Proof of Bayes Theorem

The probability of two events A and B happening, \( P(A \cap B) \), is the probability of A, \( P(A) \), times the probability of B given that A has occurred, \( P(B|A) \).

\[
P(A \cap B) = P(A)P(B|A) \tag{1}
\]

On the other hand, the probability of A and B is also equal to the probability of B times the probability of A given B.

\[
P(A \cap B) = P(B)P(A|B) \tag{2}
\]

Equating the two yields:

\[
P(B)P(A|B) = P(A)P(B|A) \tag{3}
\]

and thus

\[
P(A|B) = P(A) \frac{P(B|A)}{P(B)} \tag{4}
\]

This equation, known as Bayes Theorem is the basis of statistical inference.