Alignment of the ATLAS Inner Detector Tracking System

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on behalf of the Atlas ID Alignment group

Outline:
- Introduction
- Alignment Problem
- Possible Solutions
- Track Based Alignment
- Results
The ATLAS Detector
The ATLAS Inner Detector

Charge particle tracking system built on two technologies

Drift tubes:
- ~300,000 straw tubes
- resolution 130 μm (Rφ)
- XeCO₂O₂
- 36 hits per track

Silicon:
- ~3M Si strips
- resolution: 23 μm (Rφ) 580 μm (Z)
- 4-9 hits per track
- ~80M Si pixels
- resolution: 14 μm (Rφ) 115 μm (Z)
- 3 hits per track
Detector positions used in offline reconstruction algorithms do not correspond to the actual relative positions of the installed detector.

Why is alignment a problem?

**Tracking Goal**
Infer the position, direction, momentum of a particle given hits associated to it local measurements + assumptions about the relative location of the elements making these measurements

real track  reconstructed track
Why is alignment a problem?

Detector positions used in offline reconstruction algorithms do not correspond to the actual relative positions of the installed detector.

**Alignment Problem:**
The initial assumptions wrong

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Infer the position, direction, momentum of a particle given hits associated to it

- local measurements
- assumptions about the relative location of the elements making these measurements

**Alignment Problem:**
The initial assumptions wrong
Why is alignment a problem?

**Misalignments:**
- Will degrade quality (potentially bias) measurements made.
- Can lead to ineffective/wrong physical conclusions.

**Example:**

Spatial Misalignments

![Graph showing misaligned and aligned data with statistics](image-url)
Why is alignment a problem?

Misalignments:
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Example:

Spatial Misalignments  effect track parameters
Why is alignment a problem?

**Misalignments:**
- Will degrade quality (potentially bias) measurements made.
- Can lead to ineffective/wrong physical conclusions.

**Example:**

Spatial Misalignments → effect track parameters → compromise end physics results

![Graphs showing misaligned vs. aligned data](image-url)
The Alignment Problem in ATLAS

Large number of degrees of freedom:
   Si :  1744 align-able pixel modules, 4088 align-able SCT modules
   TRT: 176 align-able modules

Different scales of mis-alignments:
   **Relative Sub-detector**  (Si / TRT, Barrel, Endcap)
      - Largest impact on physics (Pattern Rec. / Triggering )
   **Internal Sub-detector**
      - Requires more statistics, More sensitive possible bias

**Our Alignment Objective**  - measure relative position of in-situ detectors well enough to:
   - allow for efficient track reconstruction
   - minimize degradation to track parameter resolution to < 20%
The Alignment Problem in ATLAS

Large number of degrees of freedom:

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Over 35,000 total DoF!

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Requires that the Alignment is known to the order of 10s of μms

Over 35,000 total DoF!
Solutions to the Alignment Problem

Assembly / Survey Measurements
- External measurements of as-built detector
- after/during installation

Interferometry
- laser interference monitors differences in detector positions in real time

Track Based Alignment Algorithms
- Global $\chi^2$
- Local $\chi^2$
- Robust Alignment
- External constraints
  - introduction of vertex, pT, survey, e/p constraints
to formalism of Global $\chi^2$ and Local $\chi^2$ methods
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Each of these methods have been employed in solving the ATLAS ID Alignment problem to varying degrees
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Will only concentrate on track based methods in the following
Track Based Alignment

Introspective
- Use fact that detector misalignments affect track parameters to determine the misalignments

Define a statistic sensitive to mis-alignments (ie: local measurements & our offline assumptions)

\[ \chi^2 = \sum_{\text{hits } i} \left( \frac{m_i - h_i(x)}{\sigma_i} \right)^2 \]

