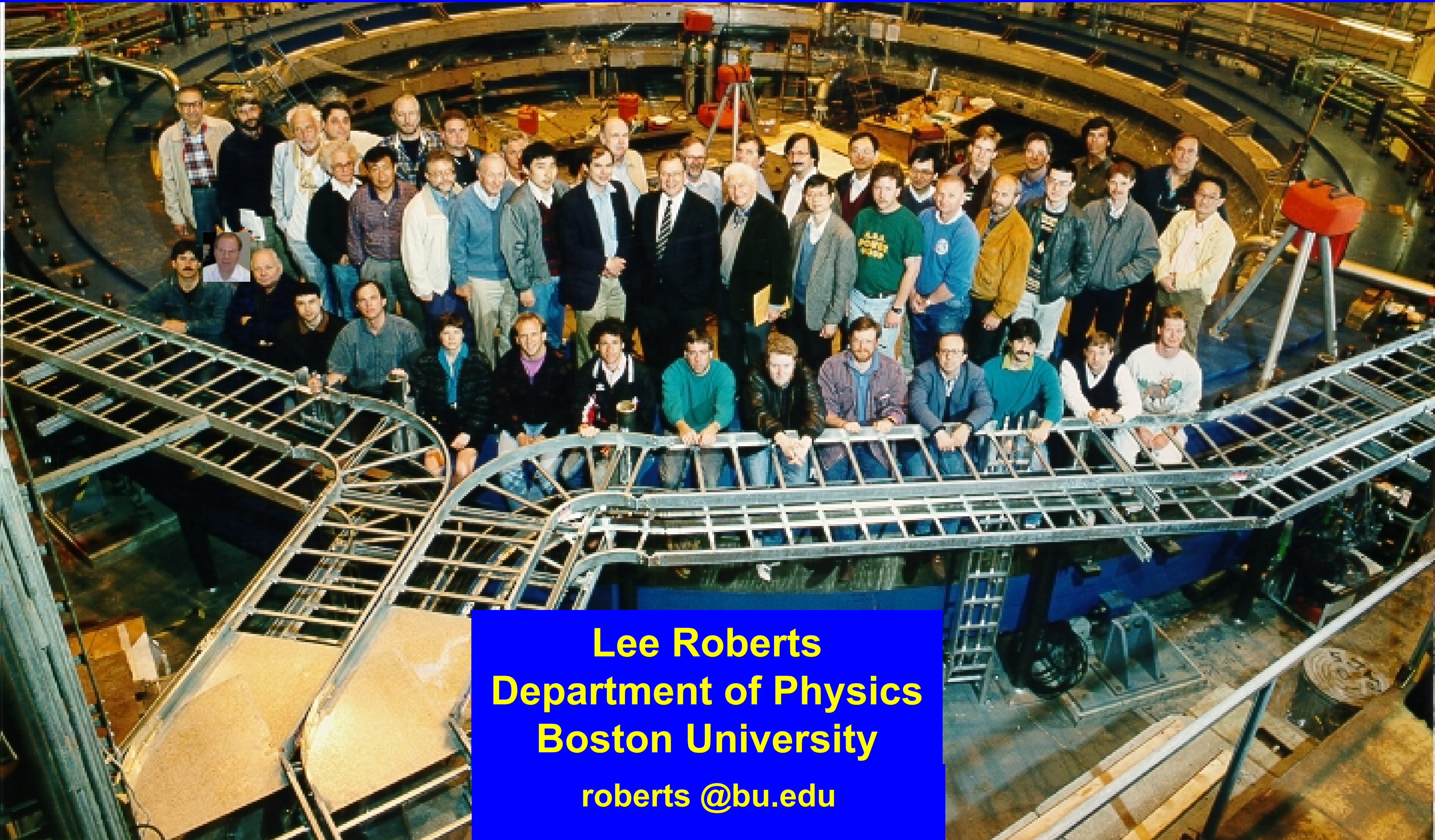
Muon (g-2): Results and Future Possibilities



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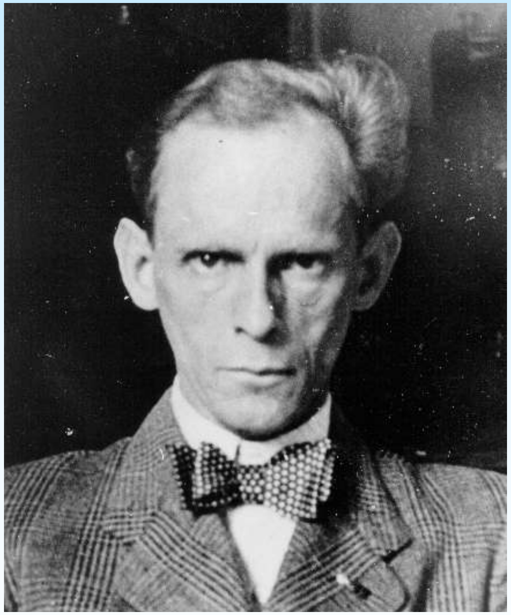
Outline

- Introduction to the muon, τ_μ and G_F
- Magnetic (a_μ) and electric (d_μ) dipole moments
  - E821 result and the SM
  - E821 EDM limit
- Limits on CPT/Lorentz Violation in muon spin precession
- Future improvements in a_μ?
- Summary and conclusions.

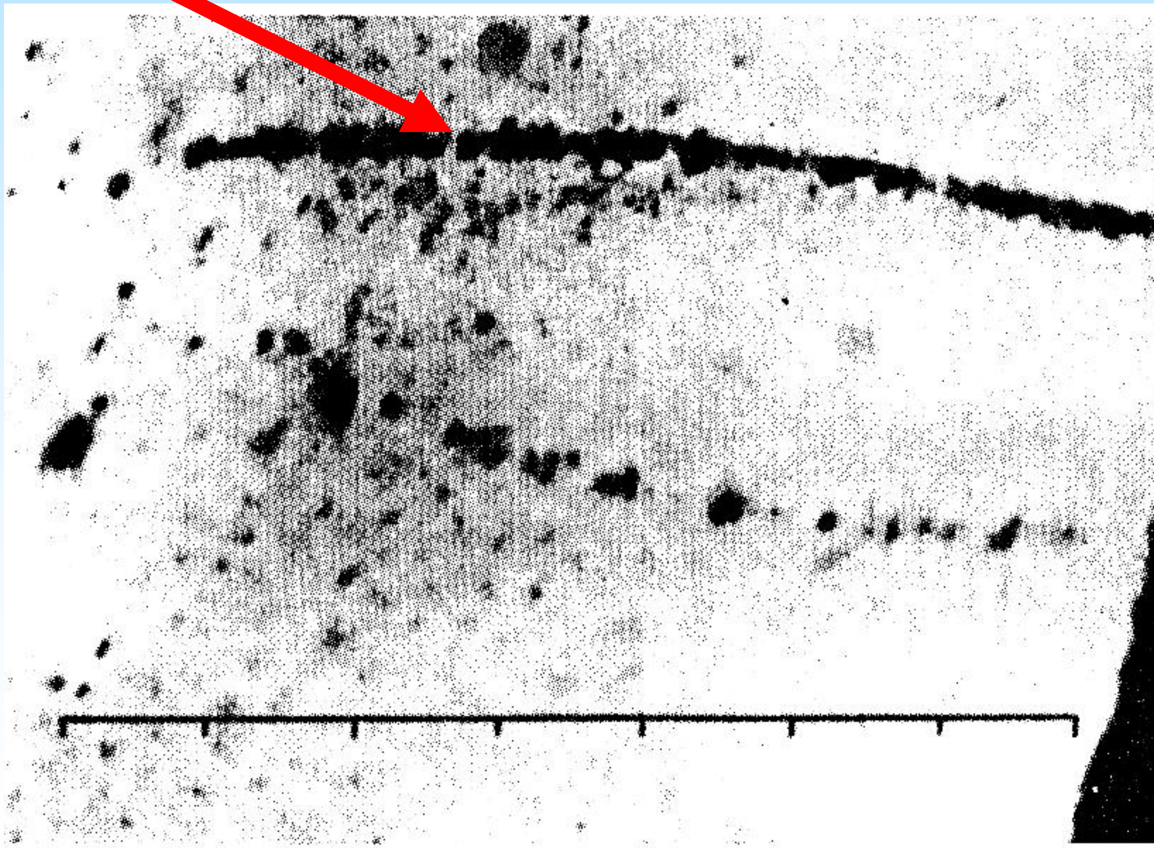
First published observation of the muon came from cosmic rays:

Paul Kunze,

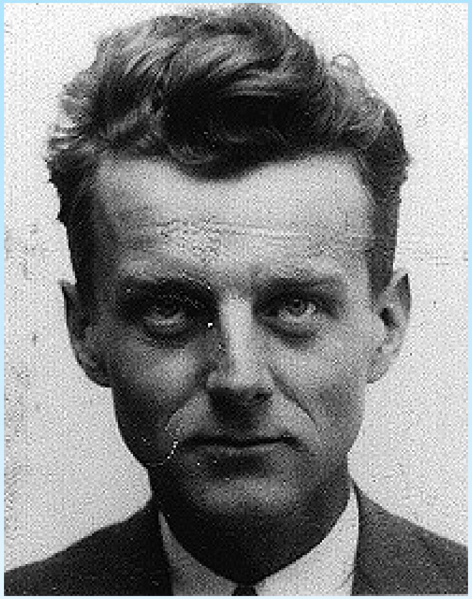
Z. Phys. 83, 1 (1933)



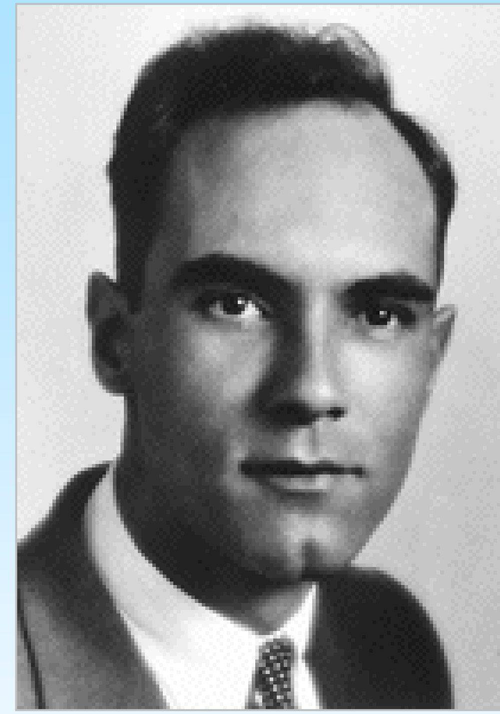
“a particle of uncertain nature”



Identified in 1936



Study of cosmic rays by Seth Neddermeyer and Carl Anderson



MAY 15, 1937 PHYSICAL REVIEW VOLUME 51

Note on the Nature of Cosmic-Ray Particles

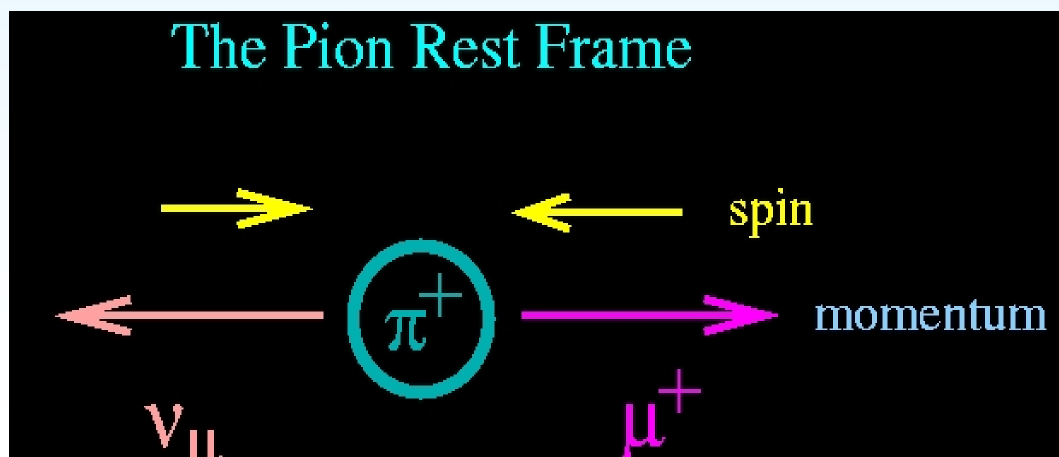
SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California

(Received March 30, 1937)

MEASUREMENTS[1] of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional massive than protons but more penetrating than electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

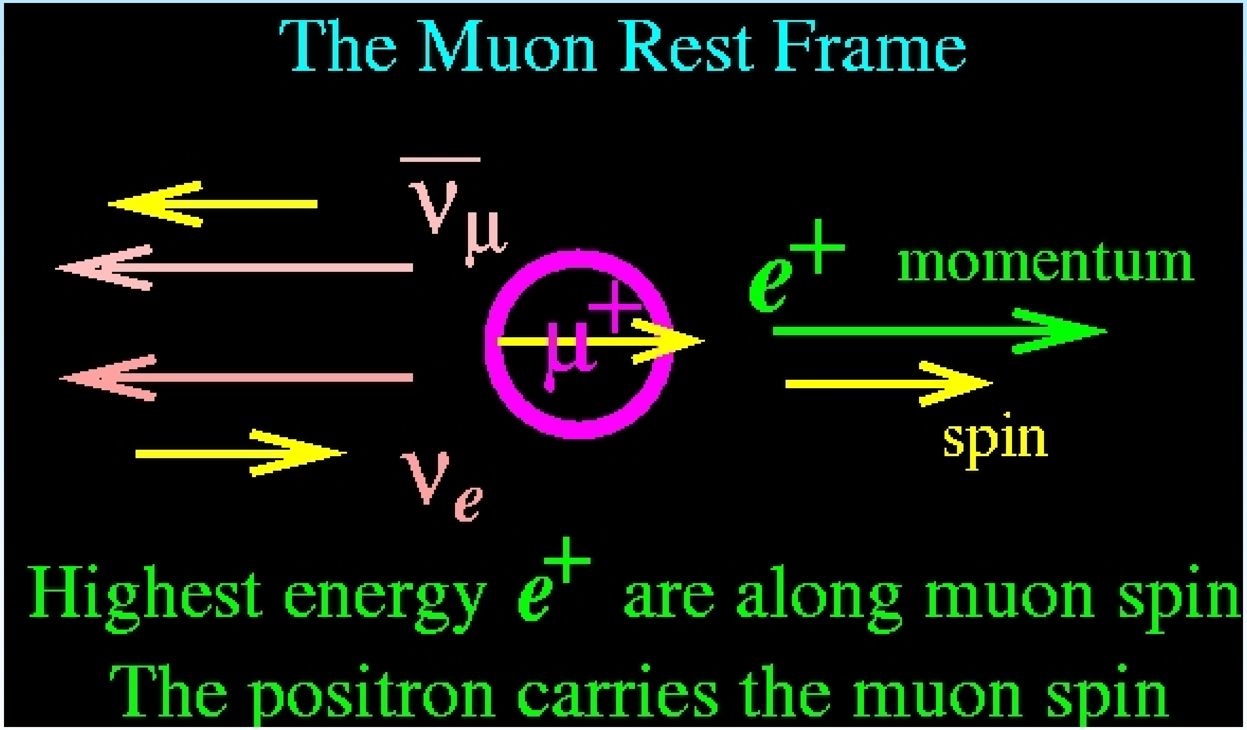
Muon properties:

- Lifetime ~2.2 μs, practically forever
- 2^{nd} generation lepton
- m_μ/m_e = 206.768 277(24)
- produced polarized
  - in-flight decay: both “forward” and “backward” muons are highly polarized
- Paul Scherrer Institut has 10^8 low-energy μ/s in a beam



Death of the Muon

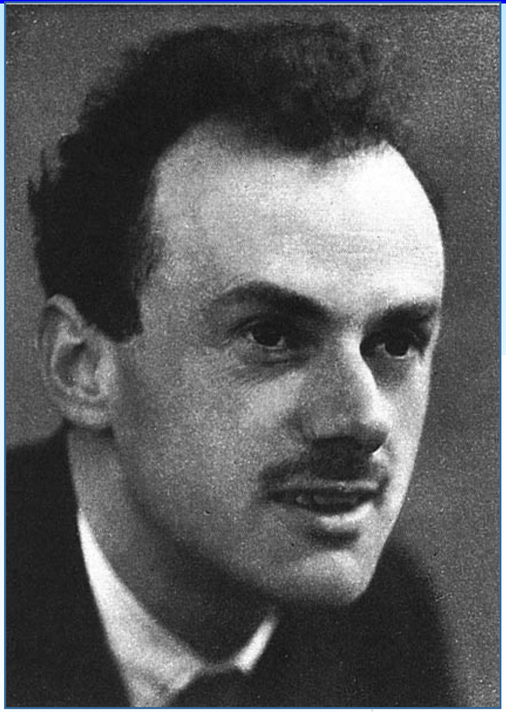
- Decay is self analyzing



What have we learned from the μ's death?

- The strength of the weak interaction
  - i.e. the Fermi constant G_F (more properly G_μ)
- The V - A nature of the weak interaction
- Lepton flavor conservation in μ-decay
- VEV of the Higgs field: $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
- Induced form-factors in nuclear μ-capture

Theory of Magnetic and Electric Dipole Moments



The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Proc. R. Soc. (London) **A117**, 610 (1928)

§ 4. *The Hamiltonian for an Arbitrary Field.*

To obtain the Hamiltonian for an electron in an electromagnetic field with scalar potential A_0 and vector potential $\mathbf{A}$, we adopt the usual procedure of substituting $p_0 + e/c \,.\, A_0$ for p_0 and $\mathbf{p} + e/c \,.\, \mathbf{A}$ for $\mathbf{p}$ in the Hamiltonian for no field. From equation (9) we thus obtain

$$\left[p_0 + \frac{e}{c} A_0 + \rho_1 \left(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}\right) + \rho_3 mc\right] \psi = 0. \tag{14}$$

This differs from (1) by the two extra terms

$$\frac{eh}{c}(\boldsymbol{\sigma}, \mathbf{H}) + \frac{ieh}{c}\rho_1(\boldsymbol{\sigma}, \mathbf{E})$$

in F. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $eh/2mc \,.\, \boldsymbol{\sigma}$ and an electric moment $ieh/2mc \,.\, \rho_1 \boldsymbol{\sigma}$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether

Magnetic and Electric Dipole Moments:

- Muon Magnetic Dipole Momoment

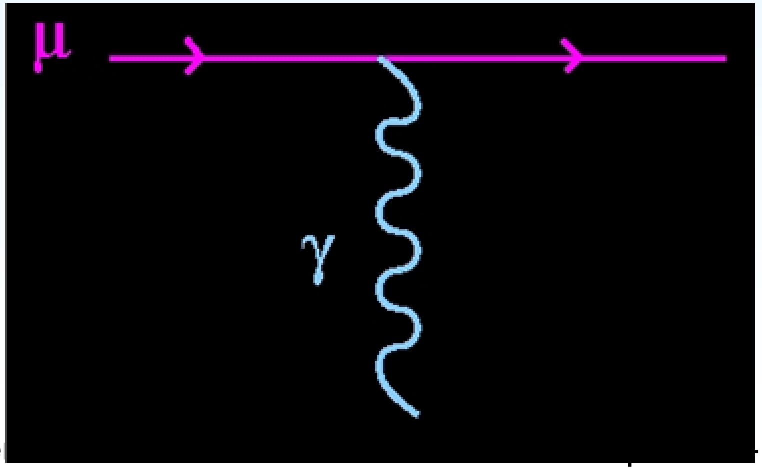
a_μ chiral changing

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m}\right) \vec{s}$$

$$\mu = (1 + a) \frac{e\hbar}{2m}$$

- Muon EDM

$$a = \frac{(g-2)}{2}$$



Magnetic and Electric Dipole Moments:

- Muon Magnetic Dipole Momoment

a_μ chiral changing

$$\bar{u}_\mu [e f_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu} f_2(q^2)\sigma_{\beta\delta}q^\delta] u_\mu$$

