

The Particle Physics and Cosmology
of $SU(4)$ Heterotic Vacua
- the B-L MSSM

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Penn HEP 2010

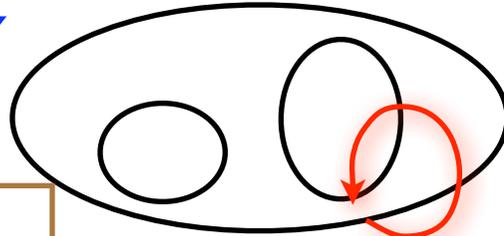
Smooth Heterotic Compactifications

$$D = 10, \quad g_{MN}, \quad A_M^a, \quad E_8$$



$$X, D = 6$$

CY



V,G

“slope” stable

W,F

$$\mathbb{R}^4$$

$N = 1$ SUSY

$$H = [E_8, G]$$

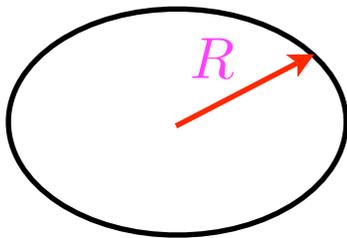
$$\mathcal{H} = [H, F]$$

$$H^1(V)^F \Rightarrow \text{matter}$$

$$H^1(V^*)^F \Rightarrow \text{conjugate matter}$$

$$H^1(\wedge^2 V)^F \Rightarrow \text{Higgs}$$

$$H^1(V \otimes V^*)^F \Rightarrow \text{Bundle Moduli}$$



$$R^{-1} = M_C \simeq 10^{16} \text{ GeV}$$

Heterotic Standard Model

$$V, G = SU(4), W, F = \mathbb{Z}_3 \times \mathbb{Z}_3$$

Braun, He, Ovrut, Pantev 2006
Anderson, Gray, He, Lukas 2009

\mathbb{R}^4 Theory Gauge Group:

Gauge connection $G = SU(4) \Rightarrow E_8 \rightarrow H = Spin(10)$

Wilson line $F = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow$

$$Spin(10) \rightarrow \mathcal{H} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

rank Spin(10)=5 plus F Abelian \Rightarrow extra gauged $U(1)_{B-L}$.

By construction $U(1)_{B-L}$ is anomaly free. Note that

$$\mathbb{Z}_2 (R - \text{parity}) \subset U(1)_{B-L}$$

\Rightarrow no rapid proton decay. But must be spontaneously broken above the scale of weak interactions.

\mathbb{R}^4 Theory Spectrum:

$$E_8 \xrightarrow{V} Spin(10) \Rightarrow$$

$$248 = (1, 45) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 10) \oplus (15, 1)$$

The Spin(10) spectrum is determined from $n_R = h^1(X, U_R(V))$.

$$Spin(10) \xrightarrow{F} SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \Rightarrow$$

The $3 \times 2 \times 1_Y \times 1_{B-L}$ spectrum is determined from

$n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3}$. Tensoring and taking invariant subspace gives **3** families of quarks/leptons each transforming as

$$Q_L = (3, 2, 1, 1), \quad u_R = (\bar{3}, 1, -4, -1), \quad d_R = (\bar{3}, 1, 2, -1)$$

$$L_L = (1, 2, -3, -3), \quad e_R = (1, 1, 6, 3), \quad \nu_R = (1, 1, 0, 3)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. Also, **1** pair of Higgs

$$H = (1, 2, 3, 0), \quad \bar{H} = (1, \bar{2}, -3, 0)$$

That is, we get exactly the matter spectrum of the **MSSM**

with **3 right-handed neutrinos!** In addition, there are

$n_1 = h^1(X, V \times V^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli $\phi = (1, 1, 0, 0)$

Note: In pre-string Spin(10) GUT theories the gauged $U(1)_{B-L}$ is spontaneously broken by adding multiplets, such as **126**, which contain $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlets for which

$$3(B - L) = \pm 2, \pm 4, \dots \Rightarrow U(1)_{B-L} \longrightarrow \mathbb{Z}_2 \text{ (} R\text{-parity)}$$

Not possible in Spin(10) heterotic strings since **45, 16, $\bar{16}$, 10, 1** are the only multiplets in the **248** of E_8 . The only singlets are ν_R

$$3(B - L) = 1 \Rightarrow U(1)_{B-L} \longrightarrow 1$$

\Rightarrow In $SU(4)$ heterotic vacua must break B-L symmetry via

$$\langle \nu_3 \rangle \neq 0 \text{ at low scale}$$

$\mathcal{N}=1$ Supersymmetry Primer

Chiral Superfield: $\Phi \sim (\phi, \psi, F)$ quarks/leptons/Higgs

Vector Superfield: $V \sim (A_\mu, \lambda, D)$ gauge fields

F and D are “auxiliary fields” which \Rightarrow potential energy is

where

$$\mathcal{V} = |F|^2 + \frac{1}{2}D^2$$

$$F = \frac{\partial W}{\partial \phi}, \quad W \text{ holomorphic "superpotential"}$$

$$D = g\phi^\dagger T\phi$$

Supersymmetric Interactions:

The most general superpotential is

$$W = \sum_{i=1}^3 (\lambda_{u,i} Q_i H u_i + \lambda_{d,i} Q_i \bar{H} d_i + \lambda_{\nu,i} L_i H \nu_i + \lambda_{e,i} L_i \bar{H} e_i) + \mu H \bar{H}$$

Note B-L symmetry forbids dangerous B and L violating terms

$$LLe, LQd, udd$$

Supersymmetry Breaking, the Renormalization Group and the LHC

Soft Supersymmetry Breaking:

Ambroso, Ovrut 2009

N=1 Supersymmetry is spontaneously broken by the moduli during compactification \Rightarrow soft supersymmetry breaking interactions. The relevant ones are

$$V_{2s} = m_{\nu_3}^2 |\nu_3|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 - (BH\bar{H} + hc) + \dots$$

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \dots$$

At the compactification scale $M_C \simeq 10^{16} GeV$ these parameters are fixed by the vacuum values of the moduli. For example

$$m_{\nu_3}^2 = m_{\nu_3}^2 (\langle \phi \rangle)$$

Can one compute $\langle \phi \rangle$?