Key properties of \( \chi^2 \) function
- its an explicit function of the alignment parameters.
- it has a minimum at the true values of the alignment parameters
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Means we can calculate \( \frac{d\chi^2}{d\alpha} \)
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- its an explicit function of the alignment parameters.
- it has a minimum at the true values of the alignment parameters

Means we can calculate \( \frac{d\chi^2}{d\alpha} \)

Means \( \frac{d\chi^2}{d\alpha} = 0 \) gives true alignment
Track Based Alignment

Solution:

Need: 
\[ \frac{d\chi^2}{d\alpha} = 0 \]

Approximate:
\[ \frac{d\chi^2(\alpha)}{d\alpha} \approx \frac{d\chi^2(\alpha_0)}{d\alpha} + \left. \frac{d^2\chi^2}{d\alpha^2} \right|_{\alpha_0} (\alpha - \alpha_0) \]

\( \alpha_0 \) - current alignment positions

Solution:

\[ \Delta \alpha \equiv \alpha - \alpha_0 = -\left( \left. \frac{d^2\chi^2}{d\alpha^2} \right|_{\alpha_0} \right)^{-1} \frac{d\chi^2(\alpha_0)}{d\alpha} \]
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\( \alpha_0 \) - current alignment positions

N x N matrix with N ~ 35K
Inversion non-trivial
Track Based Alignment

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\[ \frac{d\chi^2}{d\alpha} = 0 \]

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In general highly non-linear

Need iterations to solve.

N x N matrix
with N \sim 35K
Inversion non-trivial

\[ y = f(x) \]

moral equivalent

\[ x_i \]
Weak Modes: Detector deformations which have little if any impact on $\chi^2$

- inherently problematic for any method based only on $\chi^2$
  - $\frac{d\chi^2}{d\alpha} = 0$ Solution is blind to multiple minima / classes of minima

- physically important:
  - keep $\chi^2$ unchanged by biasing track parameters
Weak Modes: Detector deformations which have little if any impact on \( \chi^2 \)
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Example: “Clocking Effect”

pT dependent pT biasing
Weak Modes: Detector deformations which have little if any impact on $\chi^2$
- inherently problematic for any method based only on $\chi^2$
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- physically important:
  keep $\chi^2$ unchanged by biasing track parameters

Example: “Clocking Effect”

Whole class of systematic distortions which plague detectors with cylindrical symmetry
Global Vs Local

- Described Global $\chi^2$ method.
- Local $\chi^2$ method exactly the same except:

\[
\left( \frac{d^2 \chi^2}{d\alpha^2} \right)_{\alpha_0} \rightarrow \begin{pmatrix}
\frac{d\chi^2}{d\alpha_1 d\alpha_1} & \cdots & \frac{d\chi^2}{d\alpha_1 d\alpha_j} & 0 & 0 & 0 \\
\vdots & \ddots & \vdots & 0 & 0 & 0 \\
\frac{d\chi^2}{d\alpha_i d\alpha_1} & \cdots & \frac{d\chi^2}{d\alpha_i d\alpha_j} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{d\chi^2}{d\beta_1 d\beta_1} & \cdots & \frac{d\chi^2}{d\beta_1 d\beta_j} \\
0 & 0 & 0 & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \frac{d\chi^2}{d\beta_i d\beta_1} & \cdots & \frac{d\chi^2}{d\beta_i d\beta_j} \\
\vdots & & & & \ddots & \ddots
\end{pmatrix}
\]

$\alpha_i, \beta_i$ alignment parameters for physically distinct align-able modules
### Global Vs Local

- Described Global $\chi^2$ method.
- Local $\chi^2$ method exactly the same except:

\[
\left( \frac{d^2 \chi^2}{d\alpha_i d\alpha_0} \right)
\]