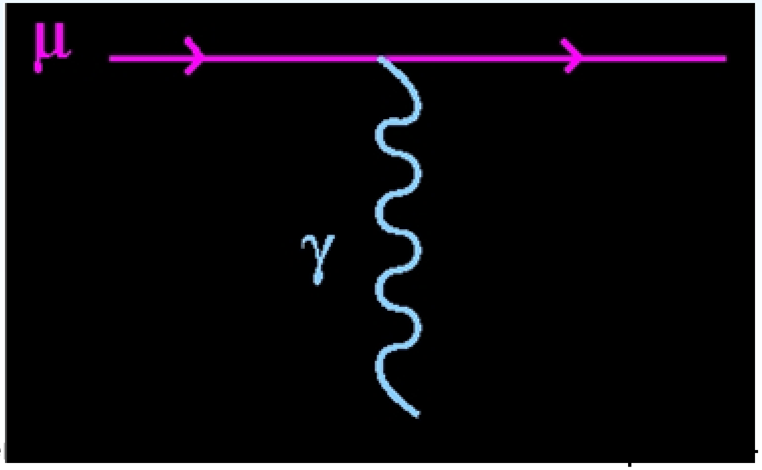
$$f_1(0) = 1 \qquad f_2(0) = a_\mu$$

- Muon EDM

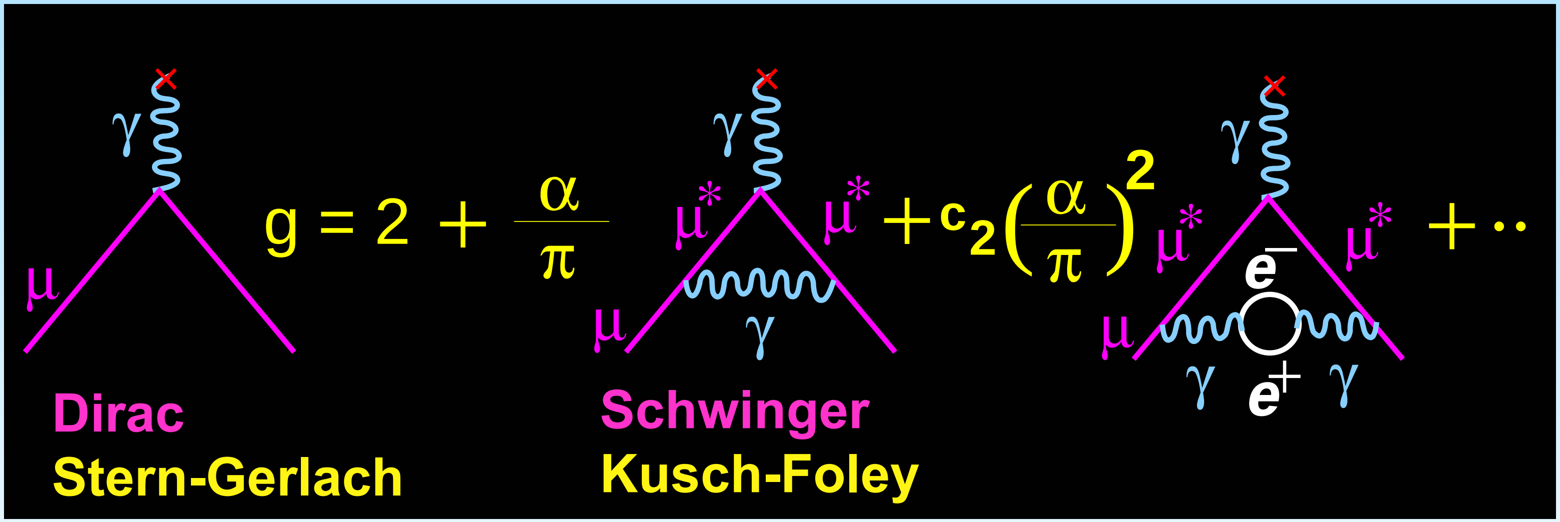
$$\bar{u}_\mu \left[\frac{ie}{2m_\mu} f_2(q^2) - f_3(q^2)\gamma_5\right]\sigma_{\beta\delta}q^\nu u_\mu$$

$$\frac{(g-2)}{2}$$

$$f_2(0) = a_\mu \qquad f_3(0) = d_\mu;\ \text{EDM}$$



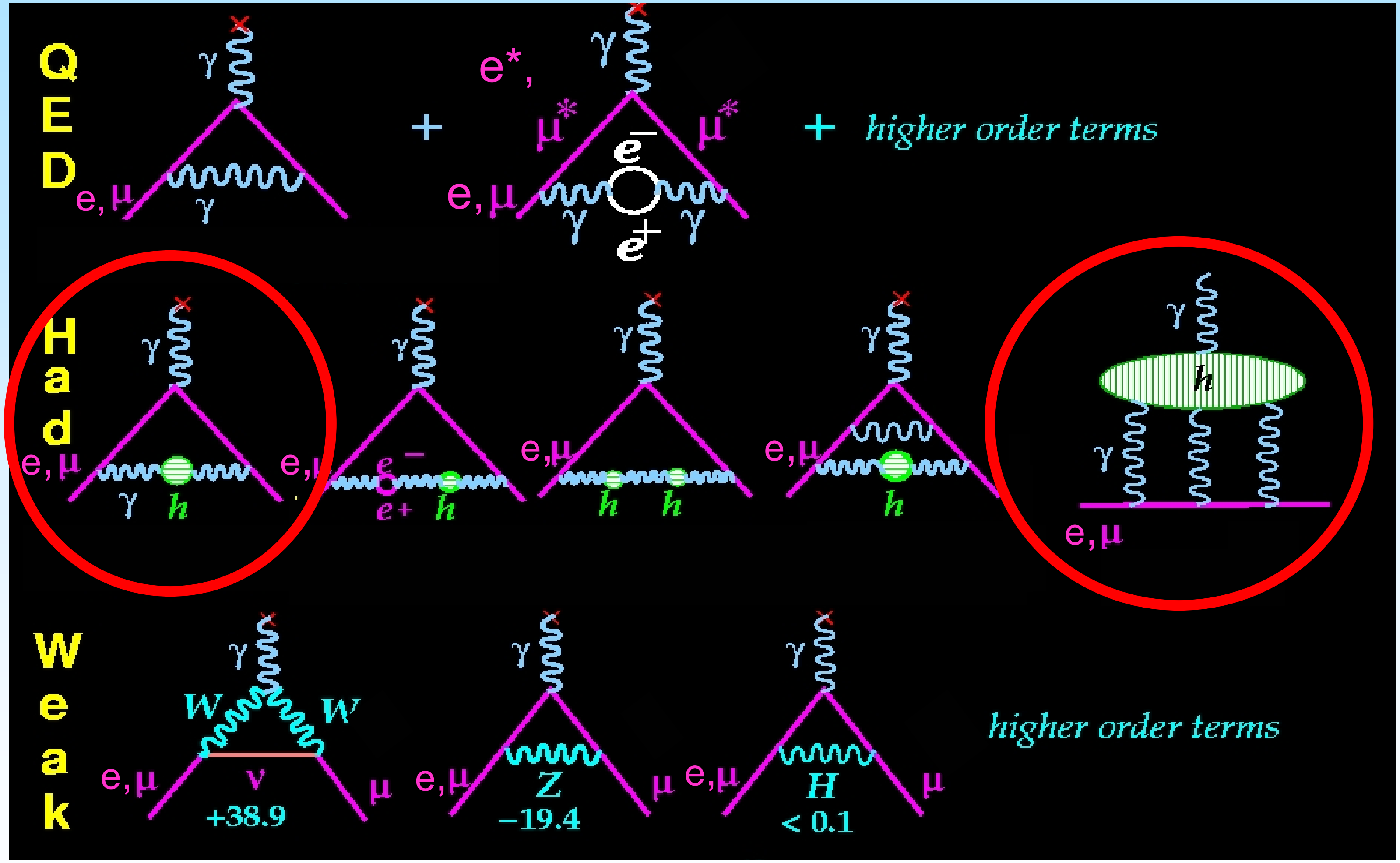
Radiative corrections change g



$$a(\text{QED}) = \frac{1}{2}\frac{\alpha}{\pi} + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

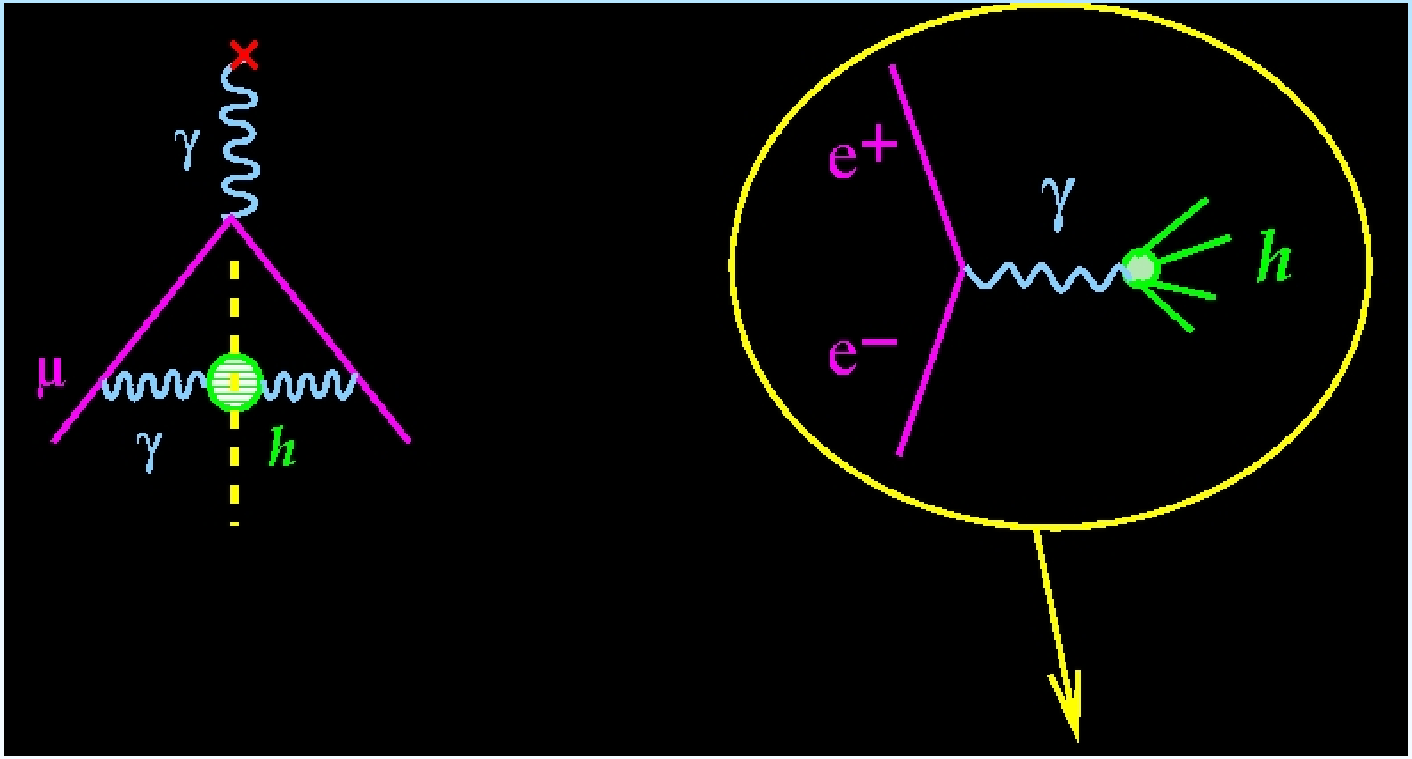
The SM Value for electron and muon anomalies

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak})$$



e vrs. μ : relative contribution of heavier things $\left(\frac{m_\mu}{m_e}\right)^2 \simeq 42,000$

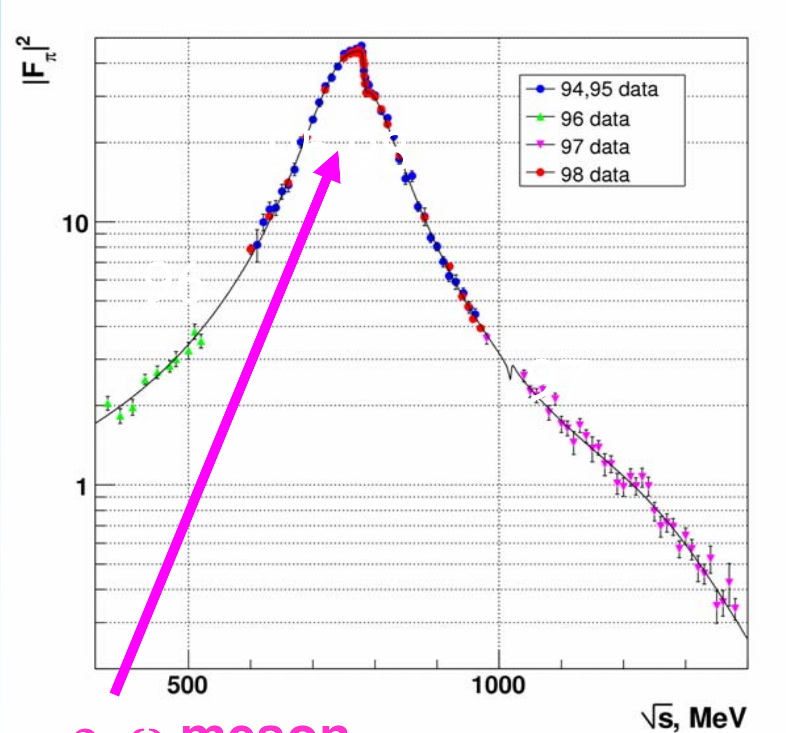
Lowest Order Hadronic from e^+e^- annihilation using analyticity and the optical theorem:



$$a_\mu(\text{had}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) \left(\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\right)$$

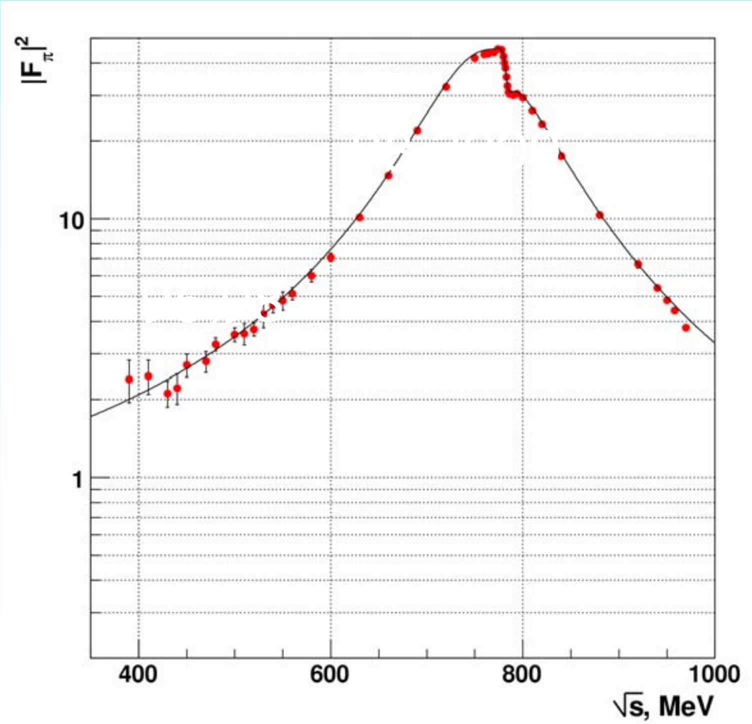
Two experiments at the Budker Insitute at Novosibirsk have measured R(s) to better than a percent. KLOE at Frascati has also measured R, and BaBar has a large data set that is being analyzed with a blind analysis.

CMD-2

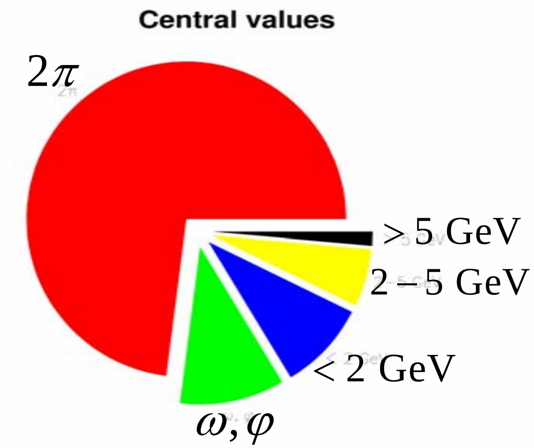


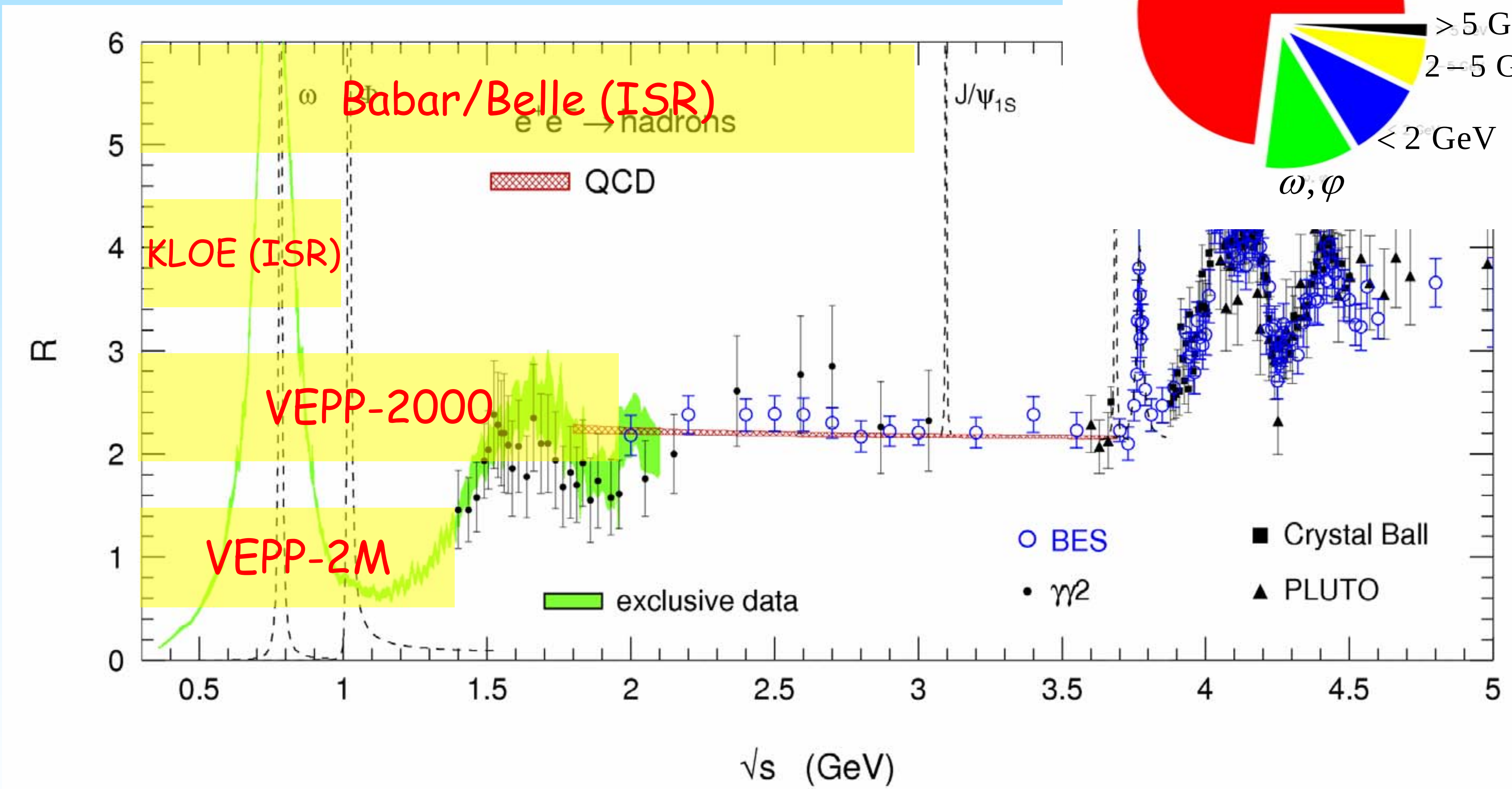
ρ–ω meson interference

SND



R(s) measurements at low s



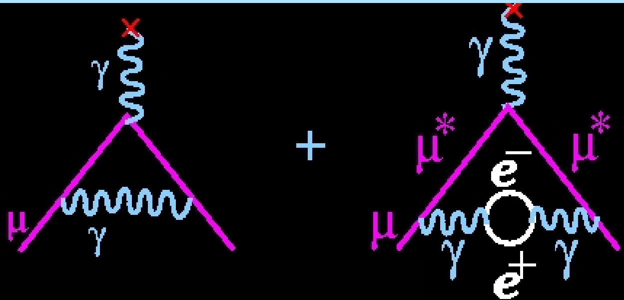


At low s the cross-section is measured independently for each final state

from Davier/Höcker

The SM Value for the muon anomaly (10^{-10})

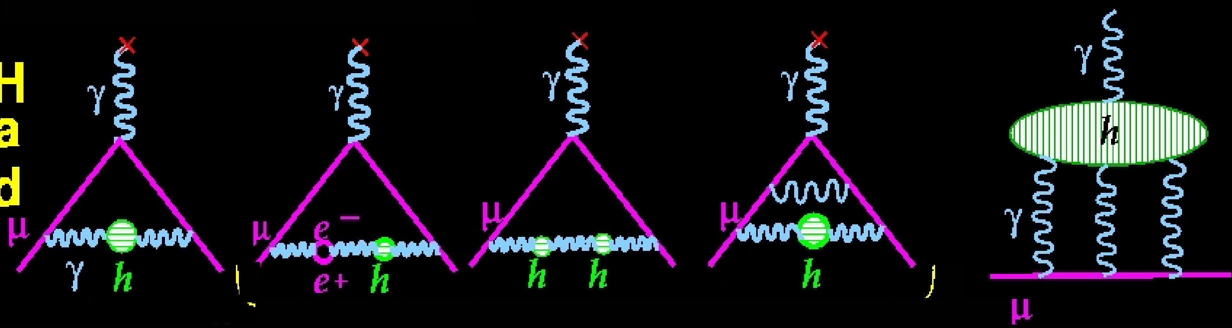
QED



$$+ \cdots + C_5 \left(\frac{\alpha}{\pi}\right)^5$$

11 658 471.809 (.016)

Had

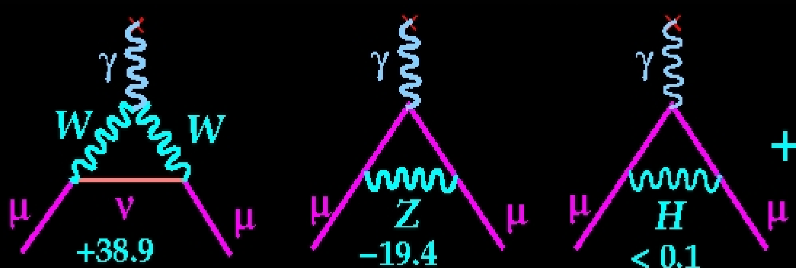


690.1 (4.7)

−9.79 (.09)

11 (4)

Weak



$$+ + \cdots$$

15.4 (.2)

---

11 659 208.0(6.3)

from Miller, de Rafael, Roberts, Rep. Prog. Phys. **70** (2007) 795–881

a_μ is sensitive to a wide range of new physics

- substructure

$$\delta a_\mu(\Lambda_\mu) \simeq \frac{m_\mu^2}{\Lambda_\mu^2}$$

- SUSY (with large tanβ)

$$a_\mu(\mathrm{SUSY}) \simeq \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W}\frac{m_\mu^2}{\tilde{m}^2}\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\tilde{m}}{m_\mu}\right)$$

$$\simeq (\mathrm{sgn}\mu)\ 13\times 10^{-10}\ \tan\beta\left(\frac{100\ \mathrm{GeV}}{\tilde{m}}\right)^2$$

- many other things (extra dimensions, etc.)