As of yet no complete theory of moduli stabilization. However

* New theory to stabilize all geometric moduli Anderson, Gray, 2010
Lukas, Ovrut

At a lower scale μ measured by $t = \ln\left(\frac{\mu}{M_C}\right)$ parameters change under the renormalization group. For example

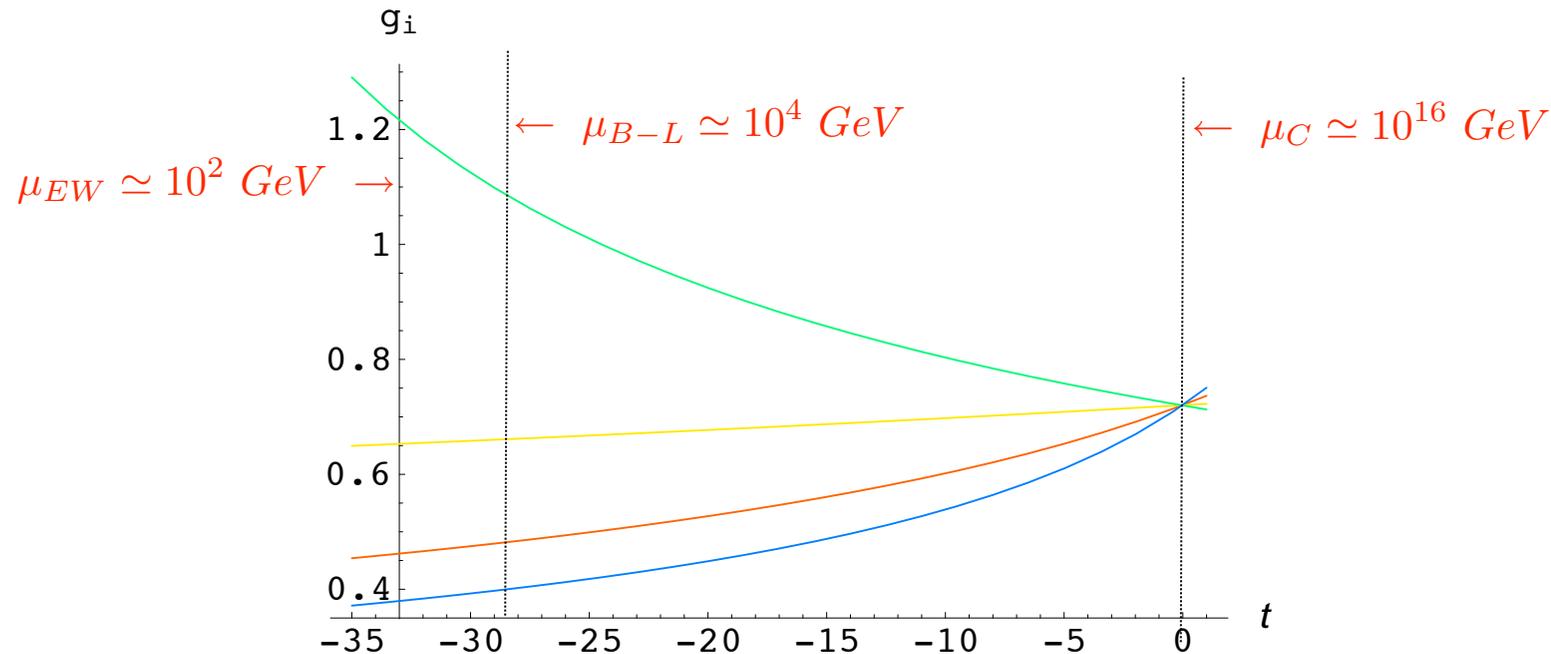


Figure 1: This plot shows the running of the gauge couplings g_1 (red), g_2 (yellow), g_3 (green) and g_4 (blue) and their subsequent unification at 2×10^{16} GeV. For this plot, $t = \text{Ln}(\mu/(2.2 \times 10^{16}))$.

The renormalization group equation for m_{ν_3} is

$$16\pi^2 \frac{dm_{\nu_3}^2}{dt} \simeq \frac{3}{4} g_4^2 \sum_{i=1}^3 (m_{\nu_i}^2 + \dots)$$

Solving the RGEs assuming

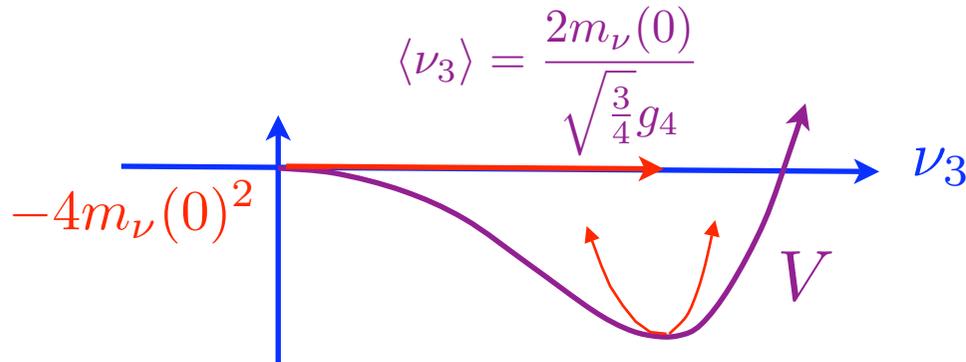
$$m_H(0)^2 = m_{\bar{H}}(0)^2, \quad m_{Q_i}(0)^2 = m_{u_j}(0)^2 = m_{d_k}(0)^2$$

$$m_{L_i}(0)^2 = m_{e_j}(0)^2 \neq m_{\nu_k}(0)^2$$

\Rightarrow at scale $\mu \simeq 10^4 \text{ GeV} \Rightarrow t_{B-L} \simeq -28.7$

$$m_{\nu_3}(t_{B-L})^2 = m_{\nu}(0)^2 - 5m_{\nu}(0)^2 = -4m_{\nu}(0)^2$$

\Rightarrow



The gauged $U(1)_{B-L}$ is **spontaneously broken** by the third family right-handed sneutrino.

