

PROOF OF BAYES THEOREM

The probability of two events A and B happening, $P(A \cap B)$, is the probability of A, $P(A)$, times the probability of B given that A has occurred, $P(B|A)$.

$$P(A \cap B) = P(A)P(B|A) \quad (1)$$

On the other hand, the probability of A and B is also equal to the probability of B times the probability of A given B.

$$P(A \cap B) = P(B)P(A|B) \quad (2)$$

Equating the two yields:

$$P(B)P(A|B) = P(A)P(B|A) \quad (3)$$

and thus

$$P(A|B) = P(A) \frac{P(B|A)}{P(B)} \quad (4)$$

This equation, known as Bayes Theorem is the basis of statistical inference.