Spin Motion in a Magnetic Field

Momentum turns with ω_C, cyclotron frequency

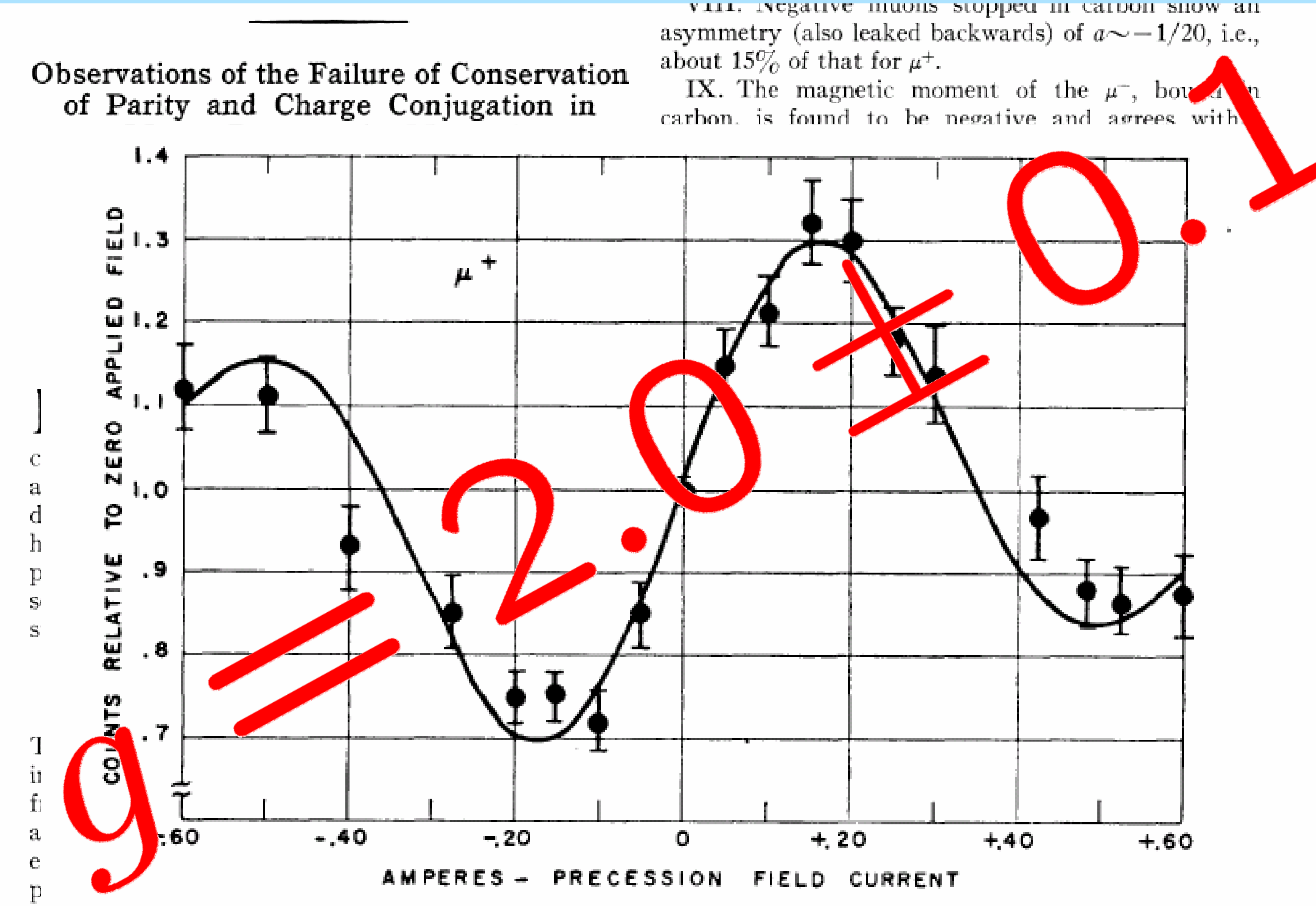
Spin turns with ω_S

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

Spin turns relative to the momentum with ω_a

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$

First muon spin rotation experiment



PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1, 1960

Accurate Determination of the μ^+ Magnetic Moment*

R. L. GARWIN,† D. P. HUTCHINSON, S. PENMAN,‡ AND G. SHAPIRO§
Columbia University, New York, New York

(Received August 4, 1959)

Note added in proof.—Experiments which have recently been reported to us [J. Lathrop, et al. and A. Bearden et al., Phys. Rev. Letters (to be published)] indicate a mass value of $M_\mu = 206.76_{-0.02}^{+0.03} M_e$. This yields a value of $g_\mu = 2(1.00113_{-0.00012}^{+0.00016})$. Although the assigned errors are now slightly greater than above, it is to be noted that the new result represents a direct measurement, rather than a lower limit. The agreement

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Subsequent (g-2) experiments measured the difference frequency, ω_a, between the spin and momentum precession

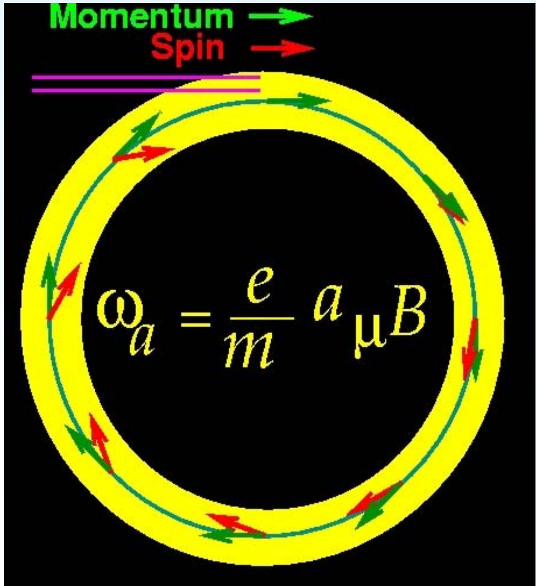
With an electric quadrupole field for vertical focusing

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}}\right]$$

$$B \Rightarrow \langle B \rangle_{\mu-\mathrm{dist}}$$

$$\gamma_{\mathrm{magic}} = 29.3$$

$$p_{\mathrm{magic}} = 3.09 \text{ GeV/c}$$



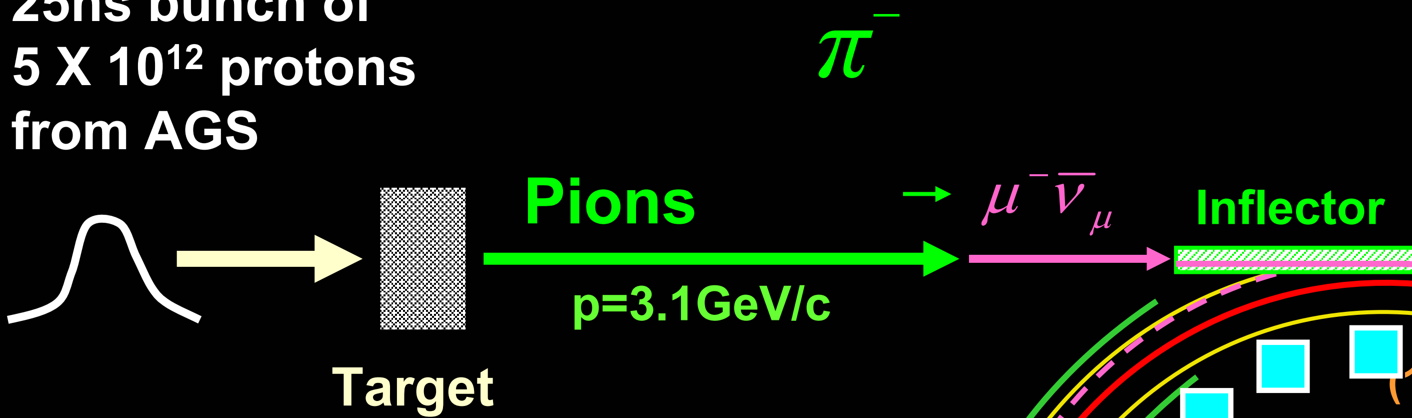
Experimental Technique

25ns bunch of
5 X 10^{12} protons
from AGS

$x_c \approx 77$ mm

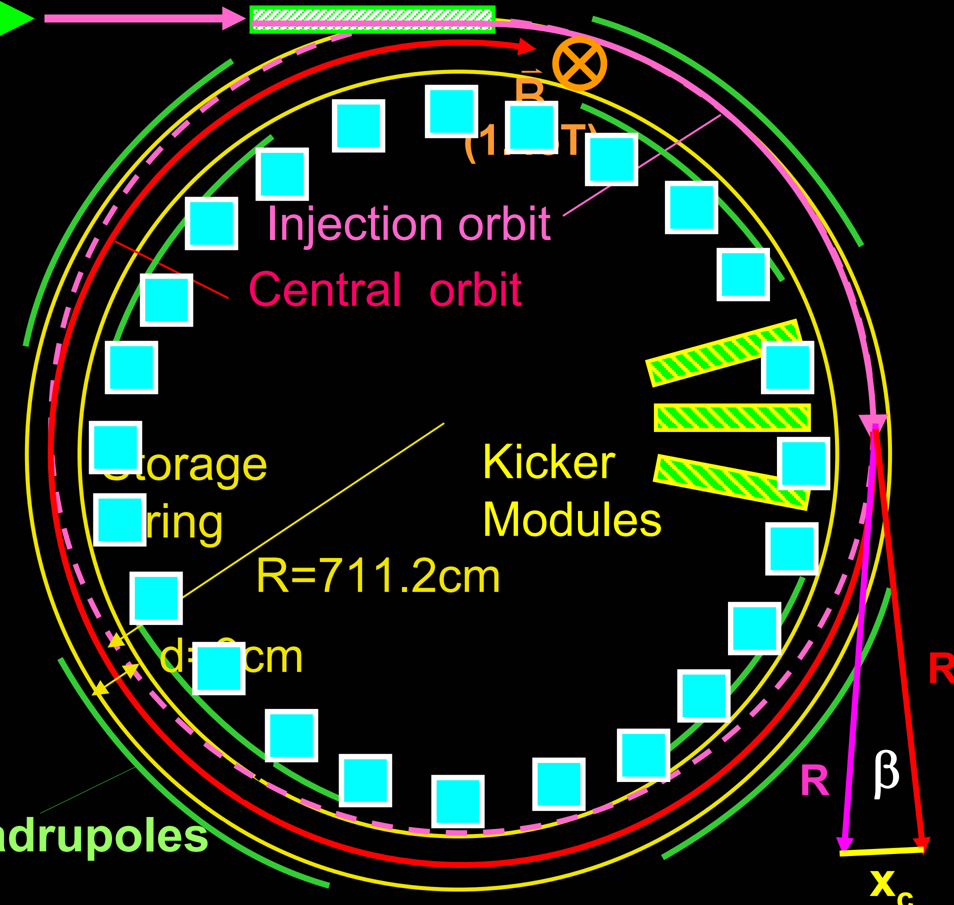
$\beta \approx 10$ mrad

$B\cdot dl \approx 0.1$ Tm



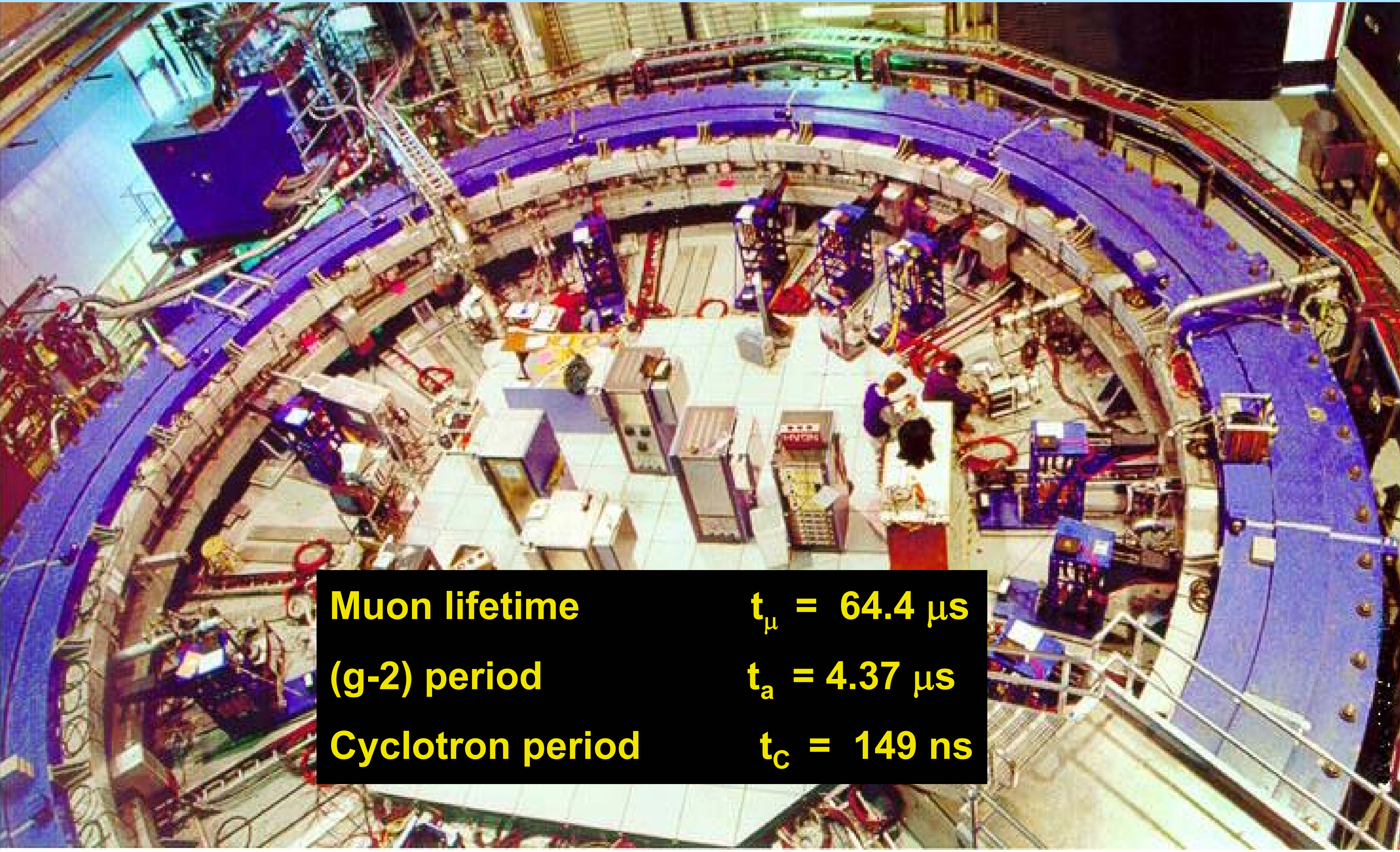
- **Muon polarization**
- **Muon storage ring**
- **injection & kicking**
- **focus with Electric Quadrupoles**
- **24 electron calorimeters**

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$



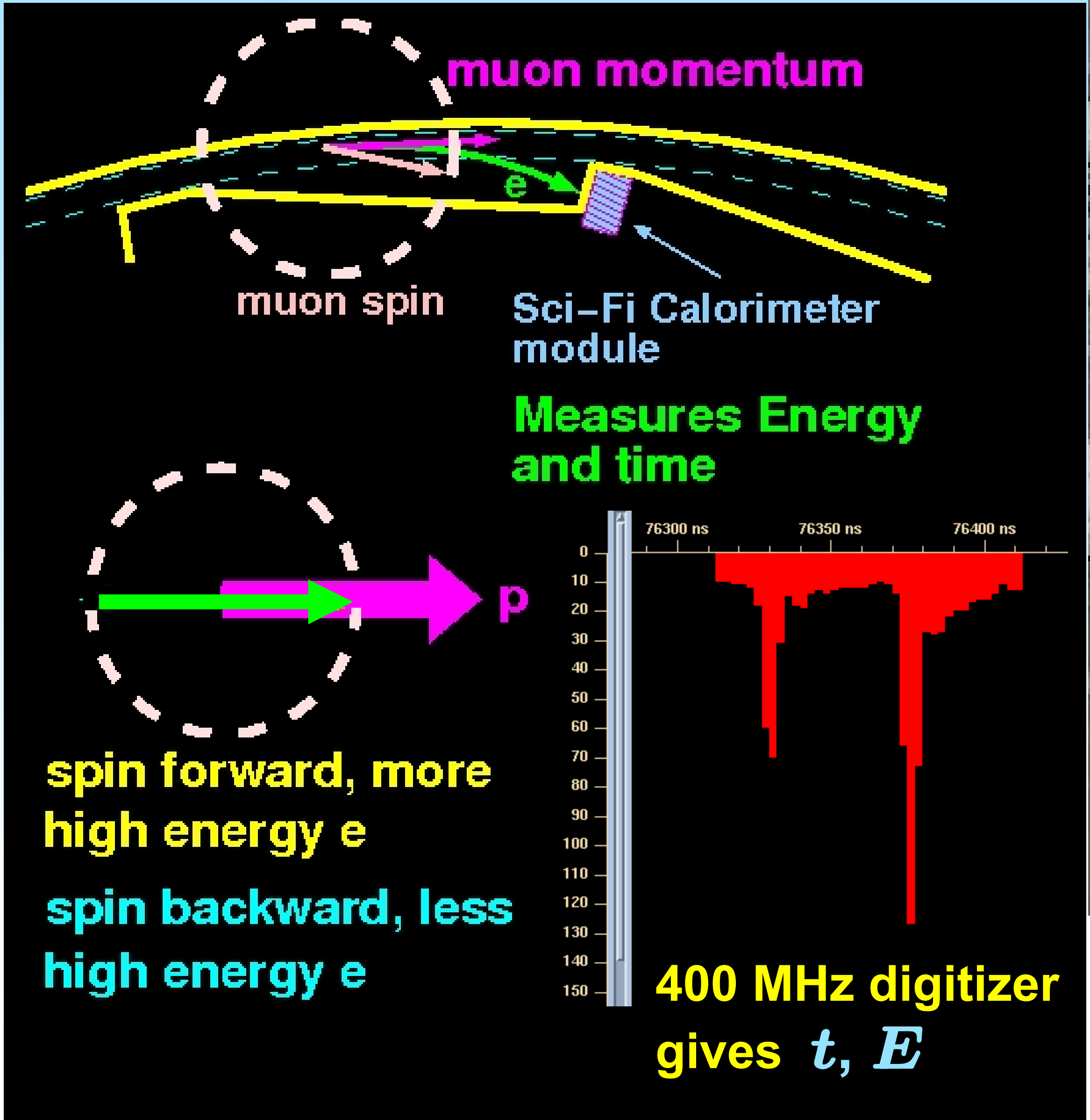
(thanks to Q. Peng)

muon (g-2) storage ring



Muon lifetime	t_μ = 64.4 μs
(g-2) period	t_a = 4.37 μs
Cyclotron period	t_C = 149 ns

To measure ω_a, we used Pb-scintillating fiber calorimeters.