⇒ a **B-L vector boson mass** of

$$M_{A_{B-L}} = 2\sqrt{2}m_\nu(0)$$

At this scale, **no** other symmetry is broken.

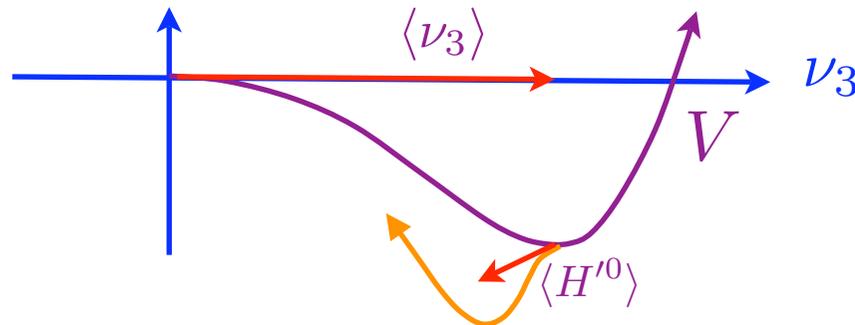
Similarly, at the electroweak scale $\mu \simeq 10^2 \text{ GeV} \Rightarrow t_{EW} \simeq -33.2$

$$m_{H'}(t_{EW})^2 \simeq -\frac{\Delta^2}{\tan\beta^2} m_H(0)^2, \quad m_{\bar{H}'}(t_{EW})^2 \simeq m_H(0)^2$$

where $\tan\beta = \frac{\langle H \rangle}{\langle \bar{H} \rangle}$ and $0 < \Delta^2 < 1$ is related to $M_3(0)$. ⇒ at t_{EW} electroweak symmetry is broken by the expectation value

$$\langle H'^0 \rangle = \frac{2\Delta m_H(0)}{\tan\beta \sqrt{\frac{3}{5}g_1^2 + g_2^2}}$$

⇒ a **Z-boson mass** of $M_Z = \frac{\sqrt{2}\Delta m_H(0)}{\tan\beta} \simeq 91 \text{ GeV}$



Search for Neutral MSSM
Higgs Bosons at LEP

ALEPH, DELPHI, L3 and
OPAL Collaborations
The LEP Working Group
for Higgs Boson Searches

