

- Event yield consistent with background only
- Fluctuated about 1σ high: slightly unlucky
- Result: The World's Best Limit!

 $\mathscr{B}(t \to Zq) < 10.6\%$  @ 95% C.L.

- Expected limit: 7.1% ± 3.0%
- 25% better than L3 (13.7%)
- 3x better than CDF Run I (33%)
- Above results assumes  $m_t = 175$ GeV/ $c^2$ , limit at  $m_t = 170$  GeV/ $c^2$ : BR(t  $\rightarrow$  Zq) < 11.2% @ 95% C.L.
- Update with 2 fb<sup>-1</sup> and improved method in the works

Selection	Observed	Expected
Base Selection	141	130±28
Base Selection (Tagged)	17	$20\pm6$
Anti-Tagged Selection	12	$7.7 {\pm} 1.8$
Tagged Selection	4	$3.2{\pm}1.1$

#### Mass $\chi^2$ (95% C.L. Upper Limit)





# **Top FCNC Searches at the LHC**



#### Large Hadron Collider (LHC):

- Proton-proton collider at 14 TeV center-of-mass energy (CERN)
- Two multi-purpuse experiments: ATLAS and CMS
- First data expected in 2008 (2009?)
- Recent ATLAS study on sensitivity for top FCNC
  - Improvement of current limits on BR (t→Zq) by 2–3 orders of magnitude
  - Entering interesting regime of 10<sup>-4</sup> to 10<sup>-5</sup>: exclusion of first theoretical models
- Caveat: background model
  - Existing MC tools not tuned to new energy regime
  - Tevatron experience: obtain backgrounds from data





## **Summary and Conclusions**





- Top flavor changing neutral current decays
  - Extremely rare in the standard model
  - Enhanced in theories beyond the standard model → any signal would indicate new physics
- First Tevatron Run II search for FCNC t → Zq in top quark decays
  - Event signature:  $Z + \ge 4$  jets
  - Main background process: standard model
     Z + jets production
  - Mass  $\chi^2$  to separate signal from background
- No evidence for top FCNC found
  - World's best limit: BR(t→Zq) < 10.6% at 95% C.L.</li>
  - Working on improvements, stay tuned!





# **Backup Slides**

Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC









## **Typical Top Selection Criteria**





#### Cylindrical coordinate system:

- θ: polar angle w.r.t. to proton direction
- φ: azimuthal angle
- Pseudorapidity:  $\eta = -\ln \tan(\theta/2)$
- Transverse energy:

$$\vec{E_T} = \sum_{\text{cal towers}} E_i(\sin \theta_i, \phi_i)$$

 $\vec{E_T} = -\sum_{\text{jets}} \vec{E_T} - \sum_{\text{leptons}} \vec{p_T}$ 

Missing transverse energy ("MET"):

• Lepton + Jets: 
$$t\bar{t} \rightarrow Wb Wb \rightarrow Ivb qq'b$$

- Isolated lepton with  $p_T > 20 \text{ GeV}/c$
- Neutrino: missing  $E_T$  ("MET") > 20 GeV
- 3 jets within  $|\eta| < 2$  with  $E_T > 15$  GeV, 4th jet:  $E_T > 8$  GeV
- 0, 1, ≥ 2 identified jets from b quarks ("btags")



## **Typical Top Selection Criteria**





#### Cylindrical coordinate system:

- θ: polar angle w.r.t. to proton direction
- φ: azimuthal angle
- Pseudorapidity:  $\eta = -\ln \tan(\theta/2)$
- Transverse energy:

E

$$\vec{E_T} = \sum_{\text{cal towers}} E_i(\sin \theta_i, \phi_i)$$

Missing transverse energy ("MET"):

$$\vec{F}_T = -\sum_{\text{jets}} \vec{E_T} - \sum_{\text{leptons}} \vec{p_T}$$

• Lepton + Jets:  $t\bar{t} \rightarrow Wb Wb \rightarrow Ivb qq'b$ 

- Isolated lepton with  $p_T > 20 \text{ GeV}/c$
- Neutrino: missing  $E_T$  ("MET") > 20 GeV
- 3 jets within  $|\eta| < 2$  with  $E_T > 15$  GeV, 4th jet:  $E_T > 8$  GeV
- 0, 1, ≥ 2 identified jets from b quarks ("btags")
- Dilepton:  $t\bar{t} \rightarrow Wb Wb \rightarrow Ivb Ivb$ 
  - Two oppositely charged leptons with  $p_T > 20 \text{ GeV}/c$
  - Two neutrinos: MET > 25 GeV
  - $\geq$  2 jets within  $|\eta| < 2.5$  with  $E_T > 15$  GeV
  - Scalar sum of lepton  $p_T$ s, jet  $E_T$ s and MET:  $H_T$  > 200 GeV
  - 0, 1, ≥ 2 b-tags