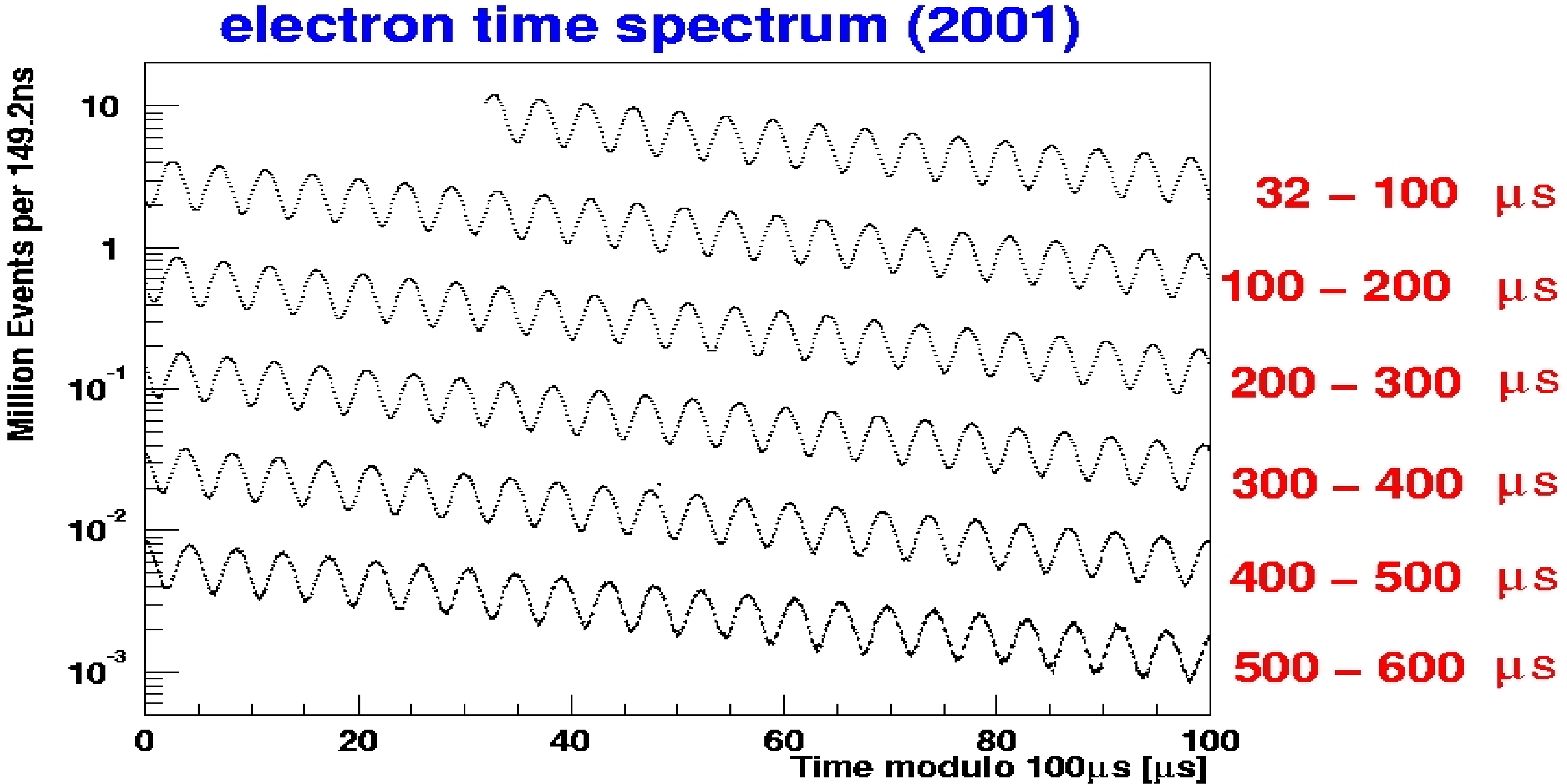


Count number of e^- with $E_e \geq 1.8$ GeV

We count high-energy electrons as a function of time.

$$4 \times 10^9 \quad e,\ E_{e^-} \geq 1.8\ \text{GeV}$$

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos\omega_a t + \phi)]$$



The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

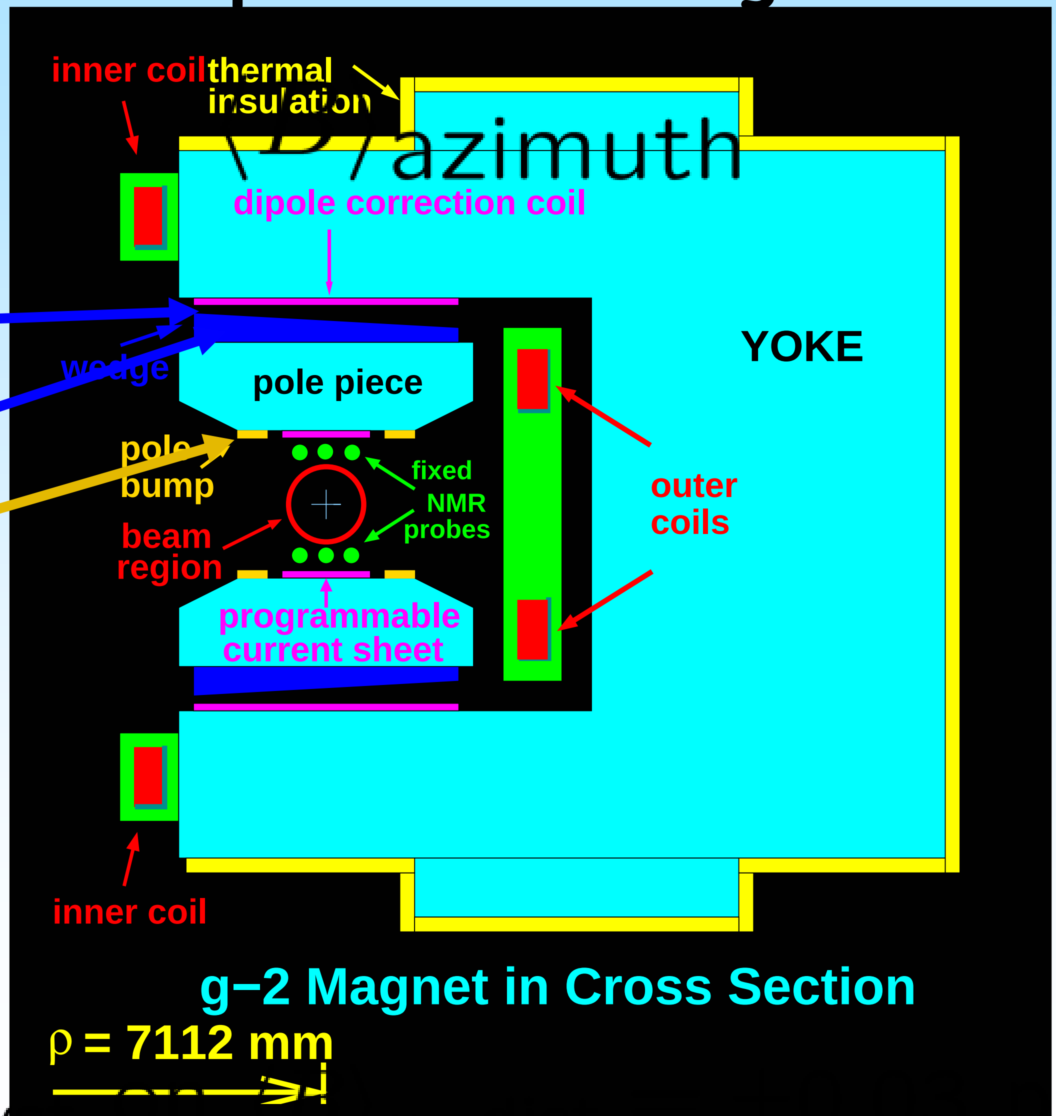
We can shim the

dipole,

quadrupole

sextupole

independently



0.5 ppm contours

The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

$\langle B \rangle_{\text{azimuth}}$

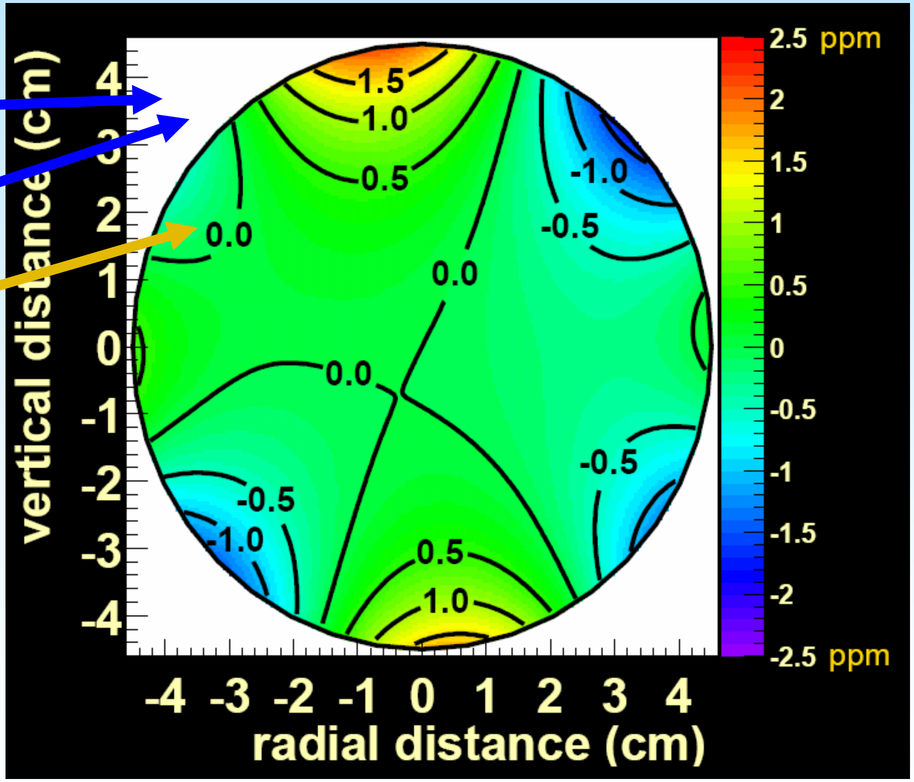
We can shim the

dipole,

quadrupole

sextupole

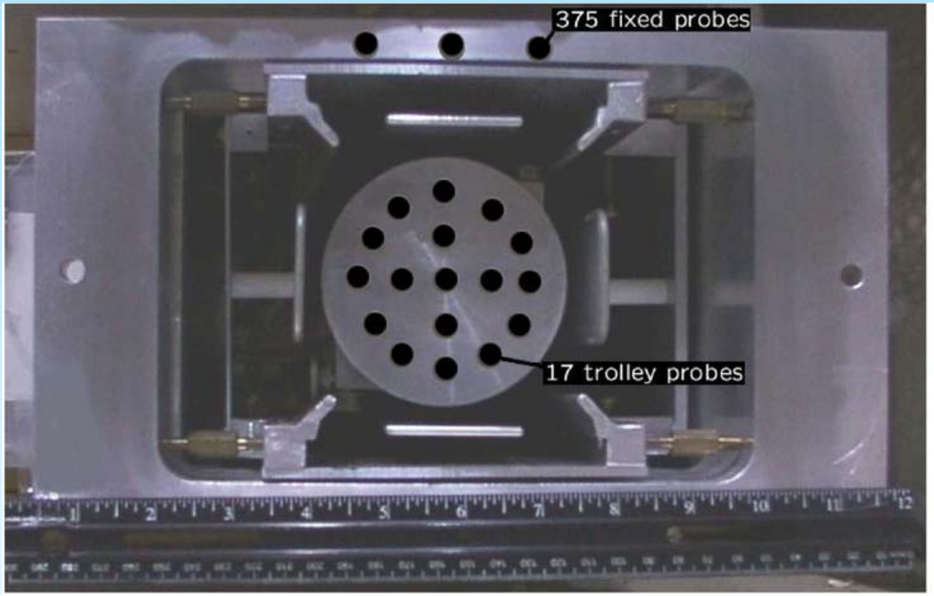
independently



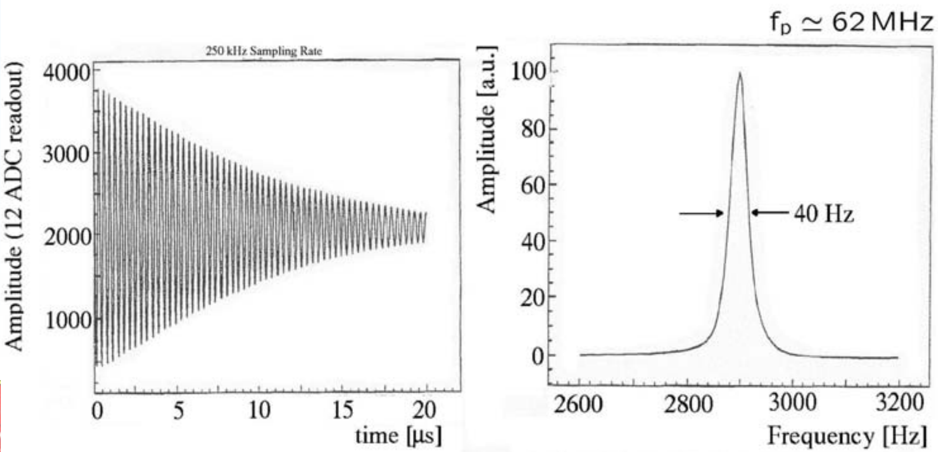
0.5 ppm contours

$$\sigma_{\text{syst}} \text{ on } \langle B \rangle_{\mu-\text{dist}} = \pm 0.03 \text{ ppm}$$

The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



Free induction decay signals:



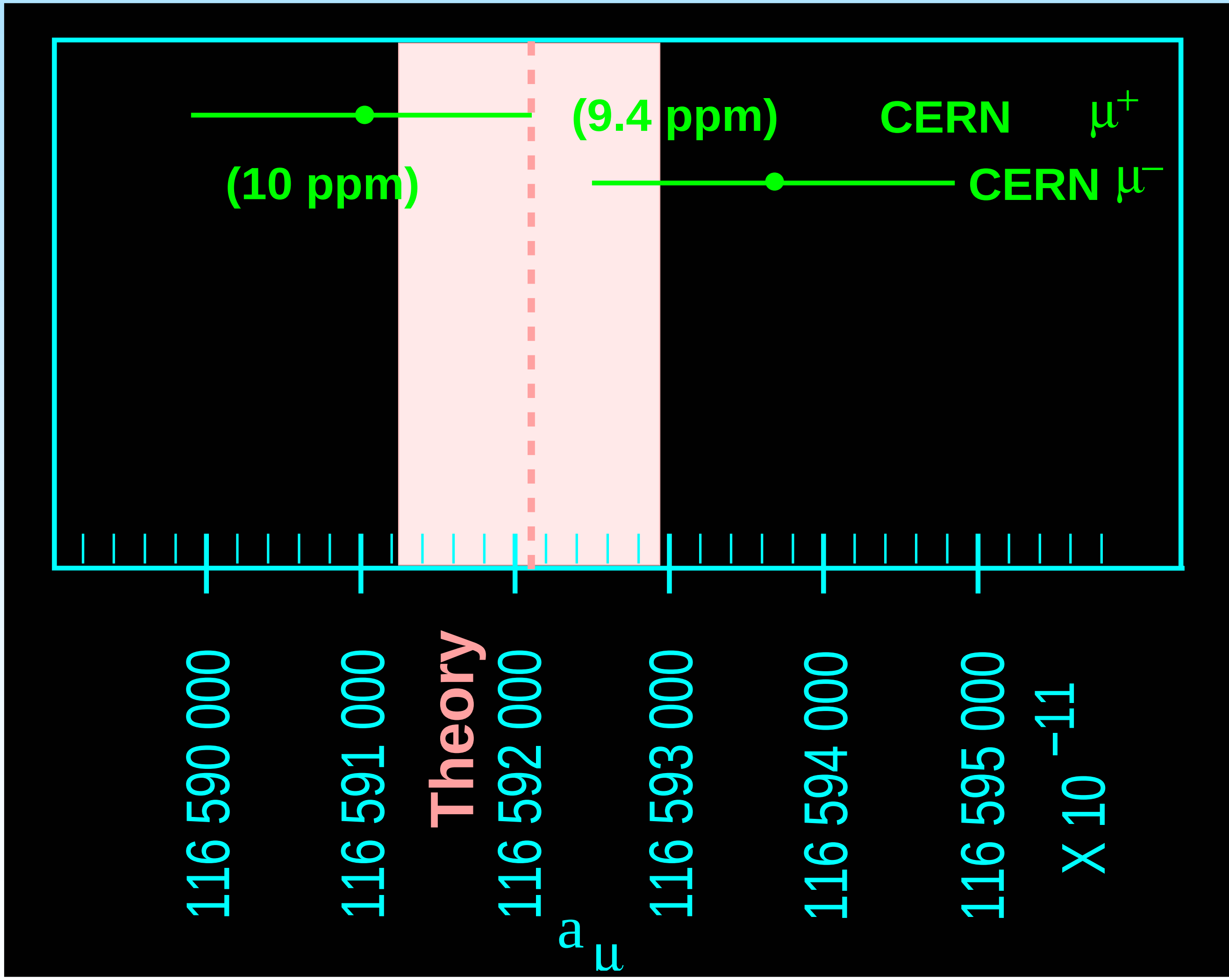
- Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.
- So we measure ω_a and ω_p

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

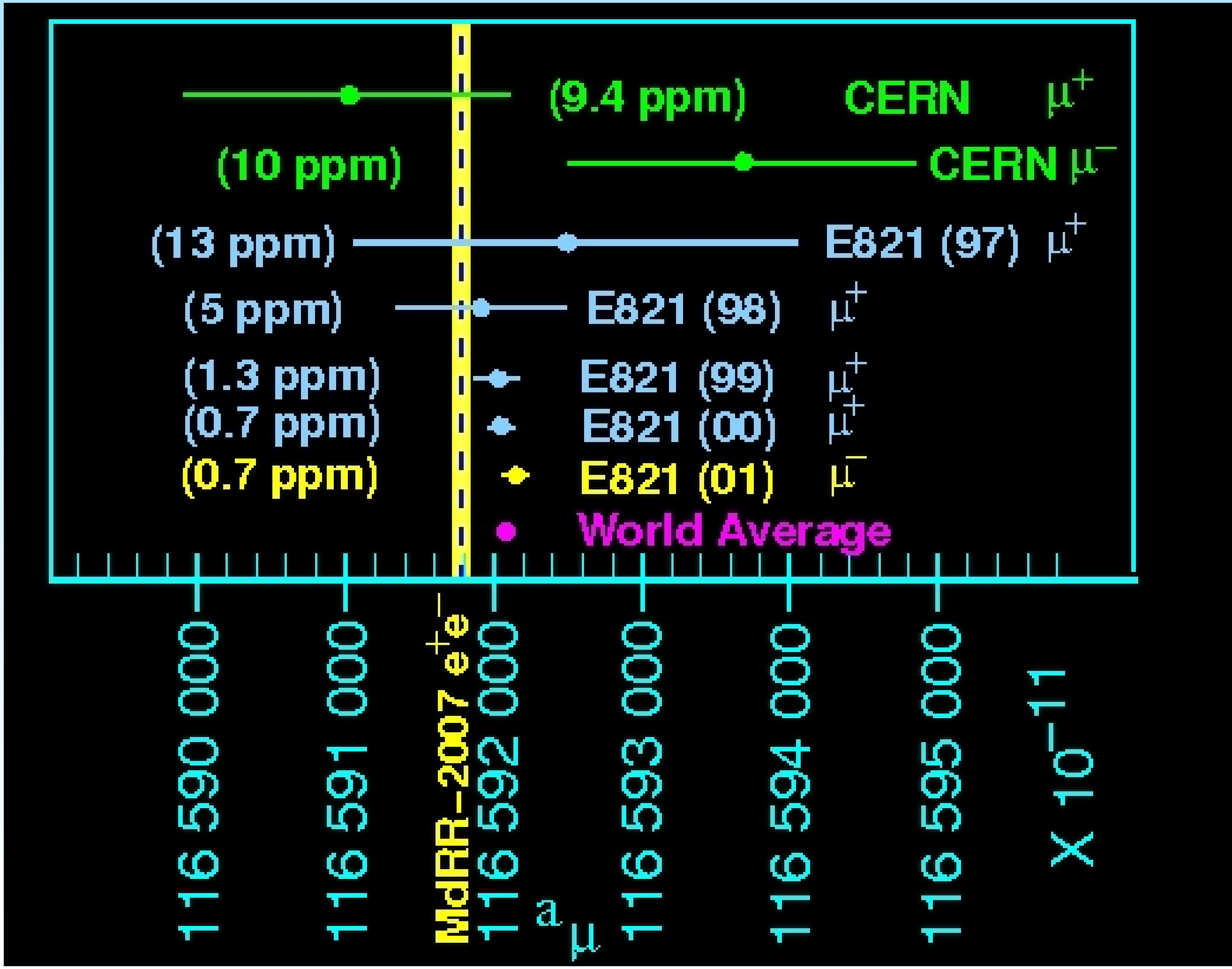
When we started in 1983, theory and experiment were known to about 10 ppm.