hep-ex/0602042

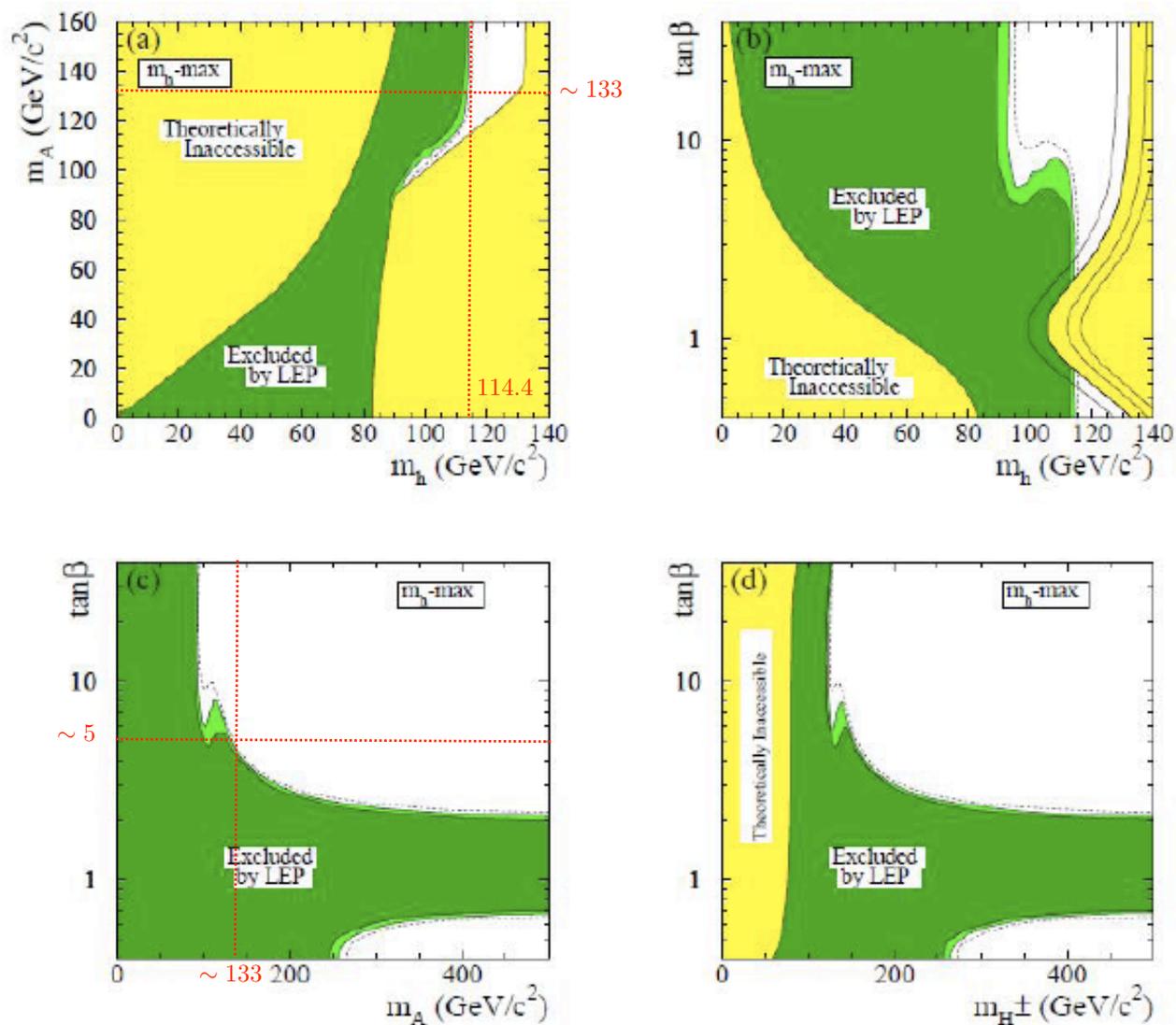


Figure 14: Exclusions at the 95% confidence level (medium-grey or light-green) and at the 99.7% confidence level (dark-grey or dark-green) for the CP conserving, $m_{h^0} - \max$ scenario and for $m_t = 174.3$ GeV/c^2 . The figure shows the theoretically inaccessible domains (light-grey or yellow) and the regions excluded in this search, in four projections of the MSSM parameters: (a): (m_{h^0}, m_{A^0}) ; (b): $(m_{h^0}, \tan \beta)$; (c): $(m_{A^0}, \tan \beta)$; (d): $(m_{H^\pm}, \tan \beta)$. The dashed lines indicate the boundaries of the region which are expected to be excluded at 95% confidence level based on Monte Carlo simulations with no signal. In the $(m_{h^0}, \tan \beta)$ projection (plot (b)), the upper boundary of the parameter space is indicated for four values of the top quark mass; from left to right: $m_t = 169.3, 174.3, 179.3,$ and 183.0 GeV/c^2 .

the stop mixing
parameter is set to a
large value

All superpartner masses are related through intertwined renormalization group equations. \Rightarrow Inputting some initial parameters allows one to **predict all sparticle and Higgs masses!**

For a **representative choice** of initial parameters $c_q(0)$, $c_{\nu_3}(0)$ the $\mu, \tan\beta$ plane can be scanned for the regime consistent with data.

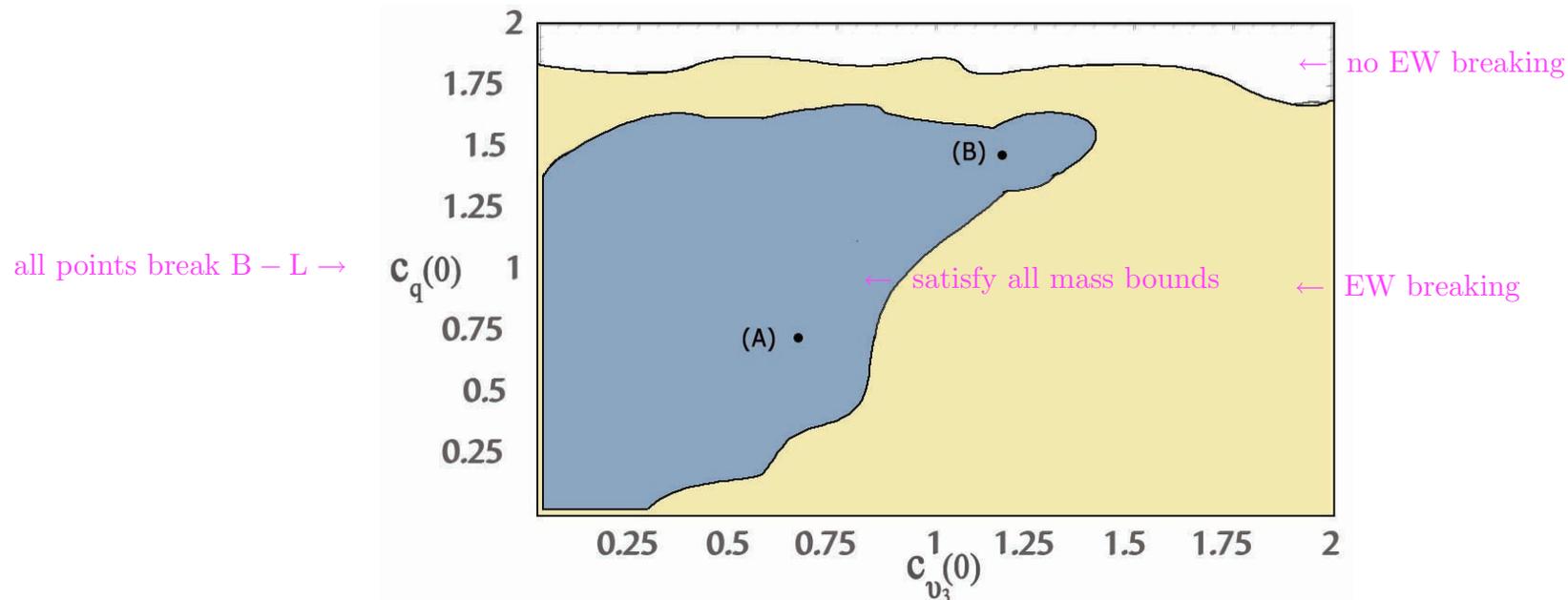
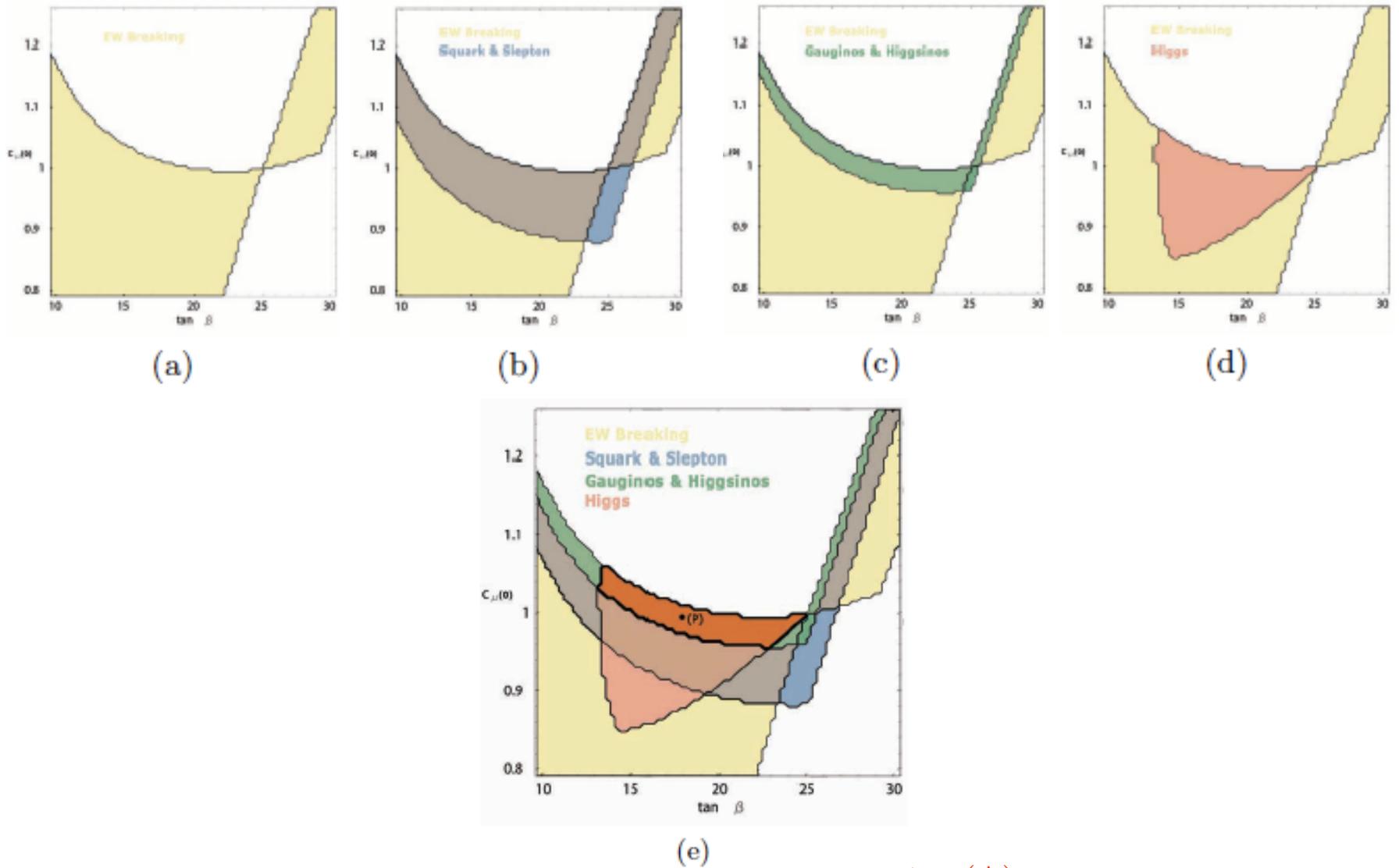