# **Secondary Vertex B-Tagging**



- CDF's standard "SecVtx" algorithm:
  - Long lifetime of B mesons: detect displaced secondary vertex
  - Discriminants: Significance of displacement in xy plane (Lxy) and impact parameter
- Further taggers based on jet probability or soft leptons from semileptonic B decays





48





- Problem: infer parton energy (hard scattering process) from measured jet energy
- Jet reconstruction by clustering algorithm with fixed cone size
- Jet energy corrected for:
  - Non-uniform detector response
  - Different response to different particles
  - Multiple pp interactions
  - Un-instrumented areas
  - Underlying event (spectators)
  - "Out-of-cone" energy
- Correction leads to large systematic uncertainties, partly compensated by in-situ calibration in data





## **Top Quark Mass**



- Top mass measurements enter the era of precision physics:
  - Three independent top decay channels
  - At least a dozen different analysis techniques
  - Measurements are very consistent
  - March 2007 Tevatron combination: 1.1% uncertainty on top mass
- 8 new or updated top mass measurements from CDF & DØ presented at Lepton-Photon 2007











## • FCNC signal MC generated with Pythia (Gen6):

Sample	Sample Size	Description	
$t\bar{t} \to Z(ll)cW(q\bar{q}')b$	539,445	$Z \rightarrow e^+ e^-, \mu^+ \mu^-$ and $W \rightarrow q \overline{q}'$	Main Sample
$t\bar{t} \to Z(ll)cW(l\nu)b$	111,181	$Z \rightarrow e^+e^-, \mu^+\mu^-$ and $W \rightarrow e\nu, \mu\nu, \tau\nu$	Additional
$t\bar{t} \rightarrow Z(incl.)cW(incl.)b$	116,573	Inclusive Z and W decays	
$t\overline{t} \to Z(ll,q\overline{q})cZ(ll,q\overline{q})c$	116,573	Double FCNC decay: $Z \rightarrow e^+e^-, \mu^+\mu^-, q\overline{q}$	
$t\bar{t} \to Z(ll)uW(q\bar{q}')b$	116,573	$Z \rightarrow e^+e^-, \mu^+\mu^- \text{ and } W \rightarrow q\overline{q}'$	t→Zu vs. t→Zc
$t\bar{t} \to Z(ll)cW(q\bar{q}')b$	116,573	As Above, $m_t = 170 \mathrm{GeV}/c^2$	) – M
$t\bar{t} \rightarrow Z(ll)cW(l\nu)b$	106,465	As Above, $m_t = 170 \mathrm{GeV}/c^2$	$170 \text{ GoV}/c^2$
$t\overline{t} \to Z(ll,q\overline{q})cZ(ll,q\overline{q})c$	116,573	As Above, $m_t = 170 \mathrm{GeV}/c^2$	

Full 1.12 fb<sup>-1</sup> run range, underlying event

- Reweight samples to get SM expected helicity of Zs from top decay: 65% longitudinal, 35% left-handed
- Signal acceptance:
  - Defined after helicity reweighting
  - Corrected for trigger efficiencies and lepton ID and reconstruction scale factors on object-by-object basis