Theory uncertainty was ~ 9 ppm

Experimental uncertainty was 7.3 ppm



E821 achieved 0.5 ppm and the e^+e^- based theory is also at the 0.6 ppm level. Difference is 3.4σ



MdRR=Miller, de Rafael, Roberts, Rep. Prog. Phys. **70** (2007) 795

$$\Delta a_\mu^{(\text{today})} = (29.5 \pm 8.8) \times 10^{-10}$$

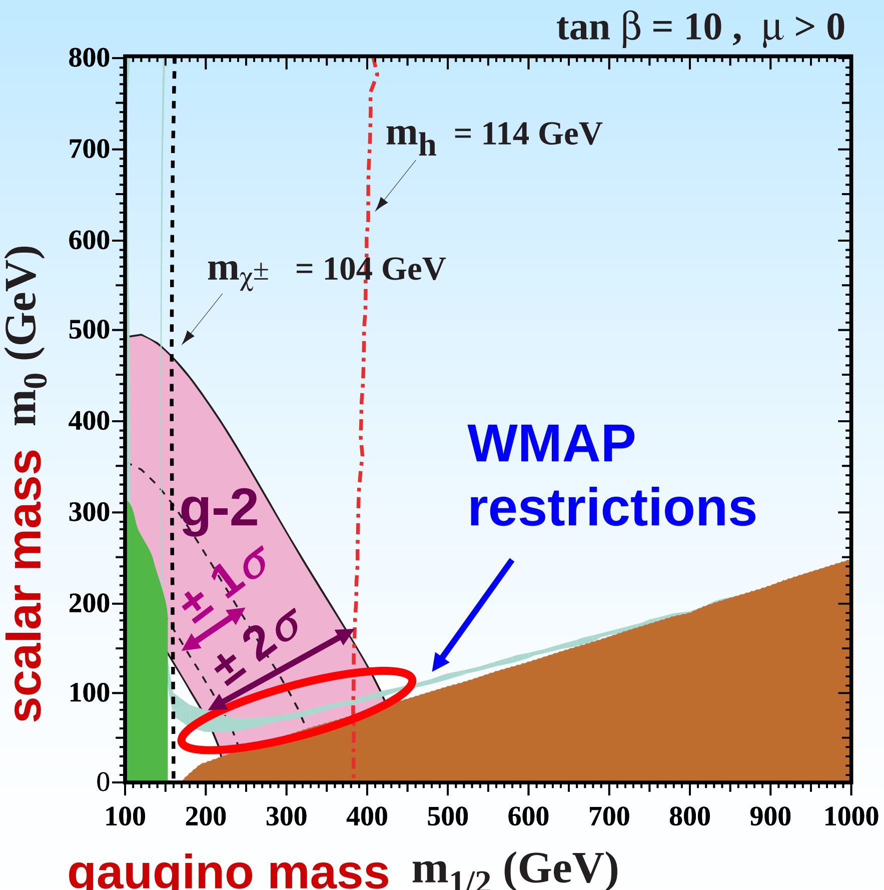
If the electroweak contribution is left out of the standard-model value, we get a 5.1 σ difference.

$$a_\mu^{EW} = 15.4(.1)(.2) \times 10^{-10}$$

$$\Delta(\text{no EW}) = 44.9(8.8) \times 10^{-10}$$

a_μ helps constrain new physics

In a constrained minimal supersymmetric model, $(g\text{-}2)_\mu$ provides an independent constraint on the SUSY LSP (lightest supersymmetric partner) being the dark matter candidate.

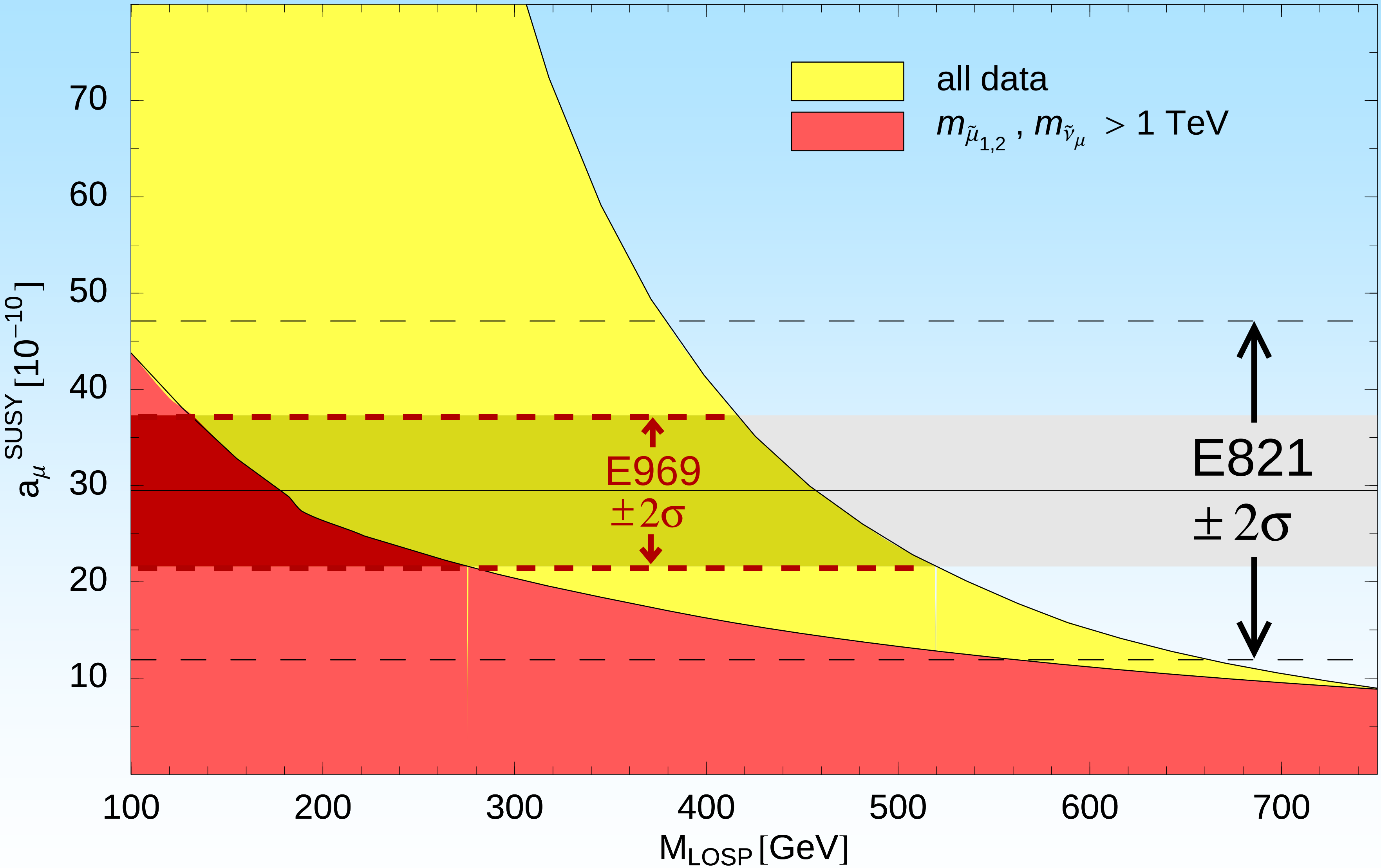


Historically muon (g-2) has played an important role in restricting models of new physics.

It provides constraints that are <u>independent and complementary</u> to high-energy experiments.

CMSSM calculation Following Ellis, Olive, Santoso, Spanos, provided by K. Olive

MSSM scan of M_{LOSP} vrs. a_μ^{SUSY}



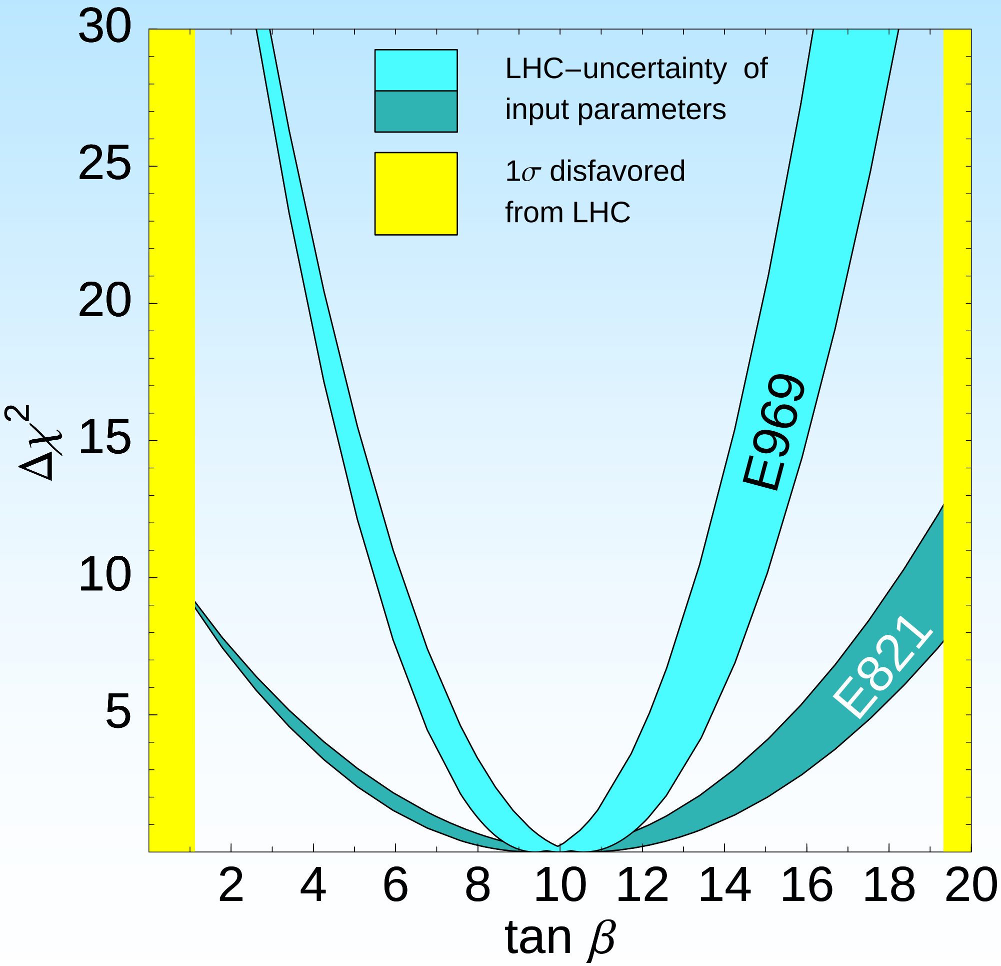
D. Stöckinger, J. Phys. G 34, R45 (2007)

a_μ will help constrain the interpretation of LHC data, e.g. tan β

MSSM reference point SPS1a

With these SUSY parameters, LHC gets tan β of 10.22 ± 9.1.

arXiv:0705.4617v1 [hep-ph]



An Intermezzo: The search for a Muon EDM and CPT/Lorentz violation

- We have two new results:
  - a new limit on the muon EDM
  - a limit on CPT/Lorentz invariance violation in muon spin precession

Electric Dipole Moment: ~~P T~~

$$\mathcal{H} = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E} \qquad \vec{\mu},\ \vec{d} \parallel \text{to } \vec{\sigma}$$

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	$-$	$+$	$+$
C	$-$	$-$	$-$
T	$+$	$-$	$-$

Transformation Properties

If CPT is valid, an EDM would imply non-standard model ~~*CP*~~.

Purcell and Ramsey: EDM would violate Parity
Proposed to search for an EDM of the neutron

"raises directly the question of parity."

On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei

E. M. PURCELL AND N. F. RAMSEY
Department of Physics, Harvard University, Cambridge, Massachusetts
April 27, 1950

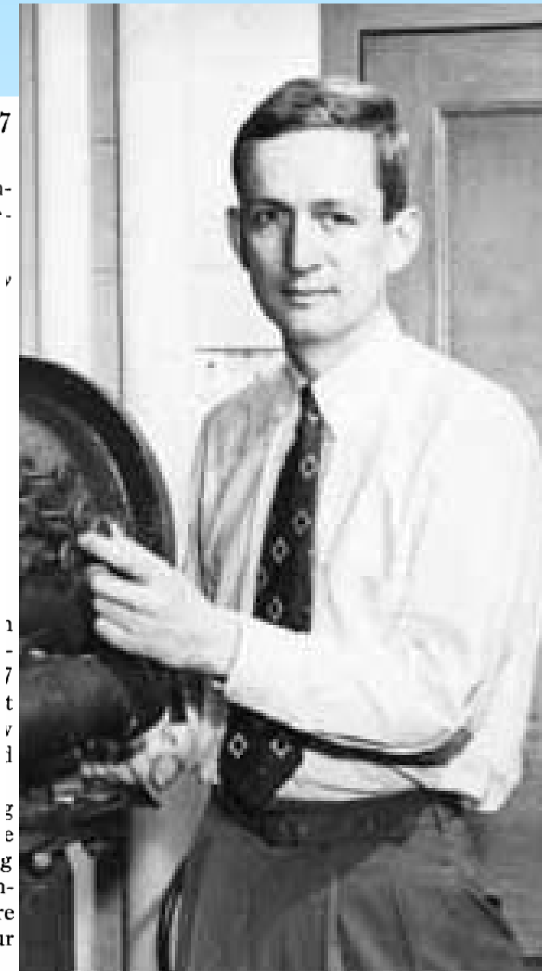
IT is generally assumed on the basis of some suggestive theoretical symmetry arguments[1] that nuclei and elementary particles can have no electric dipole moments. It is the purpose of this note to point out that although these theoretical arguments are valid when applied to molecular and atomic moments whose electromagnetic origin is well understood, their extension to nuclei and elementary particles rests on assumptions not yet tested.

One form of the argument against the possibility of an electric dipole moment of a nucleon or similar particle is that the dipole's orientation must be completely specified by the orientation of the angular momentum which, however, is an axial vector specifying a direction of circulation, not a direction of displacement as would be required to obtain an electric dipole moment from electrical charges. On the other hand, if the nucleon should spend part of its time asymmetrically dissociated into opposite magnetic poles of the type that Dirac[2] has shown to be theoretically possible, a circulation of these magnetic poles could give rise to an electric dipole moment. To forestall a possible objection we may remark that this electric dipole would be a polar vector, being the product of the angular momentum (an axial vector) and the magnetic pole strength, which is a pseudoscalar in conformity with the usual convention that electric charge is a simple scalar.

The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and right-handed coordinate systems; in one system the dipole moment

The authors wish to thank Mr. Smith for suggesting an important correction to our original calculation on the neutron-

sun, e.g., ... burning its hydr... ral temperature ... mperature of 2 to 3 ... will occur between



Phys. Rev. 78 (1950)

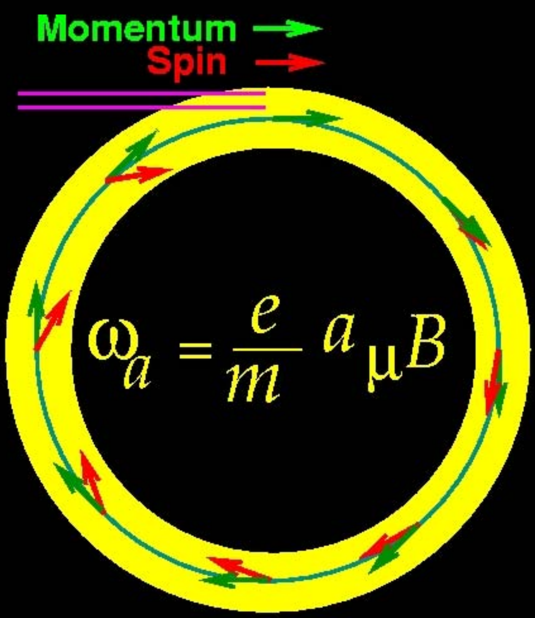
Spin Frequencies: μ in B field with MDM & EDM

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

ω_a

spin difference frequency = ω_s - ω_c



The EDI
spin to p
of plane.

The highest energy decay $e^{\pm}$ are along the muon spin direction

Spin Frequencies: μ in B field with MDM & EDM

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

$$\gamma_{\mathrm{magic}} = 29.3$$

ω_a

$$+ \quad \frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

ω_η

The motional E - field, β X B, is (~GV/m).

$$d_\mu = \frac{\eta}{2}\left(\frac{e\hbar}{2mc}\right) \simeq \eta \times 4.7 \times 10^{-14} \ e \ \mathrm{cm}$$

and

$$a_\mu = \left(\frac{g-2}{2}\right)$$

Spin Frequencies: μ in B field with MDM & EDM

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

$$\gamma_{\text{magic}} = 29.3$$

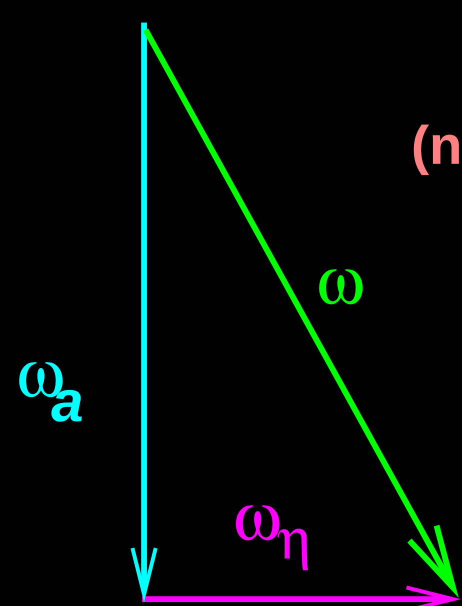
$$\omega_a + \frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

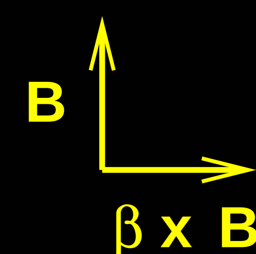
$$\omega_\eta$$

The motional E - field, β X B, is (~GV/m).

(not to scale)

The EDM causes the spin to precess out of plane.





The present EDM limits are orders of magnitude from the standard-model value

Particle	*Present EDM limit* (e-cm)	*SM value* (e-cm)
n	2.9×10^{-26}	$10^{-32} - 10^{-31}$
e^-	$\sim 1.6 \times 10^{-27}$	$< 10^{-41}$
μ	$< 10^{-18}$ (CERN) 2×10^{-19} * (E821)	$< 10^{-38}$
future μ exp	10^{-24} to 10^{-25}	

*to be finalized and submitted to PRD soon

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Dedicated EDM Experiment $\rightarrow 10^{-24} - 10^{-25}$

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right] \rightarrow 0$$

Use a radial E-field to turn off the ω_a precession

$$+ \frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right)\right]$$

With $\omega_a = 0$, the EDM causes the spin to steadily precess out of the plane.