Figure 2: A plot of the $c_q(0)$ - $c_{\nu_3}(0)$ plane showing physically relevant areas. The yellow and white indicate points whose corresponding $c_\mu(0)$ - $\tan\beta$ plane does and does not contain a region of electroweak symmetry breaking respectively. Within the yellow area, the blue shading contains all points whose $c_\mu(0)$ - $\tan\beta$ plane has a non-vanishing region satisfying all experimental sparticle and Higgs bounds and for which *all soft susy breaking masses remain positive over the entire scaling range*. (A) and (B) indicate the two points analyzed in detail in the text.



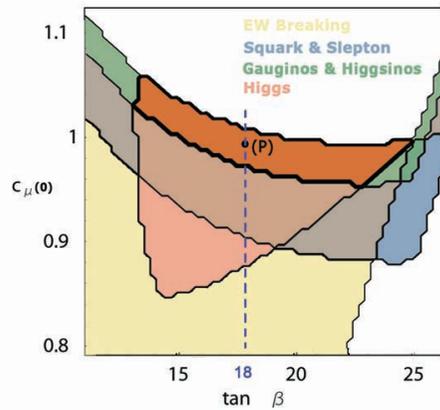
point (A)

Figure 1: The $c_\mu(0)$ - $\tan \beta$ plane corresponding to the point $c_q(0) = 0.75$, $c_{\nu_3}(0) = 0.75$. The yellow and white regions of (a) indicate where electroweak symmetry is and is not broken respectively. The individual regions satisfying the present experimental bounds for squarks and sleptons, gauginos and Higgs fields are shown in (b),(c) and (d), while their intersection is presented in (e). The dark brown area of (e) is the phenomenologically allowed region where electroweak symmetry is broken and all experimental mass bounds are satisfied. We present our predictions for the sparticle and Higgs masses at point (P).