**Pros:**
- Invert smaller matrices

**Cons:**
- Iterations needed to handle module correlations
- Explicit information loss
- More susceptible to weak modes

\[
\frac{d\chi^2}{d\alpha_1 d\alpha_1} \quad \cdots \quad \frac{d\chi^2}{d\alpha_1 d\alpha_j} \quad 0 \quad 0 \quad 0 \\
\vdots \quad \ddots \quad \vdots \quad 0 \quad 0 \quad 0 \quad \cdots \\
\frac{d\chi^2}{d\alpha_i d\alpha_1} \quad \cdots \quad \frac{d\chi^2}{d\alpha_i d\alpha_j} \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad \frac{d\chi^2}{d\beta_1 d\beta_1} \quad \cdots \quad \frac{d\chi^2}{d\beta_1 d\beta_j} \\
0 \quad 0 \quad 0 \quad \vdots \quad \ddots \quad \vdots \\
0 \quad 0 \quad 0 \quad \frac{d\chi^2}{d\beta_i d\beta_1} \quad \cdots \quad \frac{d\chi^2}{d\beta_i d\beta_j} \\
\vdots \quad \cdots \\
\alpha_i \quad \beta_i \quad \text{alignment parameters for physically distinct align-able modules}
\]
Dealing with \( \left( \frac{d^2 \chi^2}{d\alpha^2} \right)_{\alpha_0}^{-1} \)

- **Diagonalization:**
  - Most CPU intensive
  - Provides alignment parameter errors
  - Removal of “weak modes”

- **Full inversion:**
  - Still CPU intensive
  - Provides alignment parameter errors

- **Fast Solver Techniques**
  - Exploits unique properties of derivative matrix (sparseness, symmetry)
  - Iterative method, minimizes distance to solution
  - No errors provided
Dealing with \( \left( \frac{d^2 \chi^2}{d \alpha^2} \right)_{\rho_0}^{-1} \)

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\[
\left( \frac{d^2 \chi^2}{d \alpha^2} \right)_{\rho_0} = U D U^T
\]

\[
U \rightarrow \text{eigenvectors} \quad D \rightarrow \begin{pmatrix}
\lambda_1 \\
\vdots \\
\lambda_n
\end{pmatrix}
\]

\[
C(\alpha) = U D^{-1} U^T
\]

CLHEP, LAPACK
Dealing with \( \left( \frac{d^2 \chi^2}{d \alpha^2} \right)_{\alpha_0}^{-1} \)

- **Diagonalization:**
  Most CPU intensive
  Provides alignment parameter errors
  Removal of “weak modes”

- **Full inversion:**
  Still CPU intensive
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- **Fast Solver Techniques**
  Exploits unique properties of derivative matrix (sparseness, symmetry)
  Iterative method, minimizes distance to solution
  No errors provided

\[
\begin{align*}
\text{Done w/ } N &> 10,000K \\
\text{Minimize the distance defined as:} \\
\|d\| & = \left| \frac{d^2 \chi^2}{d \alpha^2} \Delta \alpha + \frac{d \chi^2}{d \alpha} \right| \\
\text{MA27 Fortran routine from HSL}
\end{align*}
\]
Dealing with Weak Modes

Weak Modes: *The real alignment problem*

Detecting weak modes:
- Diagonalization provides means of diagnosis

\[ C(\alpha) = U D^{-1} U^T \]
Dealing with Weak Modes

Weak Modes: *The real alignment problem*

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\]
Dealing with Weak Modes

Weak Modes: The real alignment problem

Detecting weak modes:
- Diagonalization provides means of diagnosis
  \[ C(\alpha) = UD^{-1}U^T \]

Removing weak modes:
- Explicitly remove modes below threshold
  - used to remove global movements,
  - can be dangerous / threshold arbitrary
- Enhancing definition of $\chi^2$
  - add terms to $\chi^2$ which depend on track parameters (eg: pT constraint, e/p)
- Event topology
  - $\chi^2$ landscape highly dependent on event properties.
  - different events = different weak mode (a good thing!)
  - cosmic rays/beam halo, long lived decays.
“Personally, I liked the university. They gave us money and facilities, we didn't have to produce anything. You've never been out of college. You don't know what it's like out there. I've worked in the private sector. They expect results.”