ω_η

Connection between MDM, EDM and the lepton flavor violating transition moment $\mu \rightarrow e$

SUSY ⇒ slepton mixing

$$\begin{pmatrix} m^2_{\tilde{e}\tilde{e}} & \Delta m^2_{\tilde{e}\tilde{\mu}} & \Delta m^2_{\tilde{e}\tilde{\tau}} \\ \Delta m^2_{\tilde{\mu}\tilde{e}} & m^2_{\tilde{\mu}\tilde{\mu}} & \Delta m^2_{\tilde{\mu}\tilde{\tau}} \\ \Delta m^2_{\tilde{\tau}\tilde{e}} & \Delta m^2_{\tilde{\tau}\tilde{\mu}} & m^2_{\tilde{\tau}\tilde{\tau}} \end{pmatrix}$$

Search for Lorentz and CPT Violation Effects in Muon Spin Precession

G.W. Bennett[2], B. Bousquet[10], H.N. Brown[2], G. Bunce[2], R.M. Carey[1], P. Cushman[10], G.T. Danby[2], P.T. Debevec[8], M. Deile[13], H. Deng[13], W. Deninger[8], S.K. Dhawan[13], V.P. Druzhinin[3], L. Duong[10], E. Efstathiadis[1], F.J.M. Farley[13], G.V. Fedotovich[3], S. Giron[10], F.E. Gray[8], D. Grigoriev[3], M. Grosse-Perdekamp[13], A. Grossmann[7], M.F. Hare[1], D.W. Hertzog[8], X. Huang[1], V.W. Hughes[13,*], M. Iwasaki[12], K. Jungmann[6,7], D. Kawall[13], M. Kawamura[12], B.I. Khazin[3], J. Kindem[10], F. Krienen[1], I. Kronkvist[10], A. Lam[1], R. Larsen[2], Y.Y. Lee[2], I. Logashenko[1,3], R. McNabb[10,8], W. Meng[2], J. Mi[2], J.P. Miller[1], Y. Mizumachi[9,11], W.M. Morse[2], D. Nikas[2], C.J.G. Onderwater[8,6], Y. Orlov[4], C.S. Özben[2,8], J.M. Paley[1], Q. Peng[1], C.C. Polly[8], J. Pretz[13], R. Prigl[2], G. zu Putlitz[7], T. Qian[10], S.I. Redin[3,13], O. Rind[1], B.L. Roberts[1], N. Ryskulov[3], S. Sedykh[8], Y.K. Semertzidis[2], P. Shagin[10], Yu.M. Shatunov[3], E.P. Sichtermann[13], E. Solodov[3], M. Sossong[8], A. Steinmetz[13], L.R. Sulak[1], C. Timmermans[10], A. Trofimov[1], D. Urner[8], P. von Walter[7], D. Warburton[2], D. Winn[5], A. Yamamoto[9] and D. Zimmerman[10]

(Muon $(g-2)$ Collaboration)

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Submitted to PRL and back from the referee

What we measure that could show CPT/Lorentz violation

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

0

$\omega_a = \omega_S - \omega_C$; where ω_C is unaffected by CPT/Lorentz to lowest order.

- BUT $\omega_a = \omega_a(B) \Rightarrow \omega_a(\omega_p)$
- Instead we have to use $\mathcal{R} = \frac{\omega_a}{\omega_p}$

CPT/Lorentz violation in the Lagrangian*

$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa\psi - b_\kappa\bar{\psi}\gamma_5\gamma^\kappa\psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi$$
$$+\frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda \psi$$

- a_κ, b_κ are CPT odd, others CPT even
- All terms violate Lorentz invariance
- In lowest-order, a_μ is insensitive to violating terms

- Two tests of CPT/Lorentz violation:
  - Difference between ω_a for μ^+ and μ^-
  - Sidereal time variation in ω_a

*Bluhm, Kostelecký, Lane, PRL 84,1098 (2000)

Difference between ω_a for μ^+ and μ^-

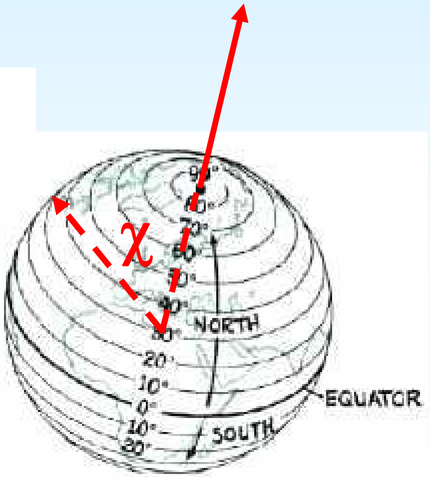
$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa \psi - b_\kappa \bar{\psi}\gamma_5\gamma^\kappa\psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi + \frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda \psi$$

$$\Delta\omega_a \equiv \langle \omega_a^{\mu^+} \rangle - \langle \omega_a^{\mu^-} \rangle = \frac{4b_Z}{\gamma}\cos\chi$$

Remember, to compare frequencies, in the experiment we must use

$$\mathcal{R} = \frac{\omega_a}{\omega_p}$$

not ω_a, since the magnetic field can vary.

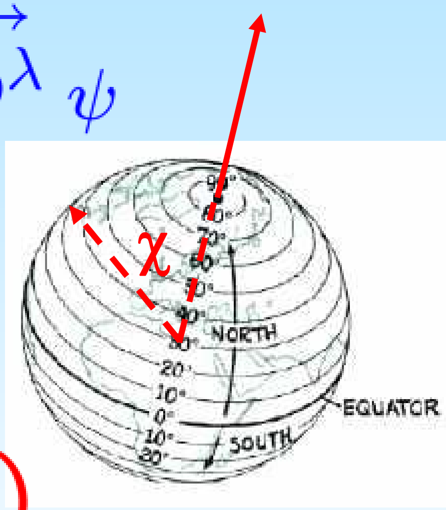


Separate studies show that any variation in ω_p is much less that our limits for ω_a. $T_{sidereal}$ = 86164.09s

T_{solar} = 86400 s

For two measurements with different colatitudes and ω_p:

$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa\psi - b_\kappa\bar{\psi}\gamma_5\gamma^\kappa\psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi + \frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda \psi$$



$$\Delta\mathcal{R} = \frac{2b_Z}{\gamma}\left(\frac{\cos\chi_1}{\omega_{p1}} + \frac{\cos\chi_2}{\omega_{p2}}\right) + 2(m_\mu d_{Z0} + H_{XY})\left(\frac{\cos\chi_1}{\omega_{p1}} - \frac{\cos\chi_2}{\omega_{p2}}\right)$$

For the difference, we find

$$\Delta\mathcal{R} = -(3.6 \pm 3.7) \times 10^{-9}$$

Bennett, et al., Phys. Rev. D73, 072003-1

$$b_Z = -(8.7 \pm 8.9) \times 10^{-24}\ \mathrm{GeV}$$

$$r_{\Delta\omega_a} \equiv \frac{\Delta\omega_a}{m_\mu} = -(1.0 \pm 1.1) \times 10^{-23}$$

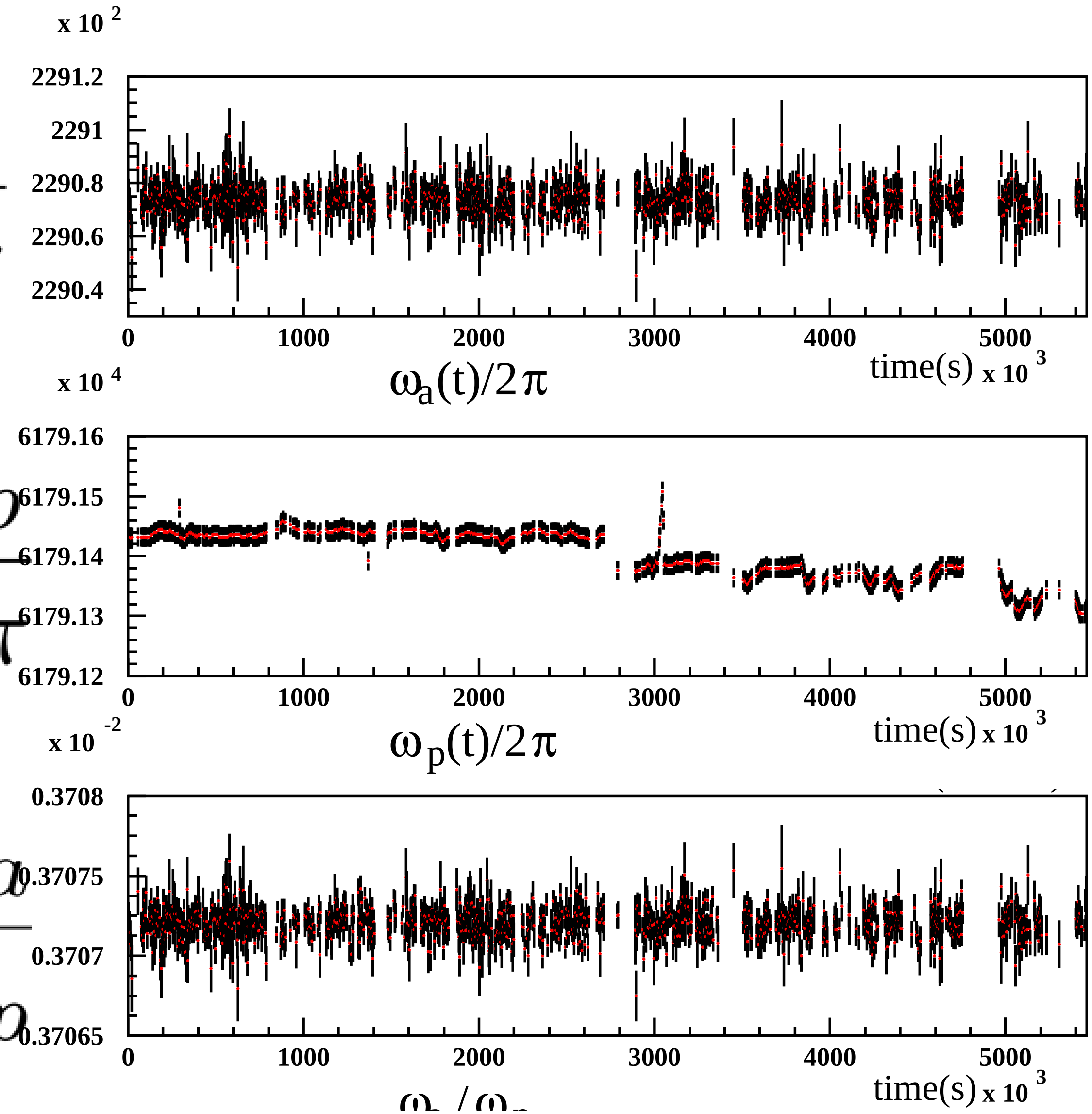
Search for a sidereal period oscillation in ω_a

2001 μ^- data

$$f_a = \frac{\omega_a}{2\pi}$$

$$f_p = \frac{\omega_p}{2\pi}$$

$$\mathcal{R}(t) \equiv \frac{\omega_a}{\omega_p}$$



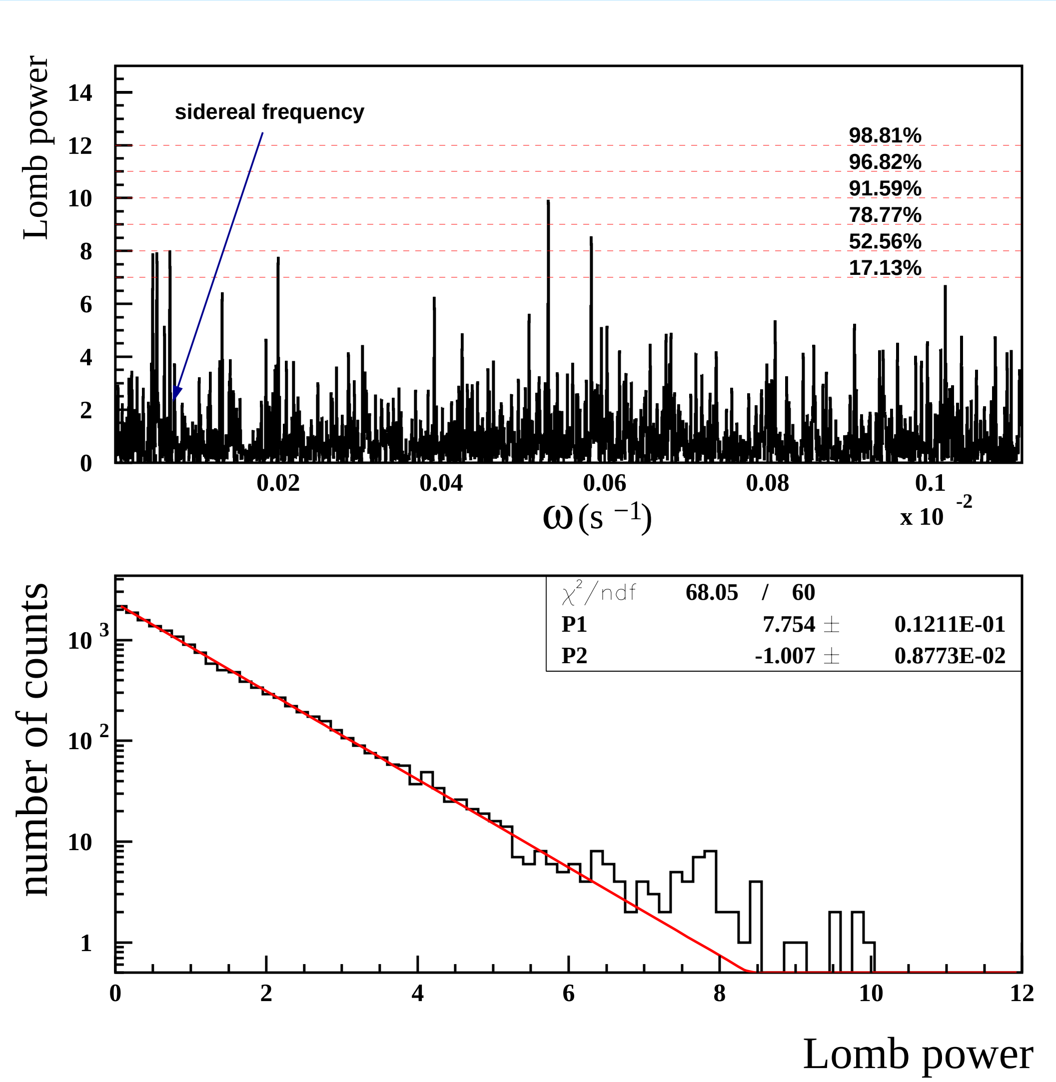
The μ^- data from 2001. Time interval ~ 3 months

Approaches to search for an oscillation signal:

- Multi-parameter fit
  - good for all data
- Fourier Transform
  - only works on equally spaced data
- Lomb-Scargle test
  - designed for unequally spaced data

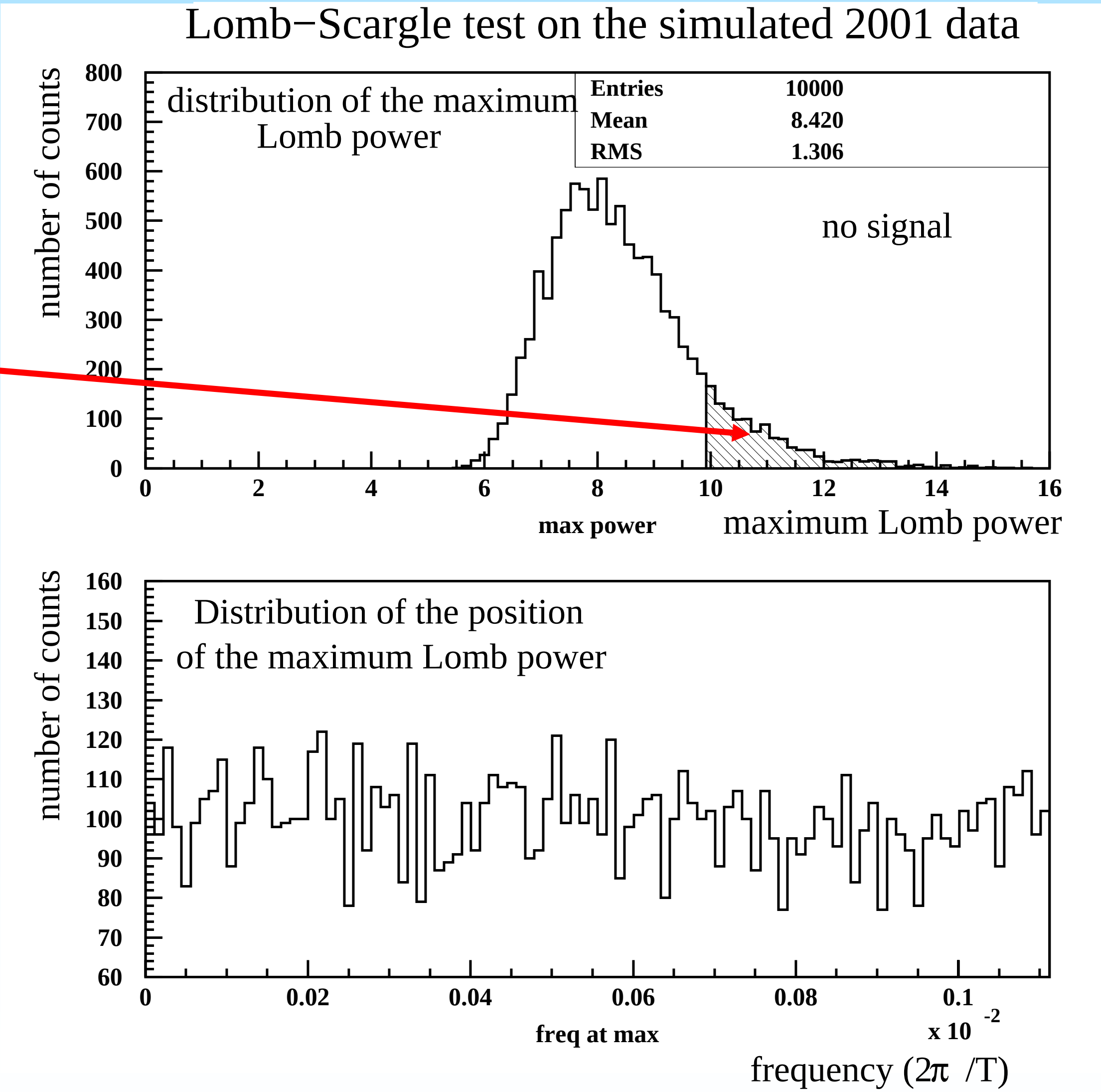
- All gave comparable results.
  - No significant oscillation

Lomb-Scargle Test: reduces to a FT for evenly spaced data.



- The exponential distribution implies that there is no statically significant frequency in the data.

Lomb-Scargle test on simulated data: no signal



fraction with Lomb power greater than that found in the real data

We conclude that no signal lies in the data

These limits translate into 95% CL limits on parameters

$$b_T^{\mu^+} = \sqrt{(\check{b}_X^{\mu^+})^2 + (\check{b}_Y^{\mu^+})^2} \leq 1.4 \times 10^{-24}\ \mathrm{GeV}$$

$$\check{b}_T^{\mu^-} = \sqrt{(\check{b}_X^{\mu^-})^2 + (\check{b}_Y^{\mu^-})^2} \leq 2.6 \times 10^{-24}\ \mathrm{GeV}$$

dividing by m_μ

$$r_{A_\Omega}^{\mu^+} \leq 2 \times 10^{-23} \qquad r_{A_\Omega}^{\mu^-} \leq 3.8 \times 10^{-23}$$

Muonium hyperfine structure $r^{\mu^+} \leq 5 \times 10^{-22}$

electron in a penning trap $r^{e} \leq 1.6 \times 10^{-21}$

note that

$$\frac{m_\mu}{M_P} = 8.7 \times 10^{-21}$$

Other tests using both CERN and E821: χ_1 χ_2

$$\Delta\mathcal{R} = \frac{2b_Z}{\gamma}\left(\frac{\cos\chi_1}{\omega_{p1}} + \frac{\cos\chi_2}{\omega_{p2}}\right)$$

$$+2(m_\mu d_{Z0} + H_{XY})\left(\frac{\cos\chi_1}{\omega_{p1}} - \frac{\cos\chi_2}{\omega_{p2}}\right)$$

$$(m_\mu d_{Z0} + H_{XY}) = (1.6 \pm 5.6 \times 10^{-23})\ \mathrm{GeV}$$

No evidence for CPT/Lorentz violation in the E821 data.