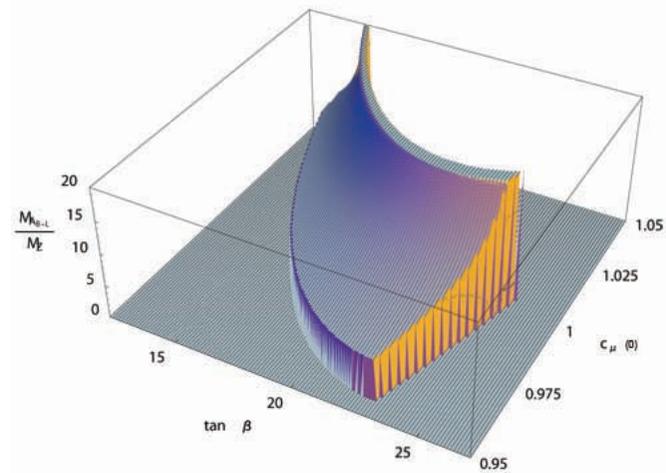
The **predicted** sparticle and Higgs masses at point (P) are

Particle	Symbol	Mass [GeV]	Particle	Symbol	Mass [GeV]
Squarks	$\tilde{Q}_{1,2}$	1080	Higgs	h^0	132
	$\tilde{t}_{1,2}, \tilde{b}_{1,2}$	1012, 1140		H^0	473
	$\tilde{b}_3^{(1)}, \tilde{b}_3^{(2)}$	884, 1055		A^0	472
	$\tilde{t}_3^{(1)}, \tilde{t}_3^{(2)}$	665, 929		H^\pm	479
Sleptons	$\tilde{L}_{1,2}$	1216	Neutralinos	\tilde{N}_1^0	147
	$\tilde{\tau}_{1,2}$	1185		\tilde{N}_2^0	286
	$\tilde{\tau}_3^{(1)}, \tilde{\tau}_3^{(2)}$	1141, 1197		\tilde{N}_3^0	523
Charginos	$\tilde{\chi}^\pm, \tilde{\chi}'^\pm$	286, 537		\tilde{N}_4^0	536
Gluinos	\tilde{g}	1074	Z'	A_{B-L}, \tilde{A}_{B-L}	1252, 1302

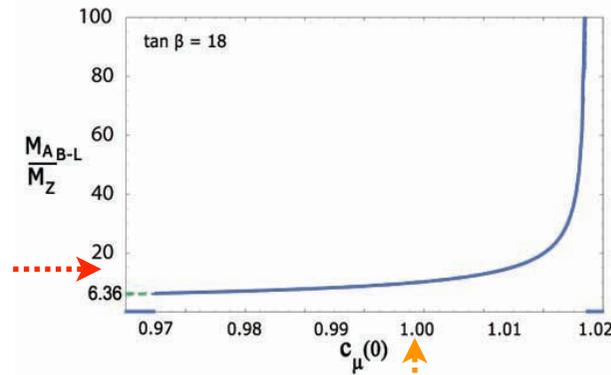
Table 2: The predicted spectrum at point (P) in Figure 1(e). The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The considerable mixing between the third family left- and right-handed scalar fields is incorporated into these results.



(a)



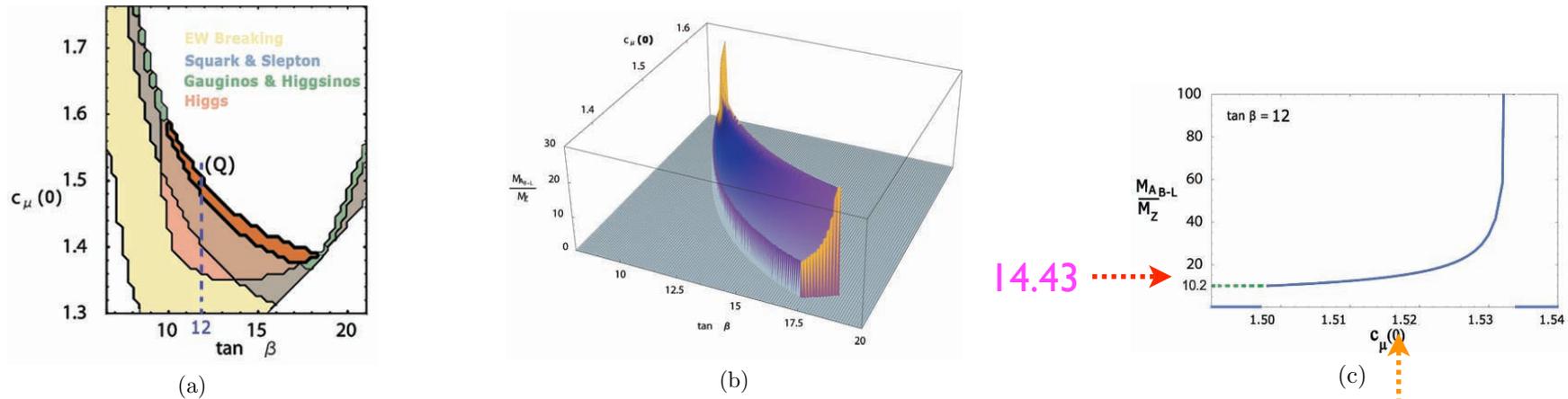
(b)



(c)

Figure 4: Plot (a) shows the $c_\mu(0)$ - $\tan\beta$ plane corresponding to point (A) in Figure 2 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (P) was presented in Table 2. A plot of the hierarchy M_{B-L}/M_Z over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_\mu(0)$ along the $\tan\beta = 18$ line passing through (P).

- $M_{A-B-L}/M_Z = 1252/91 = 13.76$ hierarchy at (P)



(a) Figure 5: Plot (a) shows the $c_\mu(0)$ - $\tan\beta$ plane corresponding to point (B) in Figure 2 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (Q) was presented in Table 3. A plot of the hierarchy M_{B-L}/M_Z over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_\mu(0)$ along the $\tan\beta = 12$ line passing through (Q).