- Dr. Ray Stanz

And now, some results.
Large sample of events simulated with realistically misaligned geometry

**GOAL:**
- Exercise alignment algorithms, test technical infrastructure
- Provided alignment constants to the wider physics community

<table>
<thead>
<tr>
<th>Type of Mis-Alignment</th>
<th>Magnitude of Mis-Alignment</th>
<th>Number Tracks Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative subsystem (Barrel / Endcap)</td>
<td>O(mm) translation O(mrad) rotation</td>
<td>20K</td>
</tr>
<tr>
<td>Si Layers/Wheels</td>
<td>O(100 μm) translation O(0.1 mrad) rotation</td>
<td>500K 50K(cosmic)</td>
</tr>
<tr>
<td>TRT Modules/Wheels</td>
<td>O(100s μm) translation O(0.1 mrad) rotation</td>
<td>20K</td>
</tr>
<tr>
<td>Si Modules</td>
<td>O(&lt; 100 μm) translation O(&lt; 0.1 mrad) rotation</td>
<td>1M</td>
</tr>
</tbody>
</table>

**Big Success:**
Both in terms of validating the alignment procedures and in understanding problems likely to arise.
Cosmic Data

- ATLAS had dedicated cosmic data taking period (late summer early fall)
- Recorded over 2 million tracks reconstructed in Inner Detector
  (both with and with out magnetic field)

Event display of a reconstructed cosmic ray as seen by Atlas Inner Detector
Cosmic Data

**Extremely Important for Alignment Algorithms**

- First data available alignment.
- Remove large subsystem misalignments
  (eliminate potential problems for pattern rec and trigger before first collisions)
- Unique module correlation important for eliminating weak modes
  - Seen in simulation needed for ultimate alignment precision

Studies of B-Field on Vs off
- measure robustness of alignment
- provide insight into weak modes
Cosmic Data

Si alignment from Cosmics

- First ever alignment of Pixels + SCT with real data.
- All within 6 days of the first cosmic the data being taken:
  - multiple sets of independent alignment constants were produced
  - results validated (shown consistent)
  - constants for “first” collisions blessed
- Convergence, Hits/Tracks increase with iteration
  - Pixels aligned to SCT
  - Alignment Barrel Si layers

Results shown w/40K tracks
no B-field
Cosmic Data

TRT Alignment from Cosmics
- TRT Barrel and both Endcaps aligned wrt the Si detectors
- Saw convergence with Hits/Tracks increasing w/iteration

<table>
<thead>
<tr>
<th>TRT Barrel Alignment</th>
<th>x (μm)</th>
<th>y (μm)</th>
<th>rotx (mrad)</th>
<th>roy (mrad)</th>
<th>rotz (mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>-142(3)</td>
<td>-196(9)</td>
<td>-0.32(1)</td>
<td>0.391(4)</td>
<td>-0.963(3)</td>
</tr>
<tr>
<td>After</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

UnBiased Residual for TRT EndCap C

Before

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>25392</td>
<td>0.05123</td>
<td>1.072</td>
</tr>
</tbody>
</table>

After

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>20521</td>
<td>0.0793</td>
<td>1.934</td>
</tr>
</tbody>
</table>

UnBiased Residual for the TRT Barrel

Before

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>615342</td>
<td>-0.003516</td>
<td>0.9203</td>
</tr>
<tr>
<td>Constant</td>
<td>7015 ± 14.8</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.009708 ± 0.000743</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>0.3527 ± 0.0010</td>
<td></td>
</tr>
</tbody>
</table>