Future Improvements in a_μ?

$\sigma_{\text{stat}} = \pm 0.46$ ppm $\sigma_{\text{syst}} = \pm 0.28$ ppm

- Theory (strong interaction part) will improve.
  - both lowest order, and light-by-light
- We proposed to upgrade E821 at BNL to reduce the total experimental error to 0.2 ppm, (2.5 X better).
  - At present, there is no funding for this upgrade.
- If money were no object, how well could we do?
  - The limit of our technique is between ~0.1 and 0.06 ppm.

The error budget for E969 represents a continuation of improvements already made during E821

Systematic uncertainty (ppm)	1998	1999	2000	2001	E969 Goal
Magnetic field – ω_p	0.5	0.4	0.24	0.17	0.1
Anomalous precession – ω_a	0.8	0.3	0.31	0.21	0.1
Statistical uncertainty (ppm)	4.9	1.3	0.62	0.66	0.2
Total Uncertainty (ppm)	5.0	1.3	0.73	0.72	0.25

- Field improvements: better trolley calibrations, better tracking of the field with time, temperature stability of room, improvements in the hardware
- Precession improvements will involve new scraping scheme, lower thresholds, more complete digitization periods, better energy calibration

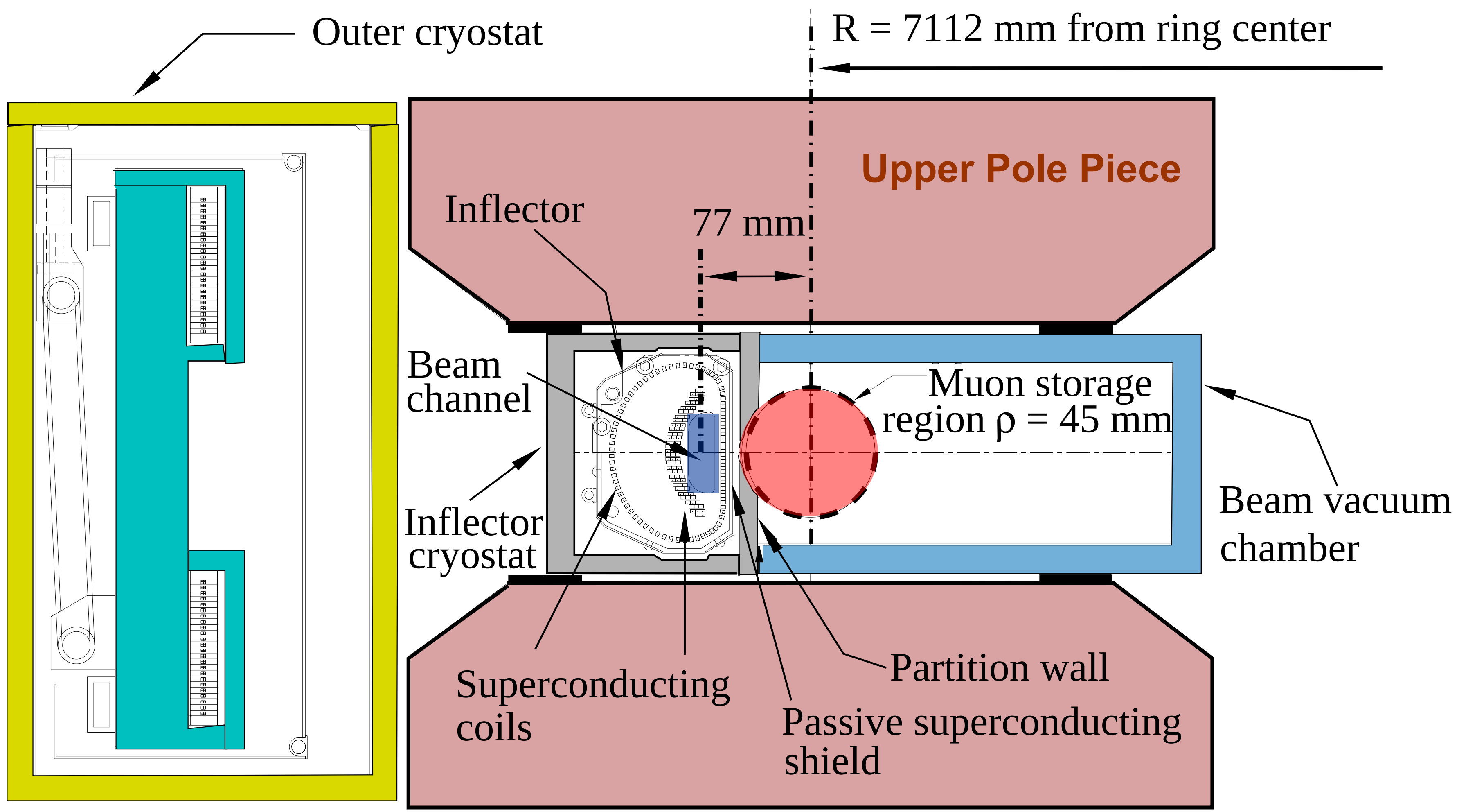
Let's spend a few minutes talking about different possible levels of inprovement.

- E969 aimed for 0.2 ppm overall error
- "Conservative" upgrade could to go 0.25 ppm
- "Legacy" effort could aim for a 0.14 ppm overall error
  - 0.1 ppm systematic and statistical errors.

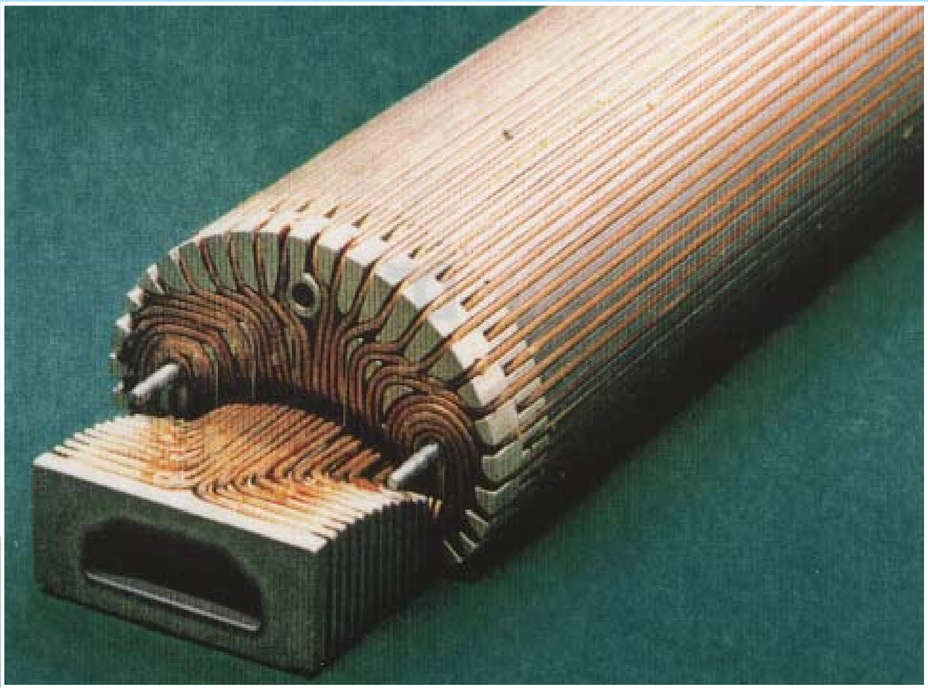
More Muons

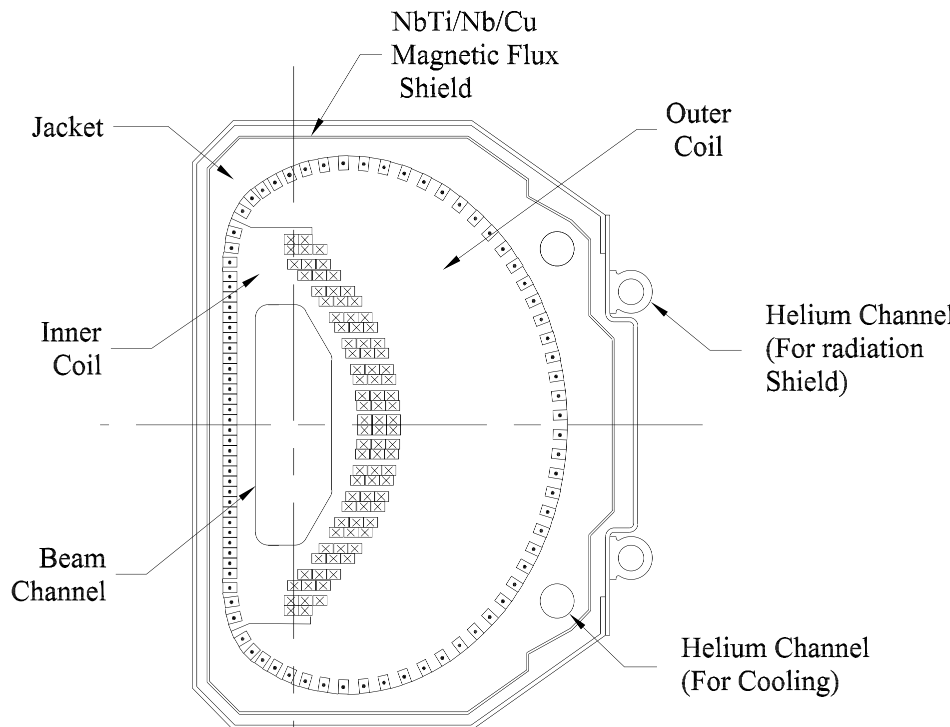
- Improve beamline acceptance X2
- Open inflector opening X2
- New beamline front-end ~X2 ?
- Other tricks ?

Space limitations prevent matching the inflector exit to the storage aperture

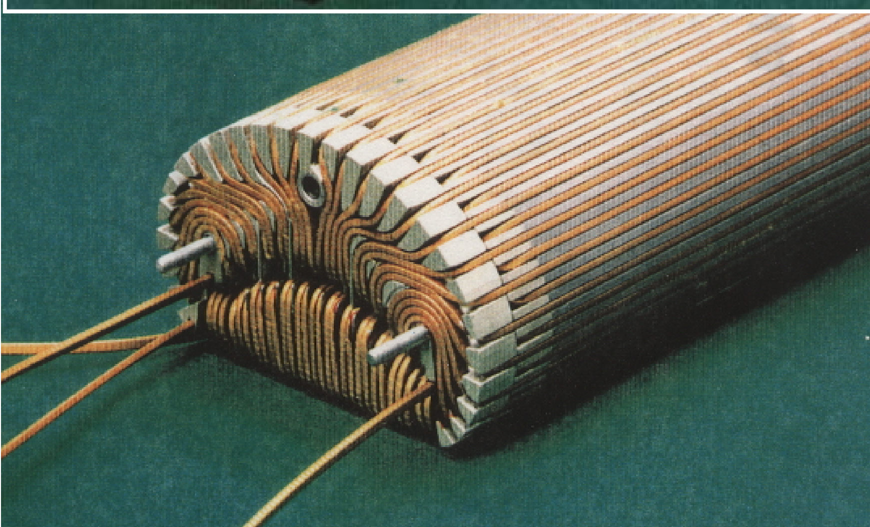


The E821 inflector magnet had closed ends which scattered away half the beam.





Length = 1.7 m; Central field = 1.45T



Open end prototype, built and tested

→X2 Increase in Beam

Instead, a few technical developments toward a next-generation experiment

For E821, a limiting factor was the hadronic flash at injection

(prompt pions, then delayed neutron captures)

Several systematics are affected by this initial pulse

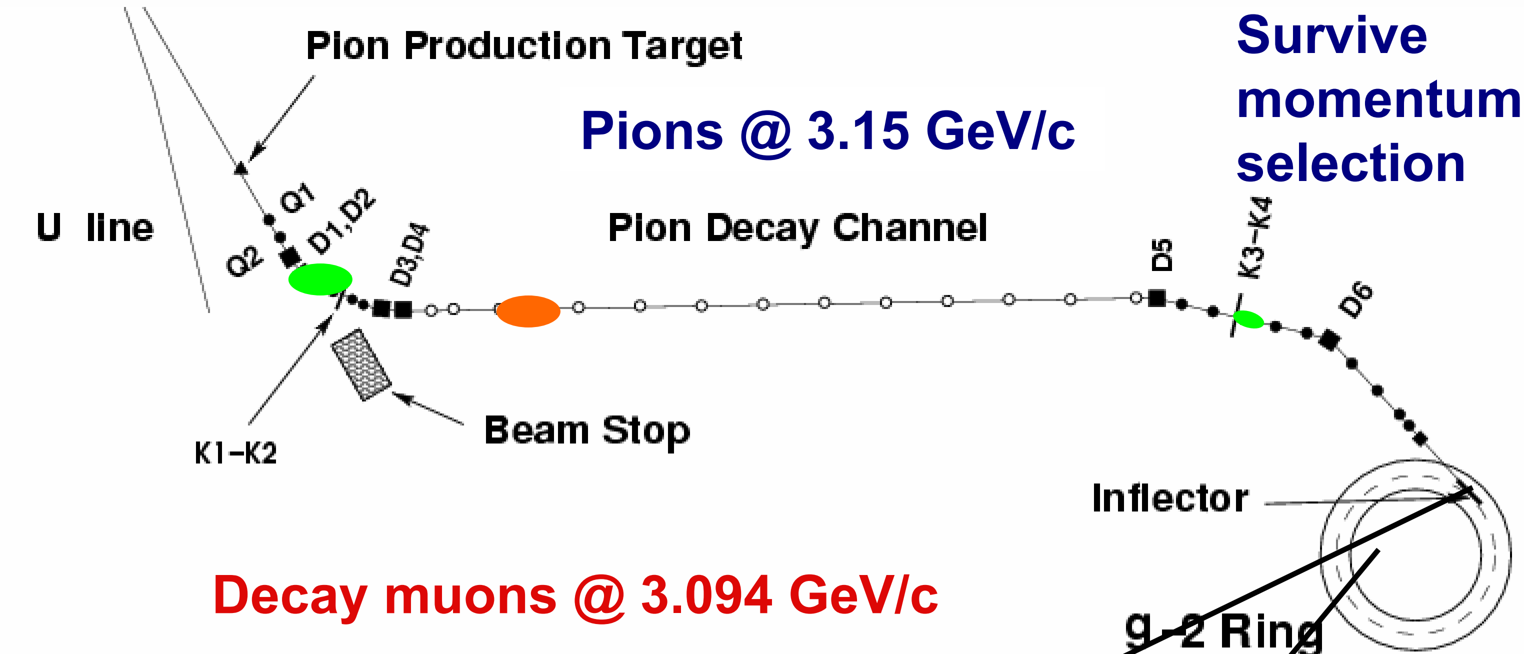
(gain, time stability; pileup extraction, start time of fits)

PMTs had to be switched off and on for every fill

Question 1:

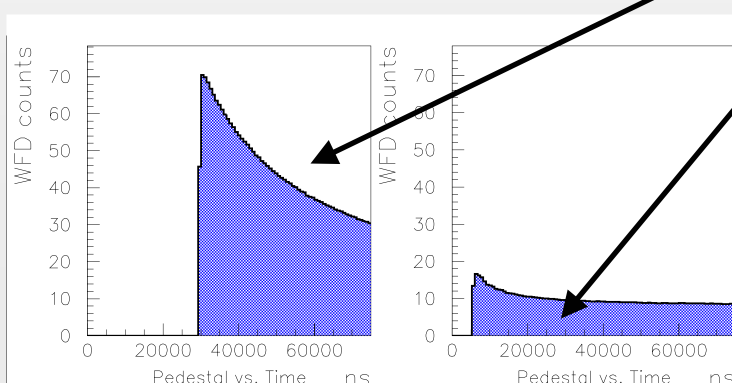
How do we get rid of the pions ?

The current “forward-decay” beam $\pi^- \longrightarrow \mu^- \bar{\nu}_\mu$



The hadronic flash background limits fit start time

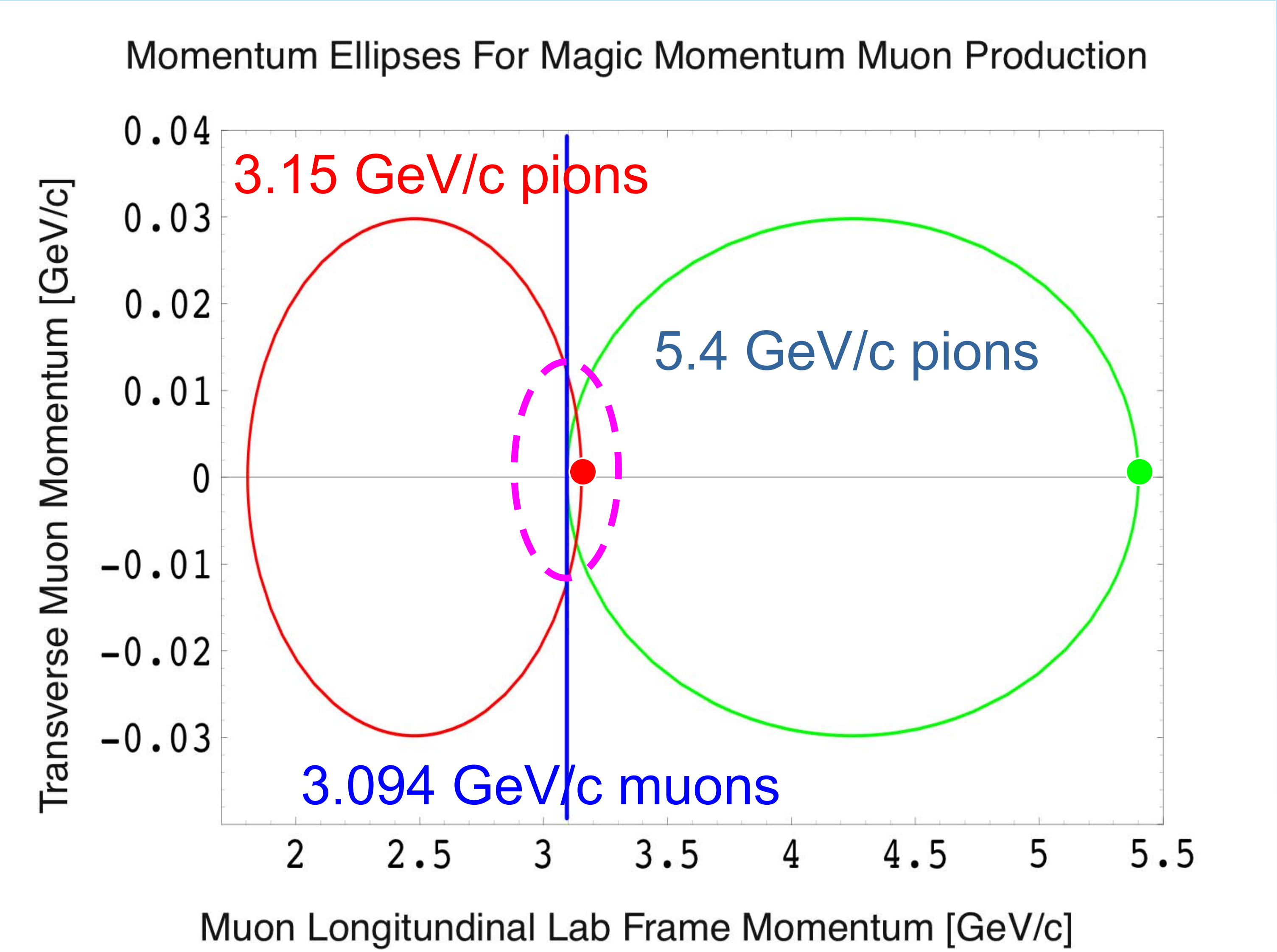
Near inflector



Far side

Pedestal vs. Time

For E969, we considered the idea of backward muon production … the advantages are appealing



Toward a next-generation experiment

E821 Final statistical error was 0.46 ppm

For 0.1 ppm "Legacy" experiment, that's > 20 times the counts

That's hard.