Particle	Symbol	Mass [GeV]	Particle	Symbol	Mass [GeV]
Squarks	$\tilde{Q}_{1,2}$	850	Higgs	h^0	127
	$\tilde{t}_{1,2}, \tilde{b}_{1,2}$	775, 953		H^0	382
	$\tilde{b}_3^{(1)}, \tilde{b}_3^{(2)}$	670, 915		A^0	381
	$\tilde{t}_3^{(1)}, \tilde{t}_3^{(2)}$	456, 737		H^\pm	390
Sleptons	$\tilde{L}_{1,2}$	1255	Neutralinos	\tilde{N}_1^0	97
	$\tilde{\tau}_{1,2}$	1237		\tilde{N}_2^0	189
	$\tilde{\tau}_3^{(1)}, \tilde{\tau}_3^{(2)}$	1217, 1246		\tilde{N}_3^0	499
Charginos	$\tilde{\chi}^\pm, \tilde{\chi}'^\pm$	190, 510		\tilde{N}_4^0	509
Gluinos	\tilde{g}	712	Z'	A_{B-L}, \tilde{A}_{B-L}	1314, 1348

Table 3: The predicted spectrum at point (Q) in Figure 3. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.

Some $\langle \nu_3 \rangle \neq 0$ Phenomenology:

Ambroso, Ovrut 2009/2010

Barger, Perez, Spinner

(A) Proton Decay

Expanding $\nu_3 = \langle \nu_3 \rangle + \nu'_3$ and diagonalizing the kinetic energy \Rightarrow the superpotential becomes

$$\mathcal{W} = W + \epsilon_3 \sum_{i=1}^3 \lambda_{e,i} L_3 L_i e_i + \epsilon_3 \sum_{i=1}^3 \lambda_{d,i} L_3 Q_i d_i + 0 u_i d_j d_k$$

lepton number violating

baryon number violating

where $\epsilon_3 = \lambda_{\nu_3} \frac{\langle \nu_3 \rangle}{\mu}$. The baryon number violating operators can only arise from B-L invariant higher-dimension terms

$$\frac{1}{M_C} \gamma_{ijk} \nu_3 u_i d_j d_k, \quad \gamma_{ijk} = -\gamma_{ikj}$$

When $\langle \nu_3 \rangle \neq 0$ these induce

$$\lambda''_{ijk} u_i d_j d_k, \quad \lambda''_{ijk} = \gamma_{ijk} \frac{\langle \nu_3 \rangle}{M_C}$$

Nucleon decay is generated from

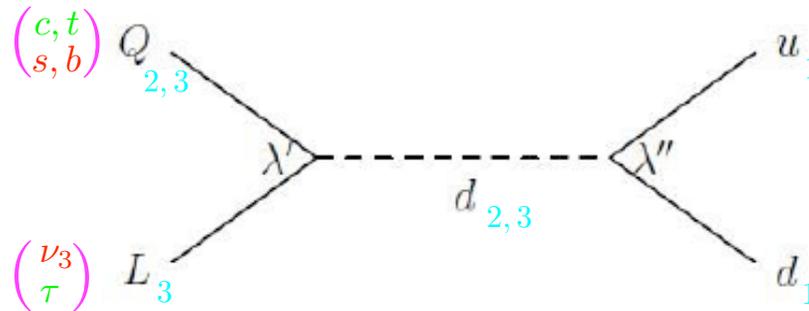


Figure 13: Effective operators generated by the dimension 4 interactions $\lambda'_{ijk} L_i Q_j d_k$ and $\lambda''_{ijk} u_i d_j d_k$. When the external fields are light families, these graphs induce nucleon decay. The solid lines represent fermions while the dashed line represents scalar propagators.

For **proton** decay, the relevant baryon violating operators are

$$\lambda''_{11k} u_1 d_1 d_k, \quad k = 2, 3$$

For the B-L MSSM with $\langle \nu_3 \rangle \neq 0$ the relevant lepton violating terms are

$$\epsilon_3 \lambda_{d,k} L_3 Q_k d_k, \quad k = 2, 3$$

Since the masses of τ^+ and B^+ exceed the proton mass, the only possible decay channel is

$$p \longrightarrow K^+ + \bar{\nu}_3$$

The product of couplings inducing this decay is

$$\lambda' \lambda'' = \epsilon_3 \lambda_{d,2} \gamma_{112} \frac{\langle \nu_3 \rangle}{M_C}$$

This decay will be suppressed below observed bound if

$$\lambda' \lambda'' < \mathcal{O}(10^{-25})$$

For example, evaluating parameters at the point (P) discussed above and taking $\gamma_{112} \sim \mathcal{O}(1)$ gives

$$\lambda' \lambda'' < 6.89 \times 10^{-16} \lambda_{\nu_3} \Rightarrow \lambda_{\nu_3} \lesssim 10^{-10}$$

- Sufficiently suppresses proton decay

Also

- Consistent with baryogenesis and gravitino dark matter

(B) Neutrino Masses

To compute the neutrino masses, must consider the full neutralino mass matrix. In the basis $(\psi_{N_3}, \psi_{\nu_3}, \lambda_{B-L}, \lambda_Y, \lambda_{W^0}, \tilde{H}^0, \tilde{H}^0)$ this is

$$\psi_{\nu_3}, \lambda_{B-L} \text{ mixing} \rightarrow \begin{pmatrix} 0 & \lambda_{\nu_3} \langle H \rangle & 0 & 0 & 0 & 0 & \lambda_{\nu_3} \langle \nu_3 \rangle \\ \lambda_{\nu_3} \langle H \rangle & 0 & \sqrt{2} g_{B-L} \langle \nu_3 \rangle & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} g_{B-L} \langle \nu_3 \rangle & M_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_1 & 0 & -\frac{g_Y \langle \tilde{H} \rangle}{\sqrt{2}} & \frac{g_Y \langle H \rangle}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & M_2 & \frac{g_2 \langle H \rangle}{\sqrt{2}} & -\frac{g_2 \langle H \rangle}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{g_Y \langle \tilde{H} \rangle}{\sqrt{2}} & \frac{g_2 \langle \tilde{H} \rangle}{\sqrt{2}} & 0 & -\mu \\ \lambda_{\nu_3} \langle \nu_3 \rangle & 0 & 0 & \frac{g_Y \langle H \rangle}{\sqrt{2}} & -\frac{g_2 \langle H \rangle}{\sqrt{2}} & -\mu & 0 \end{pmatrix} \leftarrow \text{MSSM}$$