## • Problem: $t \rightarrow Zq$ vertex unknown to PYTHIA

- Decays generated flat in cos θ\* (angle between top and lepton of same charge sign from Z decay, in Z rest frame)
- Expected helicity for pure V–A decay: 65% longitudinal (f<sup>0</sup>), 35% left-handed (f<sup>-</sup>). According to Tim Tait: "Wacky models" may mix left-handed and right handed fractions, but not longitudinal and handed:

$$\frac{d\sigma}{d\cos(\theta^*)} = f^0 \cdot \frac{3}{4} \left(1 - \cos(\theta^*)\right) + f^- \cdot \frac{3}{8} \left(1 - \cos(\theta^*)\right)^2 + f^+ \cdot \frac{3}{8} \left(1 + \cos(\theta^*)\right)^2$$

with SM prediction for f<sup>0</sup>: 
$$f^0 = \frac{{m_t}^2}{2m_Z^2 + {m_t}^2} \approx 0.65$$

Solution:

- Re-weight sample for acceptance calculation: 65% longitudinal, 35% left-handed
- Assign systematic uncertainty to unknown helicity
- To first order: acceptance for I<sup>+</sup> and I<sup>-</sup> identical → same acceptance for same fraction of left-handed/right-handed









Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC





- Blind analysis: cannot change cuts after "opening the box"
- Closer look at the data: excess of events with transverse mass around 200 GeV
- Compare cuts at 200 GeV and 220 GeV: most likely explanation of higher than expected limit



Selection	<b>Observed (Expected) Events</b>		
	$m_T > 200 {\rm ~GeV}$	$m_T > 220 \text{ GeV}$	
Anti-Tagged	12 (7.7)	7 (6.4)	
Loose Tag	4 (3.2)	3 (2.8)	
Total	16 (10.8)	10 (9.2)	
Cut Efficiency (%)	11.3 (8.3)	7.1 (7.1)	

Transverse Mass (Anti-Tagged)





- Original mass  $\chi^2$  only defined with four or more jets (mostly blind)
- Validate two out of three pieces in 3-jet bin: good agreement






- Goal: assign probability to each MC event that at least one jet is b-tagged
- MC: can match reconstructed jet to true B hadron
- Difficulty: MC simulation does not reproduce data perfectly
  - Introduce "scale factor" for b-tagging efficiency (= ratio of data to MC efficiency)
  - Derive "mistag probability" from data (= probability to assign b-tag to light flavor jet)
- Per-event tag rate: combine probabilities for all jets





-

## **Systematics: Details**



Helicity	<b>Base Selection</b>	(%) Anti-Tagged	(%) Loose Tag (%)
35% LH, 65% Long.		default	
Flat	-4.3	-4.2	-4.5
100% Longitudinal	5.0	4.7	4.5
100% Left-Handed	-9.2	-8.8	-8.3
100% Right-Handed	-8.6	-8.6	-9.5
35% RH, 65% Long.	0.2	0.1	-0.4
Total Uncertainty (%)	3.5	3.4	3.2
Sample Bas	e Selection (%)	Anti-Tagged (%)	Loose Tag (%)
More ISR	0.0	2.4	-1.6
Less FSR	0.4	-0.1	3.0
More FSR	-0.1	-0.9	2.9
Less FSR	1.3	-0.4	4.7
Total	1.3	2.6	6.5

Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC





Expected limit:

 $\sum_{n_{\rm obs}} P(n_{\rm obs}|n_{\rm back}) \cdot \operatorname{Lim}(n_{\rm obs}|A, n_{\rm back})$ 

- P: Poisson probability to observe nobs events with nback background events
- Lim: limit with *n*<sub>obs</sub> events given acceptance *A* and *n*<sub>back</sub> background events (any limit calculation machinery)
- This analysis:
  - (Faster) objective Bayesian limits for optimization
  - ("Better") Feldman-Cousins limits for final result
  - Both methods: systematic uncertainties included
  - Results track each other well



Expected 95% C.L. Upper Limit on BR(t→Zq): **7.1% ± 3.0%** 



## Why Feldman-Cousins?



- Reporting results of particle physics experiments: confidence intervals, e.g. central value and uncertainty, upper/lower limit
- Two rivaling schools on reporting confidence intervals
  - Frequentist approach: If the experiment would be repeated infinitely many times, the true value would lie within the interval in a fraction α of the experiments
  - Bayesian approach: degree of belief that the true value lies within the interval is α
- Both approaches have their advantages and disadvantages
  - New (frequentist) approach by Gary J. Feldman (Harvard) and Robert D. Cousins (UCLA)
  - Published in Phys. Rev. **D57** (1998) 3873 (quite readable)