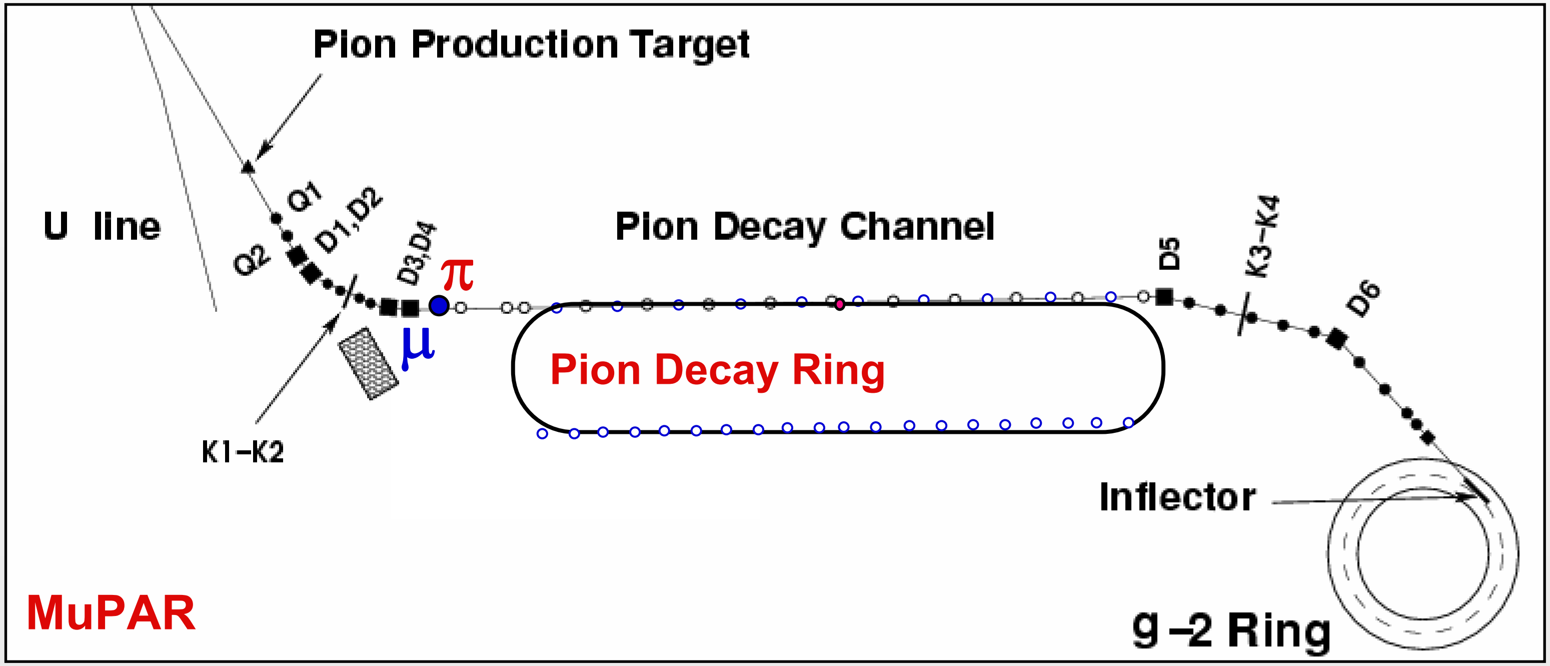
You need a new idea.

Question 2:

Where do the muons come from and how can we get (lots) more of them?

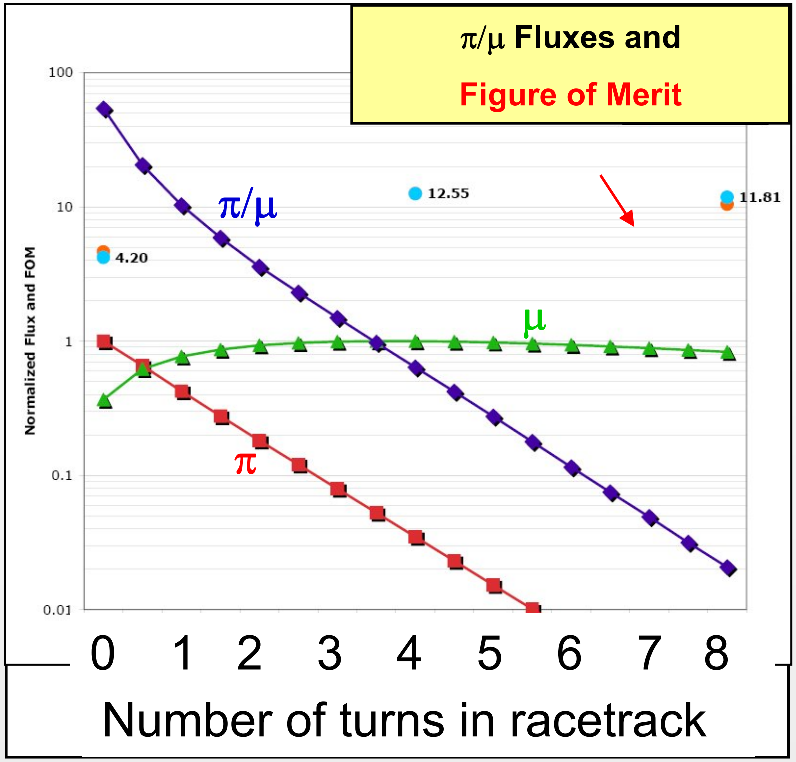
How to get more muons *AND* still avoid the flash

- The recipe is well known and simple:
  1. Take the 0-degree forward muons
     - High polarization, highest yield
  2. Make the beam line so long that all the pions decay away
- But, that's impractical, unless you recirculate



PDR: Pion Decay Ring

- **Catch most muons in first 2 turns.**
  - **Although spin precesses, it's okay**
- **Rest of turns just reduce pions by decay time**
- **Figure of Merit NP^2 increased by factor of ~12 or more**
- **Fast "kicker" magnet required to extract from the ring.**



Summary

- The measurement of e^- and $\mu^\pm$ magnetic dipole moments has been an important benchmark for the development of QED and the standard model of particle physics.
- The muon anomaly has been particularly valuable in restricting physics beyond the standard model, and will continue to do so in the LHC Era
- There appears to be a difference between a_μ and the standard-model prediction at the 3.4 σ level.

Summary

- The measurement of e^- and $\mu^\pm$ magnetic dipole moments has bee[n] ... the development of Q... f particle physics.

Topical Workshop on
The Muon Magnetic Dipole Moment $(g\text{-}2)_\mu$
25 and 26 October 2007
School of Physics and Astronomy
The University of Glasgow

- The muon anomaly ... e in restricting physics beyond the standard model, and will continue to do so in the LHC Era
- There appears to be a difference between a_μ and the standard-model prediction at the 3.4 σ level.
- Much activity continues on the theoretical front.
- The experiment can certainly be improved…

  but the future is uncertain.

Thank you

Summary

- The measurement of e^- and $\mu^\pm$ magnetic dipole moments has been [covered] the development of Q[covered] particle physics.

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- There appears to be a difference between a_μ and the standard-model prediction at the 3.4 σ level.
- Much activity continues on the theoretical front.
- The experiment can certainly be improved... but the future is uncertain.

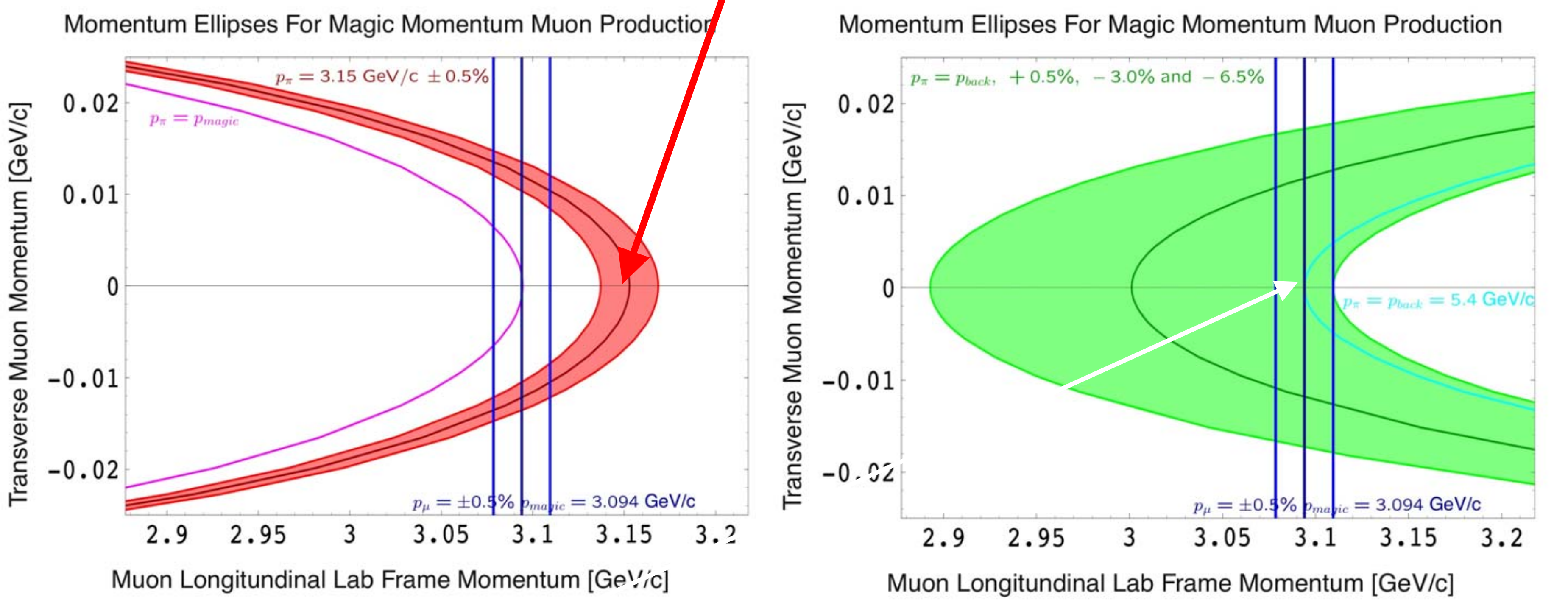
Thank you

THE END

Extra Slides

A wide momentum width, and true 180-degree decays can lead to higher polarization and more muon production … but, the Lorentz boost hurts

We could never work here at 0 degrees because the pions then enter the storage ring and swamp the detectors



But in backward mode, all the pions have very different momentum than the muons, so 180 degrees is okay

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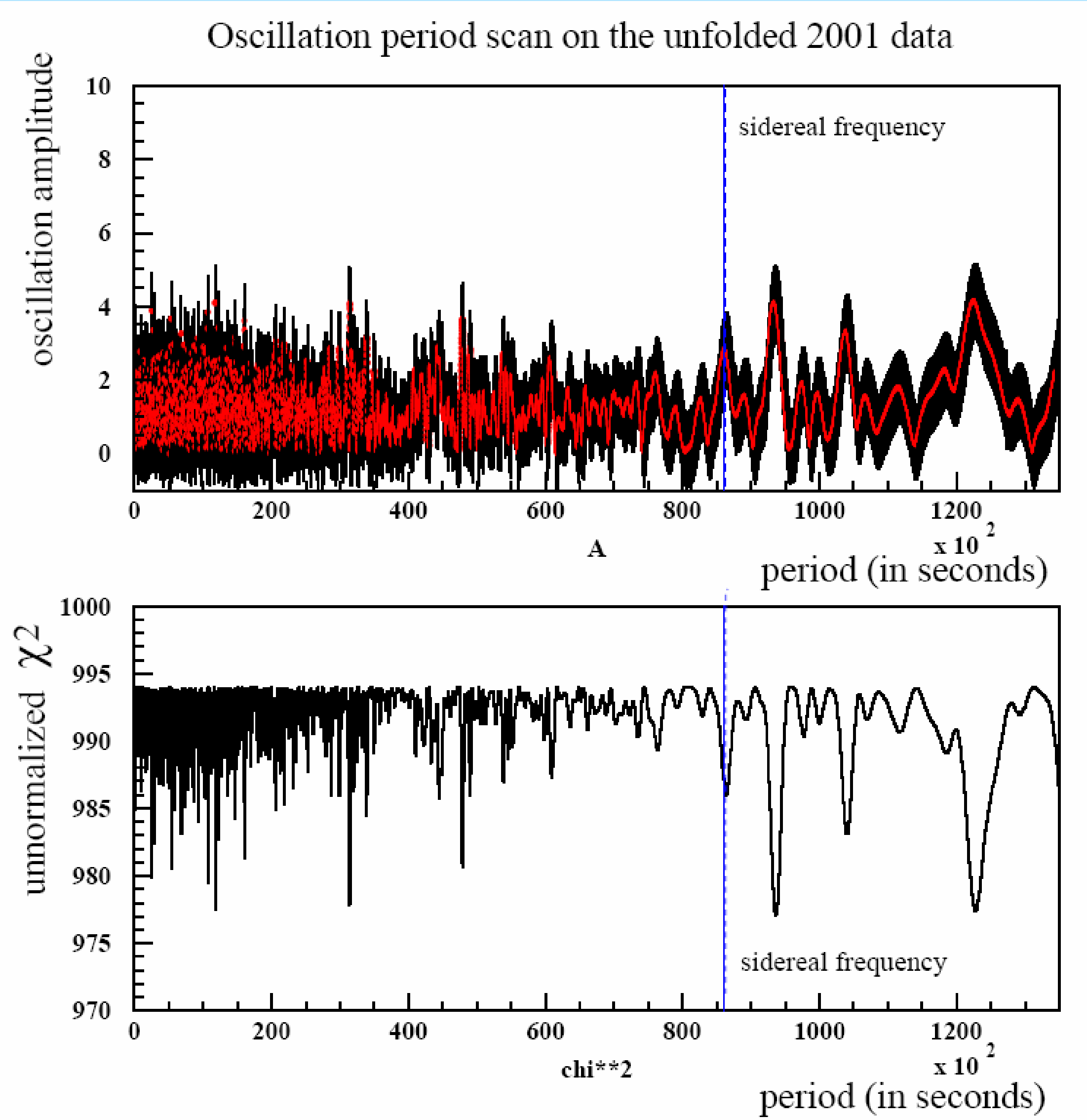
Multi-parameter fit $\mathcal{R} = C + A_\Omega \cos\left(\frac{2\pi}{T_o} + \phi\right)$



Hold frequency fixed, then scan frequency

E821 ω_p systematic errors (ppm)

Source of Uncertainty	1998	1999	2000	2001	
Absolute Calibration	0.05	0.05	0.05	0.05	(I)
Calibration of Trolley	0.3	0.20	0.15	0.09	
Trolley Measurements of B0	0.1	0.10	0.10	0.05	(II)
Interpolation with the fixed probes	0.3	0.15	0.10	0.07	(III)
Inflector fringe field	0.2	0.20	-	-	
uncertainty from muon distribution	0.1	0.12	0.03	0.03	
Other*		0.15	0.10	0.10	(iv)
Total	0.5	0.4	0.24	0.17	

*higher multipoles, trolley voltage and temperature response, kicker eddy currents, and time-varying stray fields.

Systematic errors on ω_a (ppm)

$\sigma_{systematic}$	1999	2000	2001
Pile-up	0.13	0.13	0.08
AGS Background	0.10	0.10	*
Lost Muons	0.10	0.10	0.09
Timing Shifts	0.10	0.02	0.02
E-Field, Pitch	0.08	0.03	*
Fitting/Binning	0.07	0.06	*
CBO	0.05	0.21	0.07
Beam Debunching	0.04	0.04	*
Gain Change	0.02	0.13	0.13
total	0.3	0.31	0.21

$\Sigma^* = 0.11$

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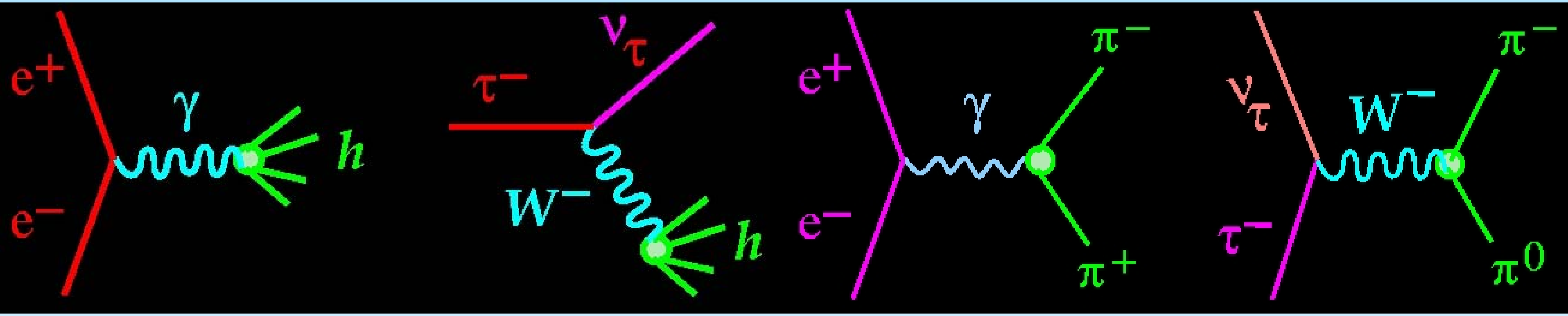
a(had) from hadronic τ decay?

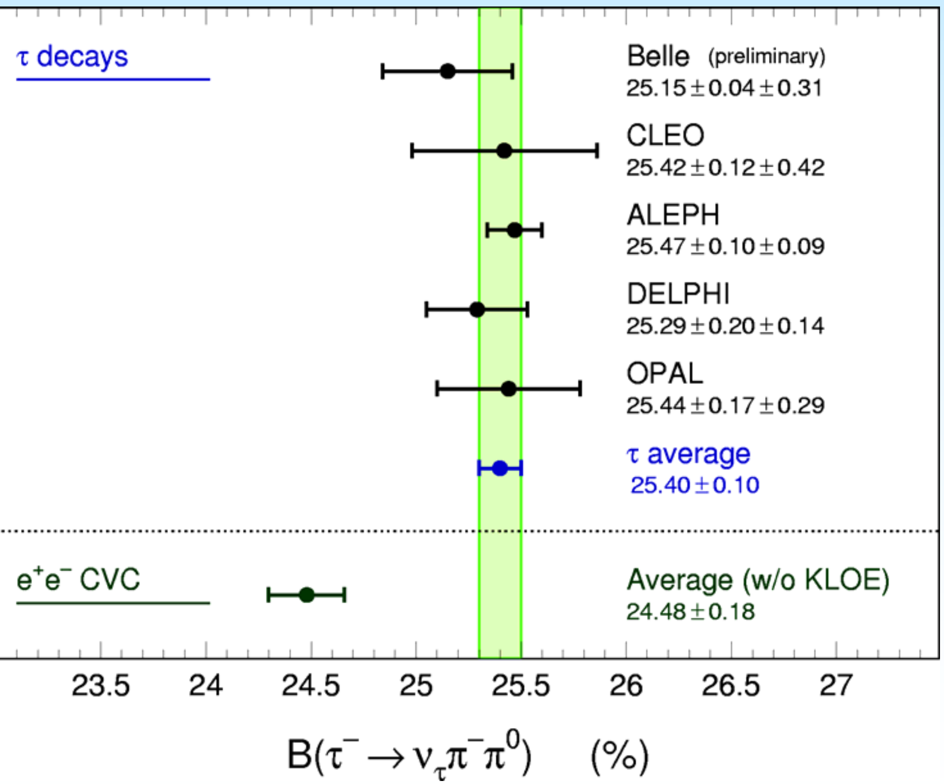


- Assume: CVC, no 2nd-class currents, isospin breaking corrections.
  - e^+e^- goes through neutral ρ
  - while τ-decay goes through charged ρ
- n.b. τ decay has no isoscalar piece, e^+e^- does
- Many inconsistencies in comparison of e^+e^- and τ decay:

Testing CVC with one number

Infer τ branching fractions (more robust than spectral functions) from e^+e^- data:

$$BR_{CVC}(\tau^- \to \pi^-\pi^0\nu_\tau) = \frac{6\pi|V_{ud}|^2 S_{EW}}{m_\pi^2}\int_0^{m_\tau} ds\, \mathrm{kin}(s)\, \nu^{SU(2)-\mathrm{corrected}}(s)$$



Difference: BR[τ] – BR[e^+e^- (CVC)]:

Mode	$\Delta(\tau - e^+e^-)$	`Sigma'
$\tau^- \to \pi^-\pi^0\nu_\tau$	+ 0.92 ± 0.21	4.5
$\tau^- \to \pi^- 3\pi^0\nu_\tau$	– 0.08 ± 0.11	0.7
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$	+ 0.91 ± 0.25	3.6

ee data on $\pi^-\pi^+\pi^0\pi^0$ not satisfactory

from Michel Davier

$\tau^- \to \pi^- \pi^0 \nu_\tau$: preliminary results from BELLE

- preliminary results from BELLE on τ ππ spectral function presented at EPS 2005
- high statistics: see dip at 2.4 GeV² for first time in τ data
- discrepancies with ALEPH/CLEO at large mass and ee data at low mass