Note that terms proportional to the left-handed sneutrino VEV have been dropped since $\langle N_3 \rangle \ll \lambda_{\nu_3} \langle \nu_3 \rangle$. In the limit $\lambda_{\nu_3} \rightarrow 0$, ψ_{N_3} has zero mass and $\psi_{\nu_3}, \lambda_{B-L}$ have diagonal masses

$$m_{\psi_{\nu_3}} = M_{AB-L} \left(1 - \frac{M_4}{2\sqrt{2}g_{B-L}\langle \nu_3 \rangle}\right), m_{\lambda'_{B-L}} = M_{AB-L} \left(1 + \frac{M_4}{2\sqrt{2}g_{B-L}\langle \nu_3 \rangle}\right)$$

Note, in the $M_4 \rightarrow 0$ supersymmetric limit

$$m_{\psi_{\nu_3}} = m_{\lambda'_{B-L}} = M_{AB-L}$$

That is, the right-handed neutrino gets a Majorana mass

$$m_{\psi'_{\nu_3}} \simeq \mathcal{O}(M_{AB-L})$$

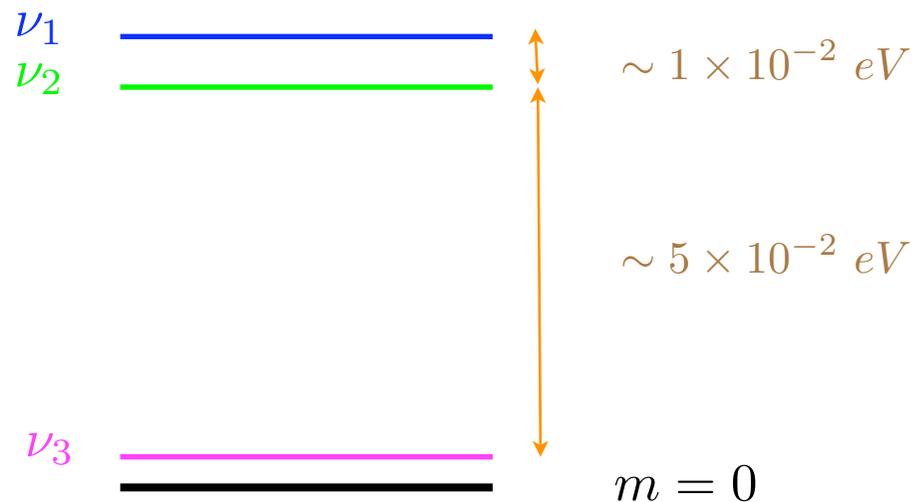
Now turning on $\lambda_{\nu_3} \neq 0 \Rightarrow$ left-handed neutrino mass

$$m_{\psi_{N'_3}} \simeq \frac{(\lambda_{\nu_3} \langle \nu_3 \rangle)^2}{m_{\bar{N}^0}}$$

Evaluating at point (P) above and taking $\lambda_{\nu_3} \lesssim 10^{-10}$ to satisfy proton decay bound \Rightarrow

$$m_{\psi'_{N_3}} \lesssim 7.2 \times 10^{-7} \text{ eV}$$

This predicts an “inverted” neutrino hierarchy



Some $\langle \nu_3 \rangle \neq 0$ Cosmology:

Ambroso, Ovrut 2009/2010

Gravitino dark matter in R-parity breaking vacua, W. Buchmuller, L. Covi, K. Hamaguchi, A. Ibarra and T. Yanagida, JHEP 0703, 037 (2007) [arXiv:hep-ph/0702184].

Baryon Asymmetry and Dark Matter

Primordial baryon asymmetry **not** erased before EW transition requires small lepton number violating terms \Rightarrow

$$\left(\frac{\epsilon_3}{10^{-6}} \right) \left(\frac{\tan\beta}{10} \right) \lesssim 1$$

where $\epsilon_3 = \lambda_{\nu_3} \frac{\langle \nu_3 \rangle}{\mu}$. Evaluated, for example, at **(P)** with $\lambda_{\nu_3} \lesssim 10^{-10}$ gives

$$\left(\frac{\epsilon_3}{10^{-6}} \right) \left(\frac{\tan\beta}{10} \right) \lesssim 0.933 \times 10^{-3} \ll 1$$

Note that a neutralino is the **lightest sparticle** in the B-L MSSM.

Assuming the **gravitino is the LSP** and the **neutralino the NLSP** \Rightarrow

gravitino lifetime:

$$\tau_{3/2} \simeq 10^{28} s \left(\frac{\epsilon_3}{10^{-7}} \right)^{-2} \left(\frac{\tan \beta}{10} \right)^{-2} \left(\frac{m_{3/2}}{10 \text{ GeV}} \right)^{-3}$$

neutralino lifetime:

$$\tau_{NLSP} \simeq 10^{-9} s \left(\frac{\epsilon_3}{10^{-7}} \right)^{-2} \left(\frac{\tan \beta}{10} \right)^{-2} \left(\frac{m_{\tilde{N}}}{200 \text{ GeV}} \right)^{-3}$$

Taking $m_{3/2} \simeq 10^2 \text{ GeV}$, the lifetimes evaluated at (P) are

$$\tau_{3/2} \simeq 3.45 \times 10^{28} s \quad , \quad \tau_{NLSP} \simeq 1.77 \times 10^{-6} s$$

The estimated lifetime of the universe is

$$\tau_{\text{universe}} \sim 13.7 \text{ billion years} \simeq 4.32 \times 10^{17} s$$

It follows that in the B-L MSSM

$$\tau_{NLSP} \ll \tau_{\text{universe}}$$

and

$$\tau_{3/2} \gg \tau_{\text{universe}} \Rightarrow \text{gravitino dark matter}$$