Bob Cousins







- Classical probability theory:
  - Probability that an element belongs to two sets A and B



 $P(A \cap B) = P(A) \cdot P(B|A)$  $P(A \cap B) = P(B) \cdot P(A|B)$ 

- In words: the probability of an element to belong to the union of two sets *A* and *B* is the probability of the element to belong to set *A* times the probability to belong to *B* given it belongs to *A* (and vice versa: probability to belong to *B* times probability to belong to *A* given it belongs to *B*)
- Result: Bayes' theorem  $P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$



Goal: measure parameter  $\mu$ , i.e. construct Bayesian confidence interval for  $\mu$  from a set of measurements **x** = ( $x_1$ ,  $x_2$ , ...,  $x_N$ )

- 1. Know probability to observe experimental value  $x_i$  for a given value of  $\mu$ :  $P(x_i | \mu)$ , e.g. Poisson distribution
- 2. Construct joint probability for **x** ("likelihood function"):

$$L(\mathbf{x}|\boldsymbol{\mu}) = \prod_{i=1}^{N} P(x_i|\boldsymbol{\mu})$$

- 3. Apply Bayes' theorem to obtain posterior probability  $P(\mu | \mathbf{x}) = \frac{L(\mathbf{x} | \mu) P(\mu)}{\int d\mu' P(\mathbf{x} | \mu') P(\mu')}$
- 4. Find confidence interval  $[\mu_1;\mu_2]$  such that

$$\int_{\mu_1}^{\mu_2} \mathrm{d}\mu' P(\mu'|\mathbf{x}) = \alpha$$



 $P(\mu|\mathbf{x})$ 



## • $\alpha$ is degree of belief that $\mu$ is in [ $\mu_1$ ; $\mu_2$ ]

Problem: Bayes' theorem requires prior probability density P(µ), i.e. prior knowledge about the the parameter to be measured (intrinsically subjective)

$$P(\mu|\mathbf{x}) = \frac{L(\mathbf{x}|\mu)P(\mu)}{\int d\mu' P(\mathbf{x}|\mu')P(\mu')}$$

 Solution for uniquely defining μ<sub>1</sub>: draw horizontal line at fraction α of area under posterior probability







64

 Likelihood function is only source of information: estimator for µ from maximum likelihood, i.e.

$$\frac{\partial L}{\partial \mu} = 0$$
, with  $L(\mathbf{x}|\mu) = \prod_{i=1}^{N} P(x_i|\mu)$ 

 Confidence interval [µ<sub>1</sub>;µ<sub>2</sub>] from Neyman construction ("confidence belt")



Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC





- Infinitely many repetitions of experiment: interval [μ<sub>1</sub>;μ<sub>2</sub>] includes true value of μ in a fraction α of the experiments
- Problem 1: freedom of choice for x<sub>1</sub>
  - Flip-flopping (as for Bayesian limit)
- Problem 2: "Under-coverage"
  - If P(x|µ) leaks into unphysical values
    (e.g. x<sub>1</sub> < 0), interval [0;x<sub>2</sub>] does not cover a fraction α
  - Over-coverage is unavoidable for discrete x
  - Generally: over-coverage tolerable, but just too "conservative"







- Use freedom in Neyman construction,
  i.e. choice of x<sub>1</sub>, to achieve
  - Smooth transition between upper/lower and central intervals ("unified" limits)
  - Correct treatment of unphysical regions
- Introduce (i.e. re-discover for highenergy physics) ordering principle based on likelihood ratio

$$R(x) = \frac{P(x|\mu)}{P(x|\mu^{\text{best}})}$$

( $\mu^{\text{best}}$ : physically allowed value of  $\mu$  for which  $P(x|\mu)$  is maximum)

Construct frequentist confidence belt







## Feldman–Cousins approach:

- Solves problems present in construction of Bayesian and frequentist confidence intervals
- Widely accepted in scientific community
- Applications: check out original paper (quite readable)
- Further developments: incorporation of systematic uncertainties (impossible in frequentist approach)
- Many examples for application in CDF: measurement of |V<sub>tb</sub>|, fraction of tt production from gluon fusion, FCNC search, ...



Exp. Particle Physics Seminar, Penn, 10/17/07– U. Husemann: Search for Top FCNC