After

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>644702</td>
<td>-0.002167</td>
<td>0.5645</td>
</tr>
<tr>
<td>Constant</td>
<td>9224 ± 18.1</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.51e-05 ± 4.56e-04</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>0.2706 ± 0.0005</td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- Overall scale and ultimate precision of ATLAS Inner Detector poses a challenging problem in terms of understanding the detector.
- Alignment crucial to reach full physics potential / detector performance
- Track based method provides means of addressing alignment problem
- Implemented in ATLAS framework
- Thorough and Stringent tests of software allowed for capitalization on early cosmic data.
- Cosmics have and continue to provide a starting point and guide the way to the ultimate alignment
- Poised meeting the alignment challenge at first collisions
Bonus.
## Alignment Levels

### Silicon Alignment Levels

<table>
<thead>
<tr>
<th>Geometry Level</th>
<th>Structures (DoFs)</th>
<th>Pixel</th>
<th>Pixel Structures (DoFs)</th>
<th>SCT</th>
<th>SCT Structures (DoFs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 (24)</td>
<td>complete pixel detector</td>
<td>1 (6)</td>
<td>1 barrel + 2 endcaps</td>
<td>3 (18)</td>
</tr>
<tr>
<td>1.5</td>
<td>7 (42)</td>
<td>2 barrel half-shells + 2 endcaps</td>
<td>4 (24)</td>
<td>1 barrel + 2 endcaps</td>
<td>3 (18)</td>
</tr>
<tr>
<td>1.6</td>
<td>11 (66)</td>
<td>3*2 barrel half-shells + 2 endcaps</td>
<td>8 (48)</td>
<td>1 barrel + 2 endcaps</td>
<td>3 (18)</td>
</tr>
<tr>
<td>2</td>
<td>31 (186)</td>
<td>3 barrel layers + 2*3 endcap discs</td>
<td>9 (54)</td>
<td>4 barrel layers + 2*9 discs</td>
<td>22 (132)</td>
</tr>
<tr>
<td>2.1</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
</tr>
<tr>
<td>2.3</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
</tr>
<tr>
<td>2.5</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
<td>-</td>
<td>- (-)</td>
</tr>
<tr>
<td>3</td>
<td>5832 (34992)</td>
<td>1456 barrel + 2*144 endcap</td>
<td>1744 (10464)</td>
<td>2112 barrel + 2*988 endcap</td>
<td>4088 (24528)</td>
</tr>
</tbody>
</table>

### TRT Alignment Levels

<table>
<thead>
<tr>
<th>Geometry Level</th>
<th>TRT</th>
<th>TRT DoFs</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 barrel + 2 endcaps</td>
<td>17</td>
<td>no alignment correction around the global Z-coordinate in the barrel</td>
</tr>
<tr>
<td>2</td>
<td>32<em>3 barrel modules+ 40</em>2 endcap wheels</td>
<td>(32x3) x 5 Dof + (40x2) x 6 Dof = 960</td>
<td></td>
</tr>
</tbody>
</table>
Alineator

Computing Cluster at Valencia, running SCALAPACK

<table>
<thead>
<tr>
<th>Description</th>
<th>Vtx constr</th>
<th>Align Level</th>
<th>Modules</th>
<th>Dofs</th>
<th>Matrix size</th>
<th>System</th>
<th>Solving time</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single PC (CPU Pentium IV Dual Core 3 GHz, Memory 3 GB)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double Cone</td>
<td>yes</td>
<td>2</td>
<td>7</td>
<td>42</td>
<td>-</td>
<td>single PC</td>
<td>0.01 min</td>
<td>CSC Multimuons</td>
</tr>
<tr>
<td>SR1</td>
<td>no</td>
<td>3</td>
<td>468</td>
<td>2808</td>
<td>31 MB</td>
<td>single PC</td>
<td>5 min</td>
<td>CSC Multimuons</td>
</tr>
<tr>
<td>Cone</td>
<td>no</td>
<td>3</td>
<td>1030</td>
<td>6180</td>
<td>146 MB</td>
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Unofficial successful tests w/ ~32k DoF
More Weak Modes

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<td><strong>Φ</strong></td>
<td><strong>Z</strong></td>
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